Field line helicity as a tool for coronal physics

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with

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What is magnetic helicity?

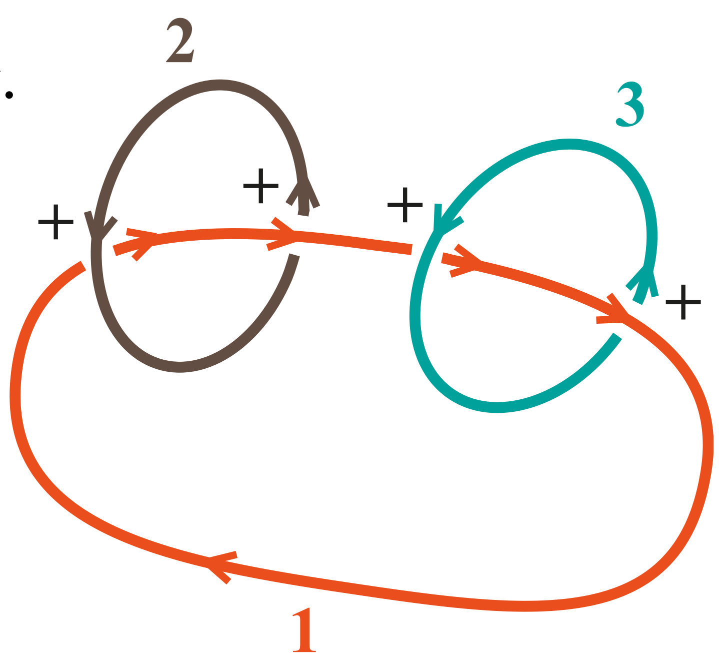
- The average pairwise linking of magnetic flux (e.g. **Moffatt & Ricca, *PRSL* 1992**).

Discrete flux tubes

$$H = \sum_i \sum_j \Phi_i \Phi_j \ell_{ij}$$

$$\ell_{ij} = \frac{N_+ - N_-}{2}$$

e.g.



$$H = \ell_{12} + \ell_{21} + \ell_{13} + \ell_{31}$$
$$= 4$$

Continuous field

$$\ell_{ij} = \frac{1}{4\pi} \oint_{\mathbf{x}(s)} \oint_{\mathbf{y}(s)} \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}s} \cdot \frac{\mathrm{d}\mathbf{y}}{\mathrm{d}s} \times \frac{\mathbf{x}-\mathbf{y}}{|\mathbf{x}-\mathbf{y}|^3} \,\mathrm{d}s\,\mathrm{d}s'$$

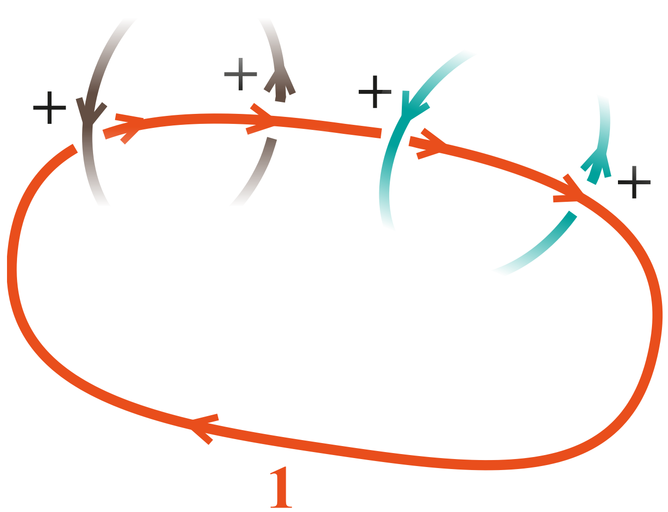
$$\implies \quad H = \frac{1}{4\pi} \int_V \int_V \mathbf{B}(\mathbf{x}) \cdot \frac{\mathbf{B}(\mathbf{y}) \times (\mathbf{x}-\mathbf{y})}{|\mathbf{x}-\mathbf{y}|^3} \,\mathrm{d}^3x\,\mathrm{d}^3y = \int_V \mathbf{A}\cdot\mathbf{B}\,\mathrm{d}^3x$$

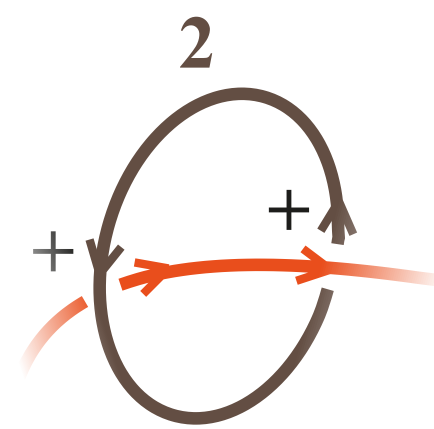
What is field line helicity?

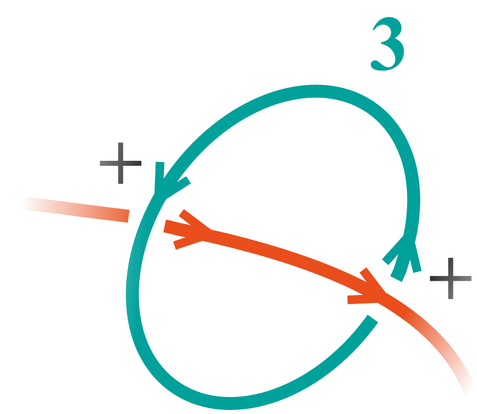
- **Field line helicity** is just the flux linked with a *single* field line.

Discrete flux tubes

$$H = \sum_i \Phi_i \mathcal{A}_i \quad \text{where} \quad \mathcal{A}_i = \sum_j \Phi_j \ell_{ij}$$







$\mathcal{A}_1 = \ell_{12} + \ell_{13} = 2$ $\qquad$ $\mathcal{A}_2 = \ell_{21} + \ell_{23} = 1$ $\qquad$ $\mathcal{A}_3 = \ell_{31} + \ell_{32} = 1$

Continuous field

$$H = \int_L \mathcal{A}(L)\,\mathrm{d}\Phi \quad \text{where} \quad \mathcal{A}(L) = \begin{bmatrix} \text{flux} \\ \text{through L} \end{bmatrix} = \oint_L \mathbf{A}\cdot\mathrm{d}\mathbf{l}$$

Why compute field line helicity?

Field line helicity $\mathcal{A}(L) = \int_L \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{l}$

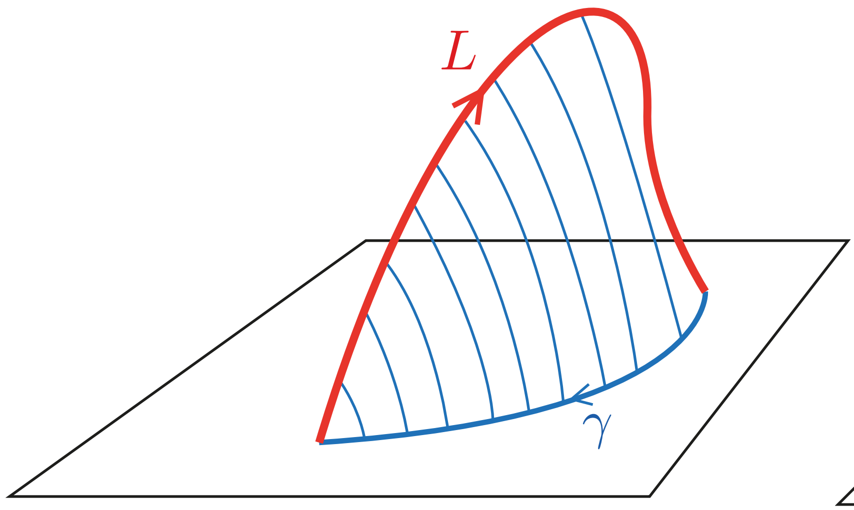
1. **Invariant** under an ideal evolution (by Alfvén's frozen-flux theorem).
2. Captures **topological** information (linking of flux).
3. Tells you **where helicity is located** within a domain.
4. "Finest possible" topological quantity.

History

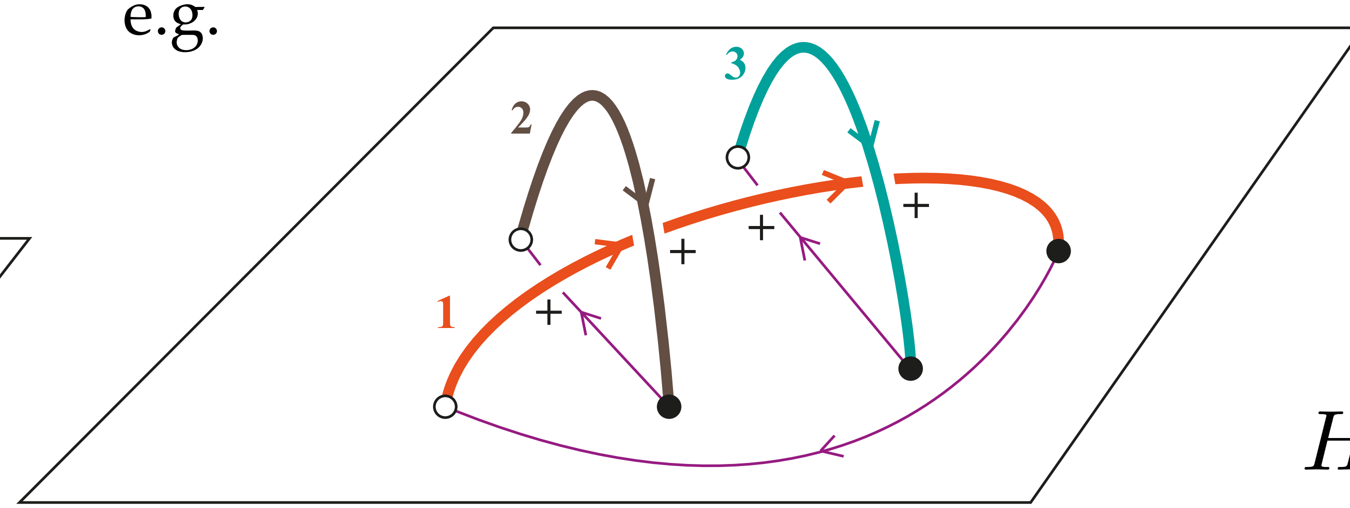
- **Taylor (*PRL*, 1974)** - "When the field lines are closed there is one invariant for each line (the volume V then being an infinitesimal flux tube)… The state [of minimum magnetic energy] is some force-free configuration $\nabla \times \mathbf{B} = \lambda(L)\mathbf{B}$."
- **Berger (*A&A*, 1988)** - application to "coronal" fields:
  - "field line helicity measures mean angle, weighted by flux, through which other field lines twist about L".
    [definition of "angle" depends on the gauge - cf. **Prior & Yeates (*ApJ*, 2014)**]
  - formula for energy of a force-free field - in a particular gauge:
    $$\int_V B^2 \,\mathrm{d}x = \frac{1}{2}\int_S \alpha(\mathbf{a})\mathcal{A}(\mathbf{a})|\mathbf{B}(\mathbf{a})\cdot\mathbf{n}|\,\mathrm{d}^2a + \int_S (\mathbf{A}^\mathrm{p}\times\mathbf{B})\cdot\mathbf{n}\,\mathrm{d}^2a$$
- **Aly (*J. Phys. Conf.*, 2014); Yeates *et al.* (*A&A*, 2014)** - use this to bound energy injection into the corona.
- **Yeates & Hornig (*PoP 2013, J. Phys. Conf.*, 2014)** - prove "completeness" for fields in a cylindrical domain - in another particular gauge.
- **Russell, Yeates, Hornig & Wilmot-Smith (*PoP* 2015)** - evolution of FLH under reconnection: efficiently redistributed but not destroyed on dynamical timescales.

Open boundaries

- Gauge dependence corresponds to choice of closing curve.



e.g.



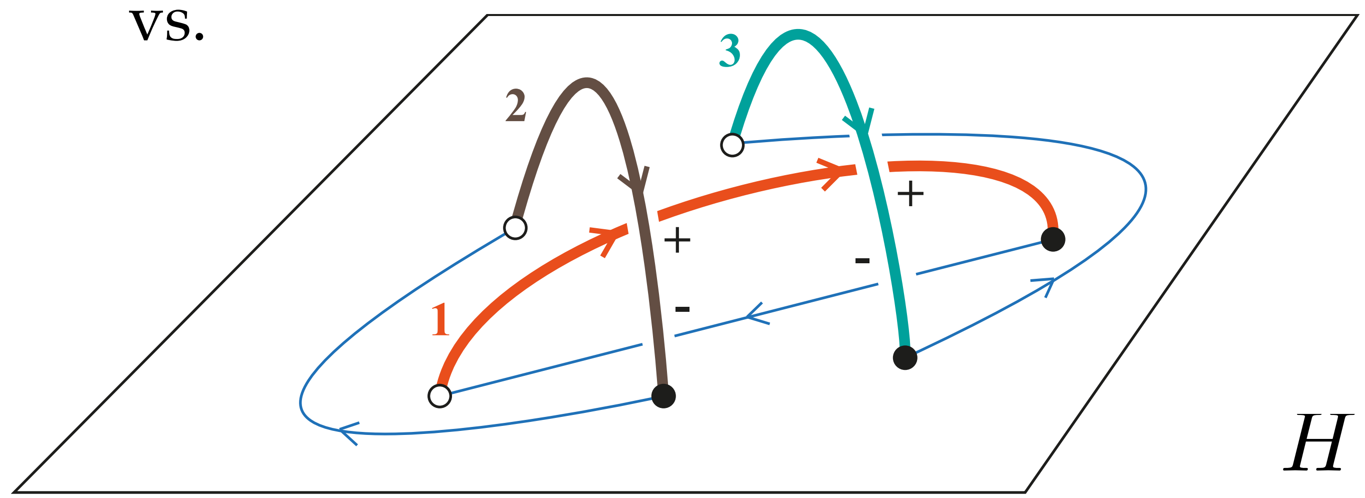
$\mathcal{A}_1 = 2$

$\mathcal{A}_2 = 1$

$\mathcal{A}_3 = 1$

$H = 4$

vs.



$\mathcal{A}_1 = 0$

$\mathcal{A}_2 = 0$

$\mathcal{A}_3 = 0$

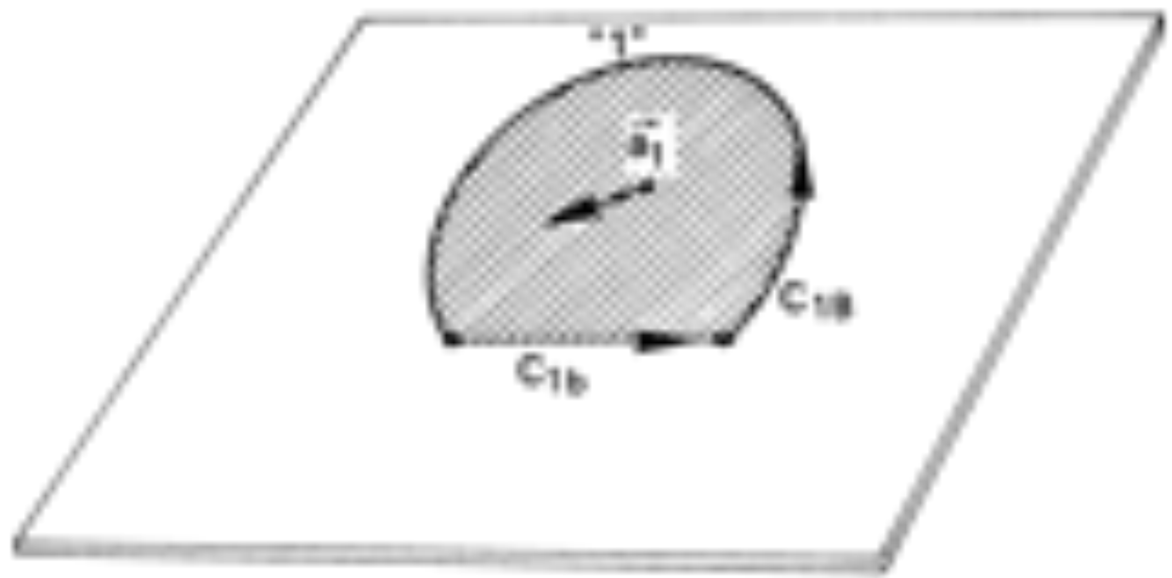
$H = 0$

- Best choice depends on the application.

Two options

1. Choose the curves ɣ and find a suitable A.

e.g. ɣ is a straight line (**Antiochos, *ApJ* 1987** - "flux per field line").



e.g. ɣ is a projection of the field line.

2. Choose a convenient A (and define ɣ implicitly).

e.g. following slides use **DeVore-Coulomb gauge**…

$$\mathbf{A}(x,y,z) = \mathbf{A}_0(x,y) + \int_{z_0}^{z} \mathbf{B}_\perp(x,y,z') \times \hat{\mathbf{e}}_z \, dz' \qquad \nabla \cdot \mathbf{A}_0 = 0$$

Example 1 - NLFFF model of active region

NLFFF extrapolation for AR 10930 (12/13 Dec 2006)

- Extrapolation by Mike Wheatland from *Hinode/SOT* magnetogram. (best performing method in **Schrijver et al., *ApJ*, 2008**)

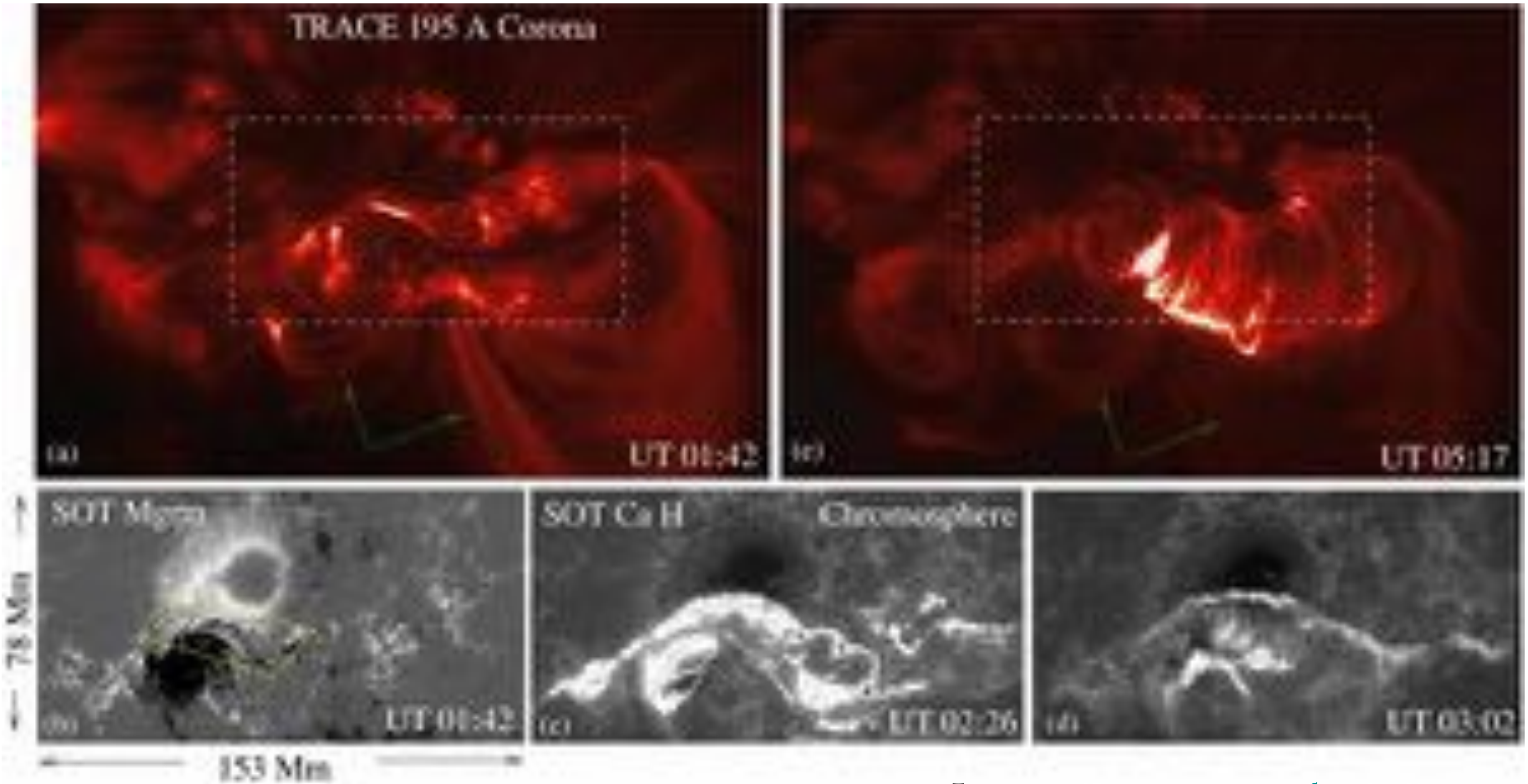
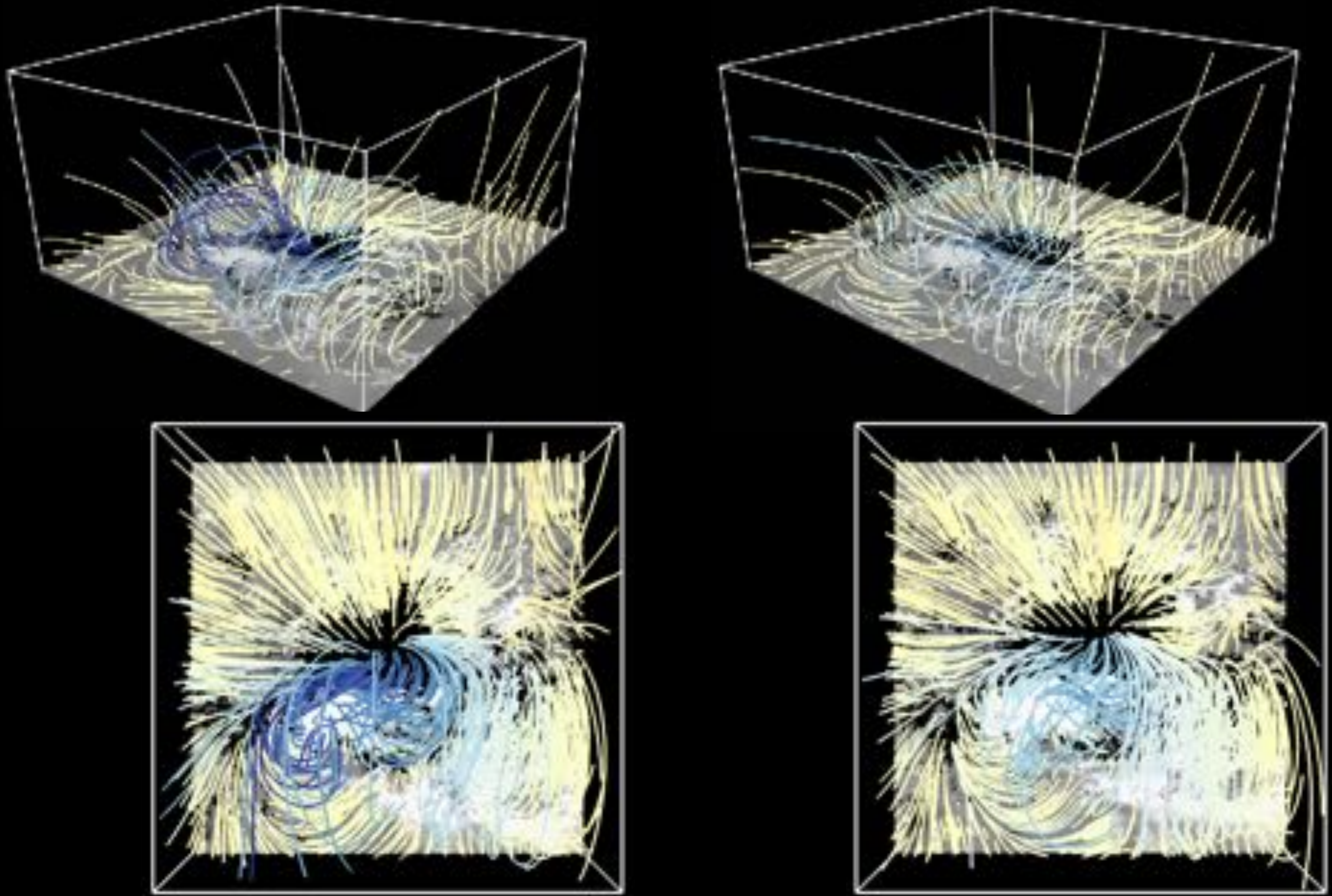
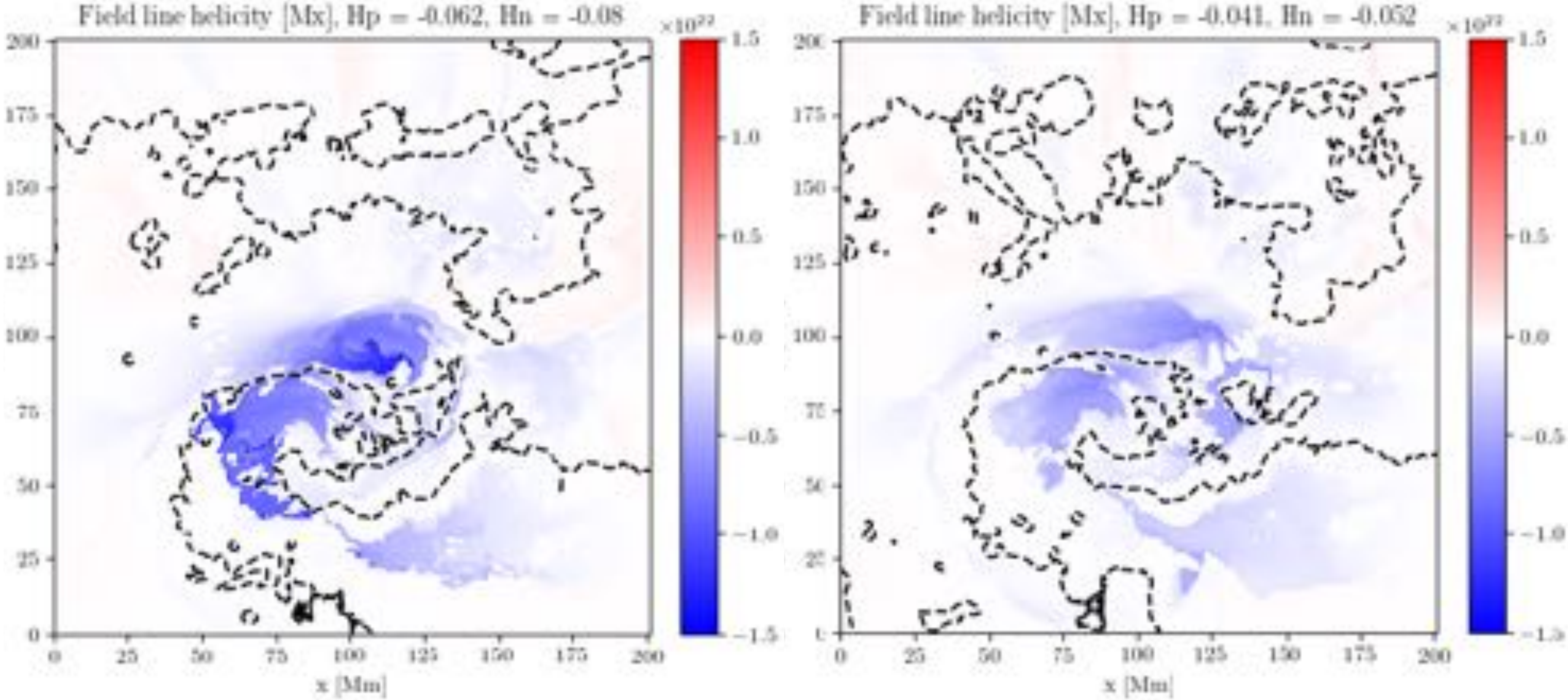


Image: **Ryutova et al., *ApJ*, 2011**

Before and after the flare…



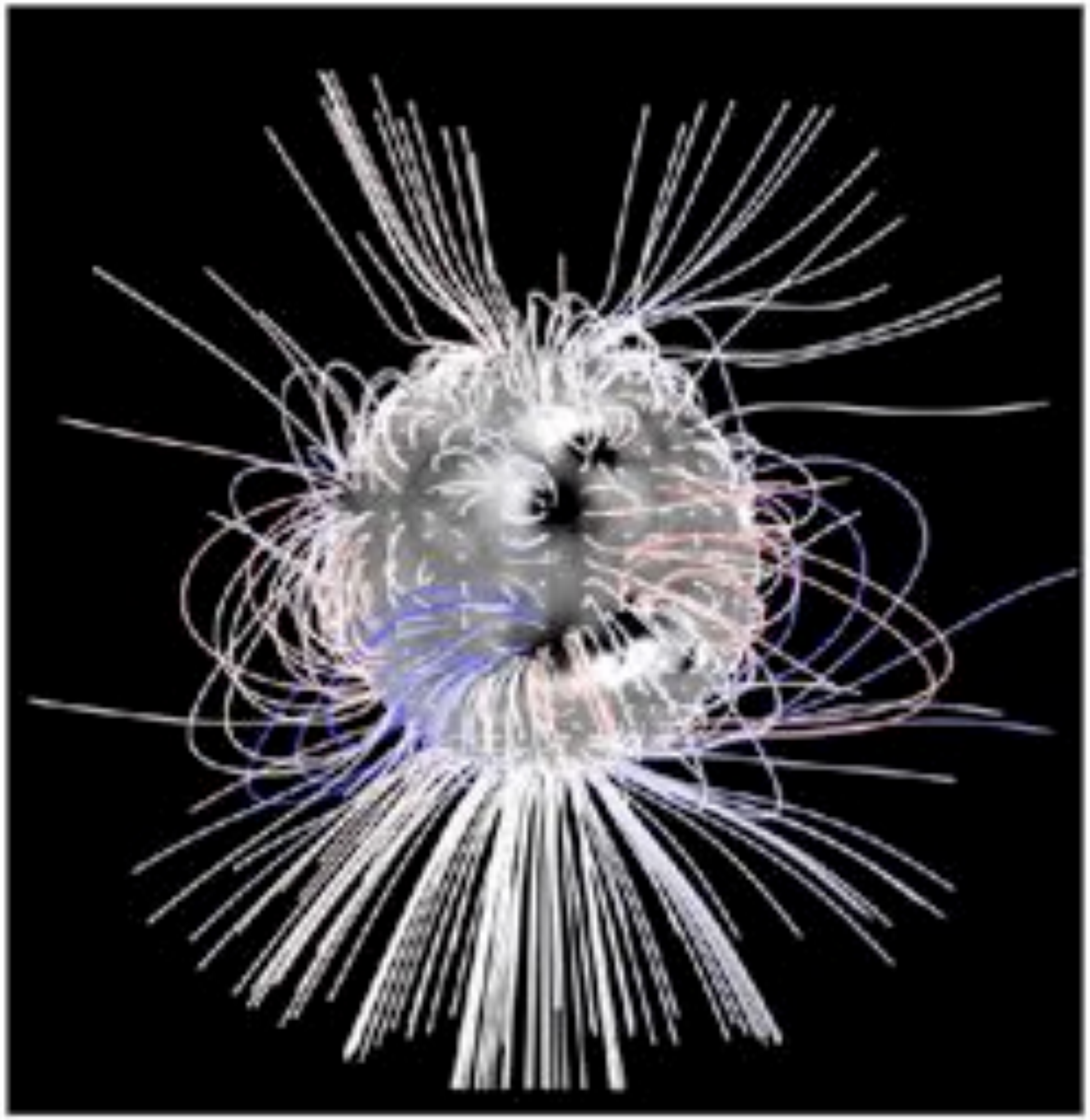
Before and after the flare…



- Significant decrease in helicity during the flare (**about 33%**).

Example 2 - Global simulations

Magneto-frictional model



Yeates & Hornig (*A&A*, 2016)

- Time-dependent coronal simulation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \Big(\mathbf{v} \times \mathbf{B}\Big) + \eta \nabla^2 \mathbf{B}$$

$$\mathbf{v} = \nu \frac{\mathbf{j} \times \mathbf{B}}{B^2}$$

- Driven by surface flux transport model.

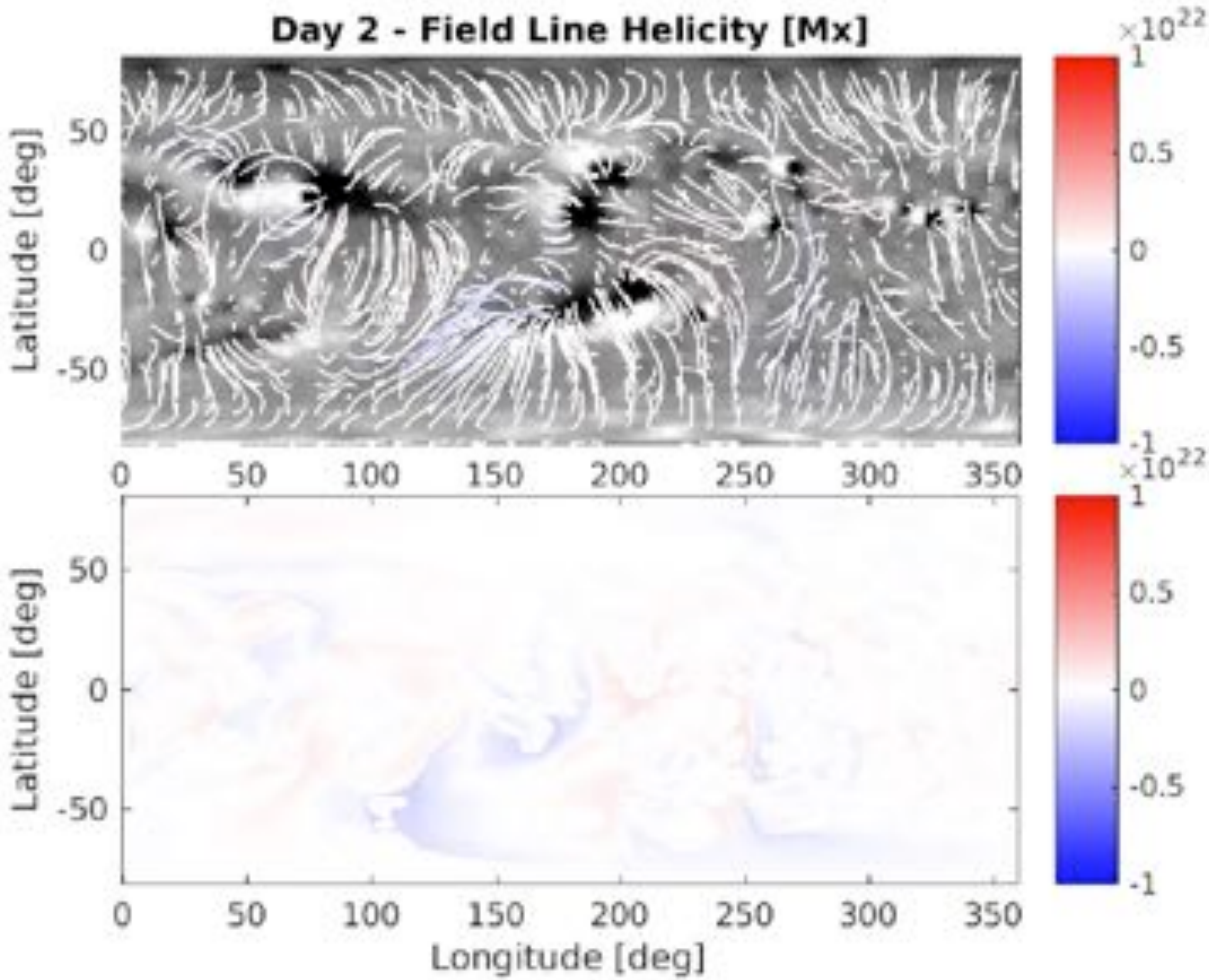
- Injects helicity by footpoint shearing.
- Concentrates into twisted flux ropes.
- Ejection of twisted ropes.

- Used spherical DeVore-Coulomb gauge:

$$r\mathbf{A}(r,\theta,\phi) = r_0\mathbf{A}_0(\theta,\phi) + \int_{r_0}^{r} \mathbf{B}(r',\theta,\phi) \times \mathbf{e}_r \, r' \, \mathrm{d}r' \qquad \nabla \cdot \mathbf{A}_0 = 0$$

Example with no emergence

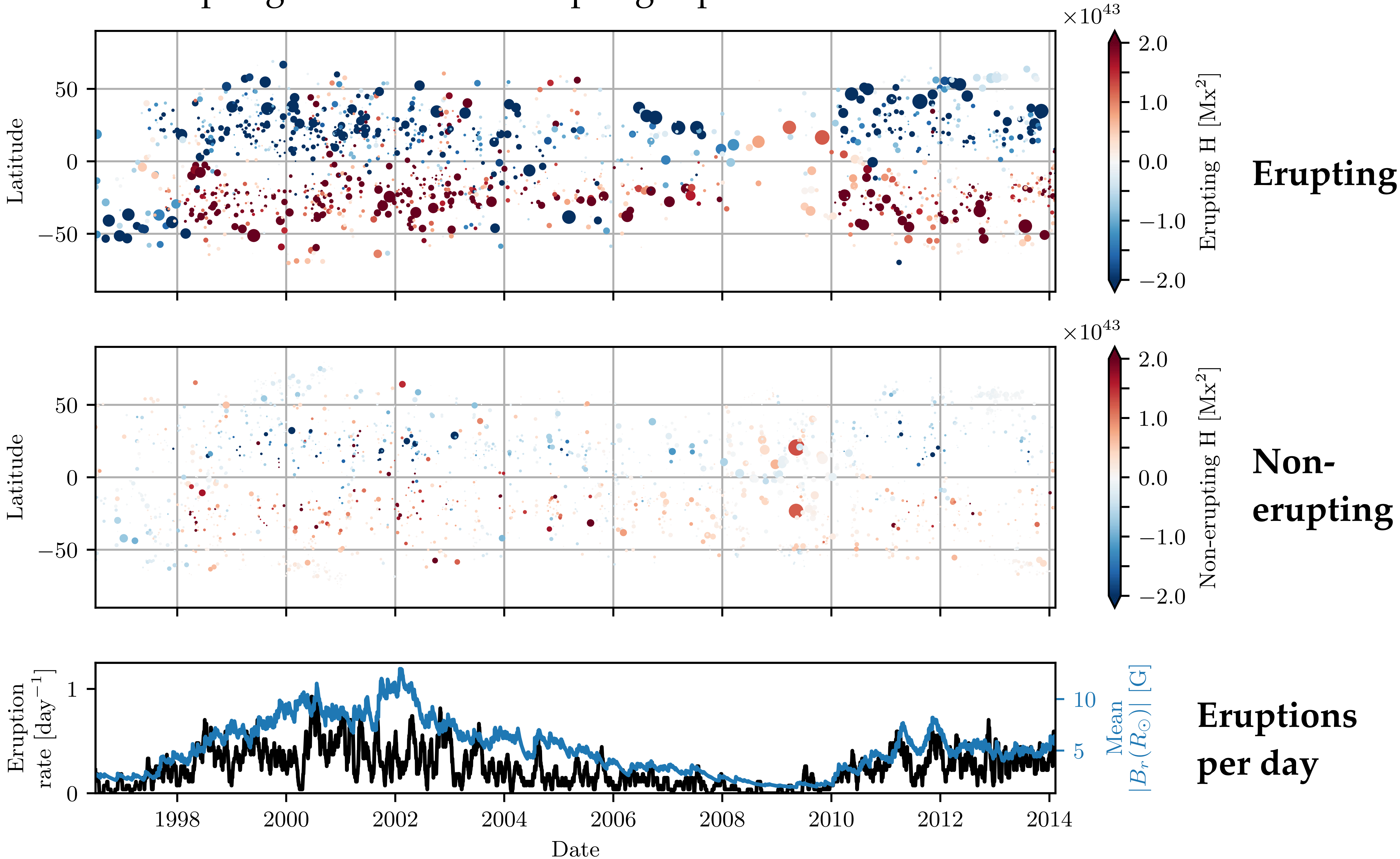
Yeates & Hornig (*A&A*, 2016)



Solar cycle (with emergence)

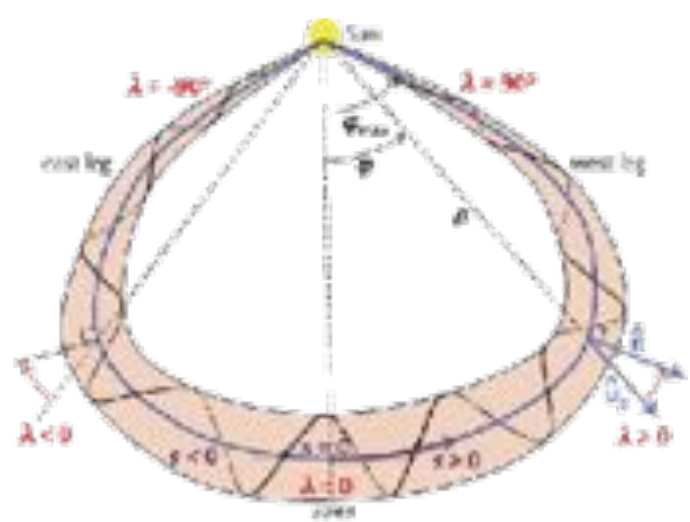
- 1561 erupting and 2099 non-erupting ropes.

Lowder & Yeates (*ApJ*, 2017)

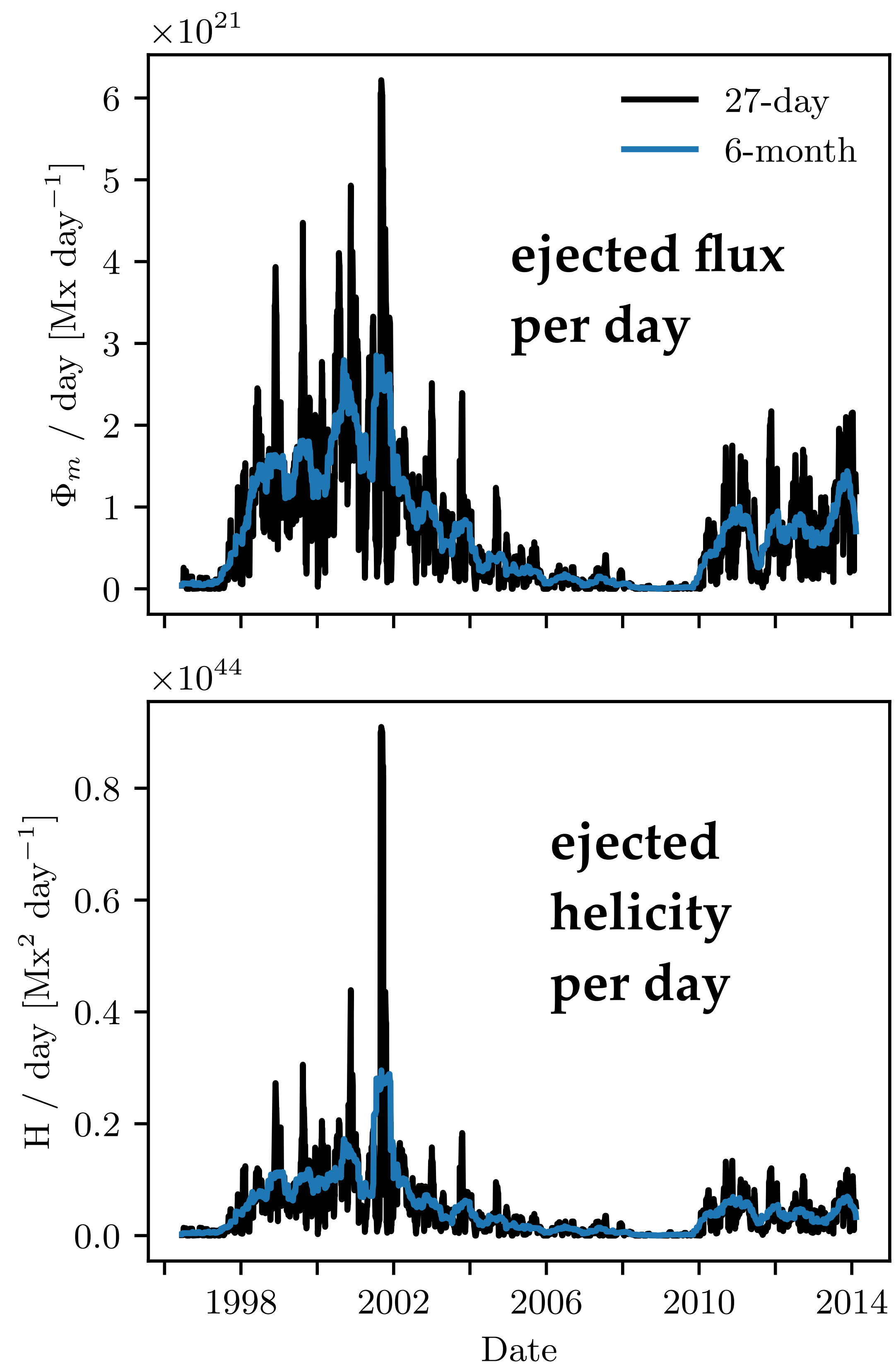


Estimated ejection rates

- At peak strength of *pre-erupting* rope.
- Totals ejected over Solar Cycle 23:
  - 3.5×10^{24} Mx magnetic flux
  - 2.4×10^{46} Mx^2 magnetic helicity
- Similar to magnetic cloud estimates by **Démoulin *et al.* (*Solar Phys.* 2016)** ~ 3×10^{24} Mx, ~ 2.5×10^{46} Mx^2.



- But our ejection rate of **0.24 per day** is lower than LASCO CME rate.



Conclusion

Maybe field line helicity can be useful in your application!

References

- Yeates & Hornig, *Phys. Plasmas* 20, 012102 (2013) - completeness theorem.
- Yeates & Hornig, *J. Phys. Conf. Ser.* 544, 012002 (2014) - more mathematical info.
- Prior & Yeates, *Astrophys. J.* 787, 100 (2014) - physical interpretation of gauges.
- Yeates, Bianchi, Welsch & Bushby, *Astron. Astrophys.* 564, A131 (2014) - lower bounds on Poynting flux.
- Russell, Yeates, Hornig & Wilmot-Smith, *Phys. Plasmas* 22, 032106 (2015) - evolution under reconnection.
- Yeates & Hornig, *Astron. Astrophys.* 594, A98 (2016) - application to global simulations.
- Lowder & Yeates, *Astrophys. J.* 846, 106 (2017) - application to flux rope identification.

http://www.maths.dur.ac.uk/~bmjg46/

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