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Large scale flows and stellar magnetic cycles

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Retreat of the Stellar Astrophysics Centre Skagen, October 20, 2015

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The Helio- and Asteroseismology Group at KIS

- **Helioseismology**
  - Forward modelling for local and global helioseismology
    - Large-scale flows, mangnetic fields
  - Inversions for large-scale flows
  - Numerical Simulations
    - Wave propagation in complex solar atmosphere
  - Instrument Development
    - HELLRIDE
    - SPRING
- **Asteroseismology**
  - Forward Modelling
    - Large-scale flows
  - Data Analysis
    - **Frequency shifts, magnetic activity cycles**

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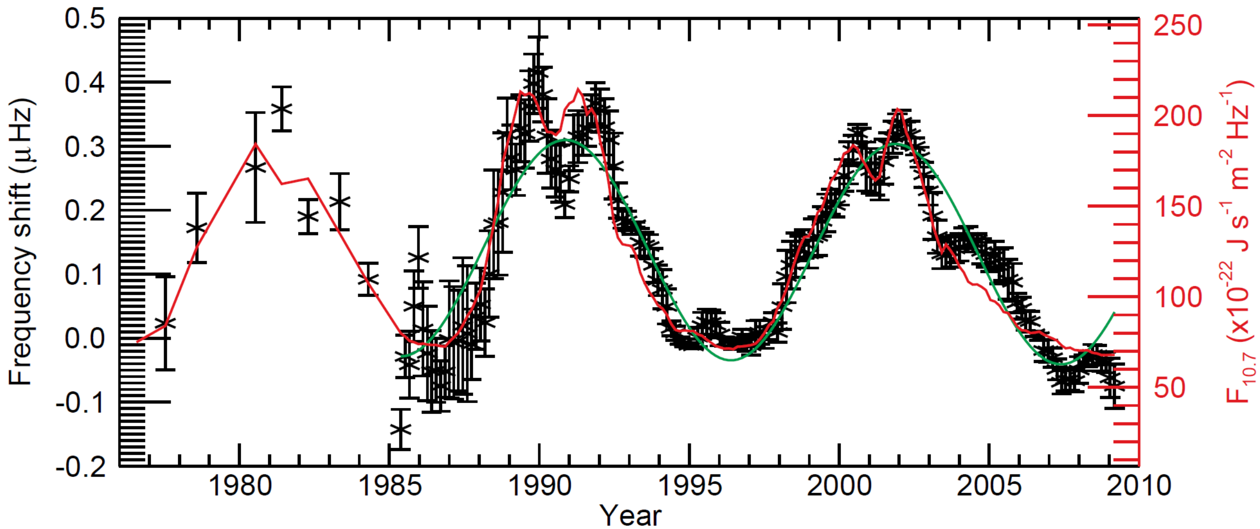
The Solar Acoustic Cycle



(Broomhall et al., MNRAS 2009)

- Low-degree oscillation frequencies are 0.4μHz higher at solar maximum compared to solar minimum
  - **What about other stars?**
- *First detection:*
  Woodard & Noyes 1985, with SolarMaximum data

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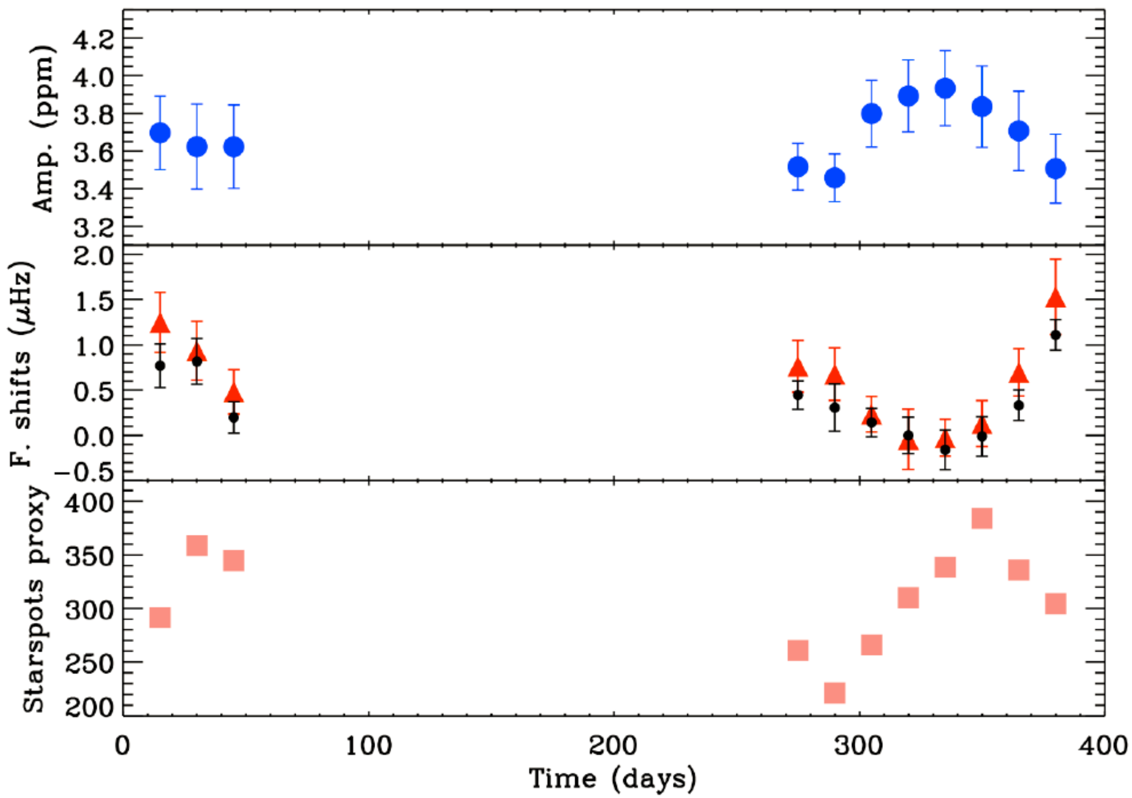
The Acoustic Stellar Cycle of HD49933



García et al. 2012

Period of cycle: 120 d

Since then not many more

Besides:
Vida et al. 2014 reports activity cycles in 9 stars

Fig. 1. Time evolution –beginning February 6, 2007– of the mode amplitude **(top)**, the frequency shifts using two different methods **(central)**: cross correlations (red triangles) and individual frequency shifts (black circles); and a starspot proxy **(bottom)** built by computing the standard deviation of the light curve (7). All of them are computed using 30-day long subseries shifted every 15 days (50% overlapping). The corresponding 1σ error bars are shown.

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Our Analyis of Kepler Data

We analysed 25 Kepler stars

- time series is divided into shorter sub-series
- compromise between frequency resolution and number of independent sub-series: length of 150 days for the subseries.
- From one sub-series to the next, the starting point was shifted by 50 days

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The Acoustic Stellar Cycle of HD49933

Example clearly visible frequency shift in KIC 8006161

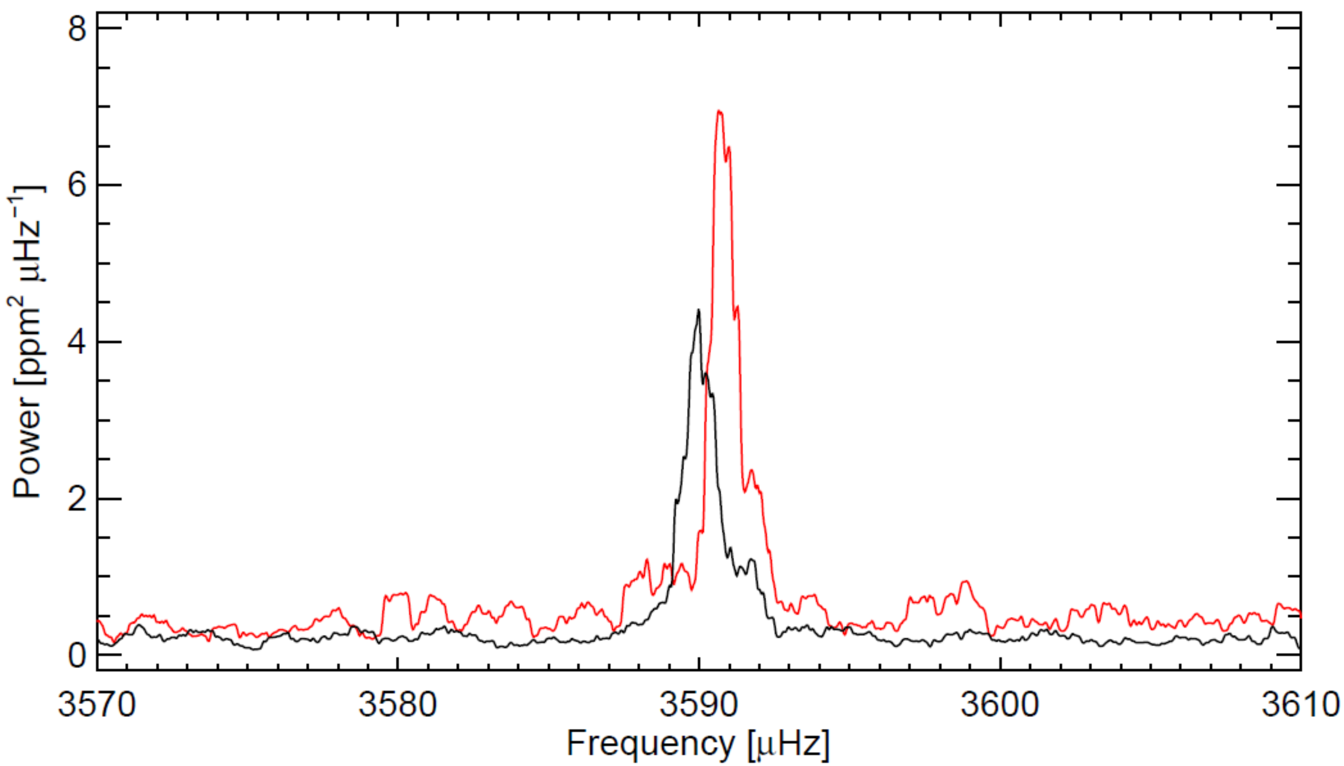

Fig. 1. Section of the periodograms of the first and nineteenth sub-series of KIC 8006161 in black and red colour, respectively. The periodograms are boxcar smoothed over 0.87 μHz (30 bins).

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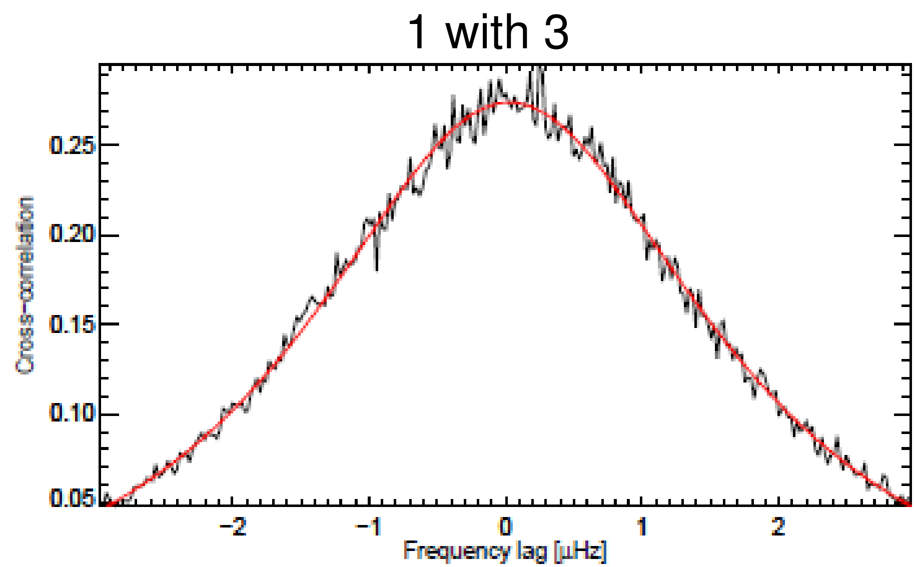
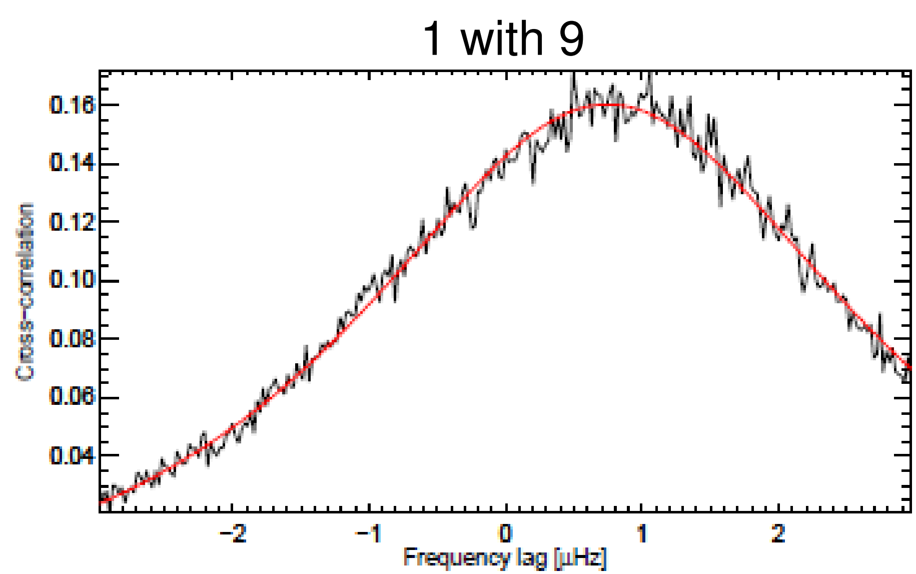
Oversampling and Cross-Correlations

- Time series have gaps ?Lomb-Scargle with oversampling by factor 4

- Cross-correlation of periodograms of sub-series with reference periodogram to determine frequency lag





Determination of Error

- Statistical properties: χ^2 distributed

$$\hat{P}(\nu) \approx \frac{1}{2}\hat{S}(\nu)\,\chi_2^2$$

- Simulate 200 realisations:
  - Estimate $S(\nu)$ by smoothing original periodogram
  - Draw χ^2 distributed random numbers and generate new periodogram
- Cross-correlating determines error-bar

Sanity Check with BiSON data

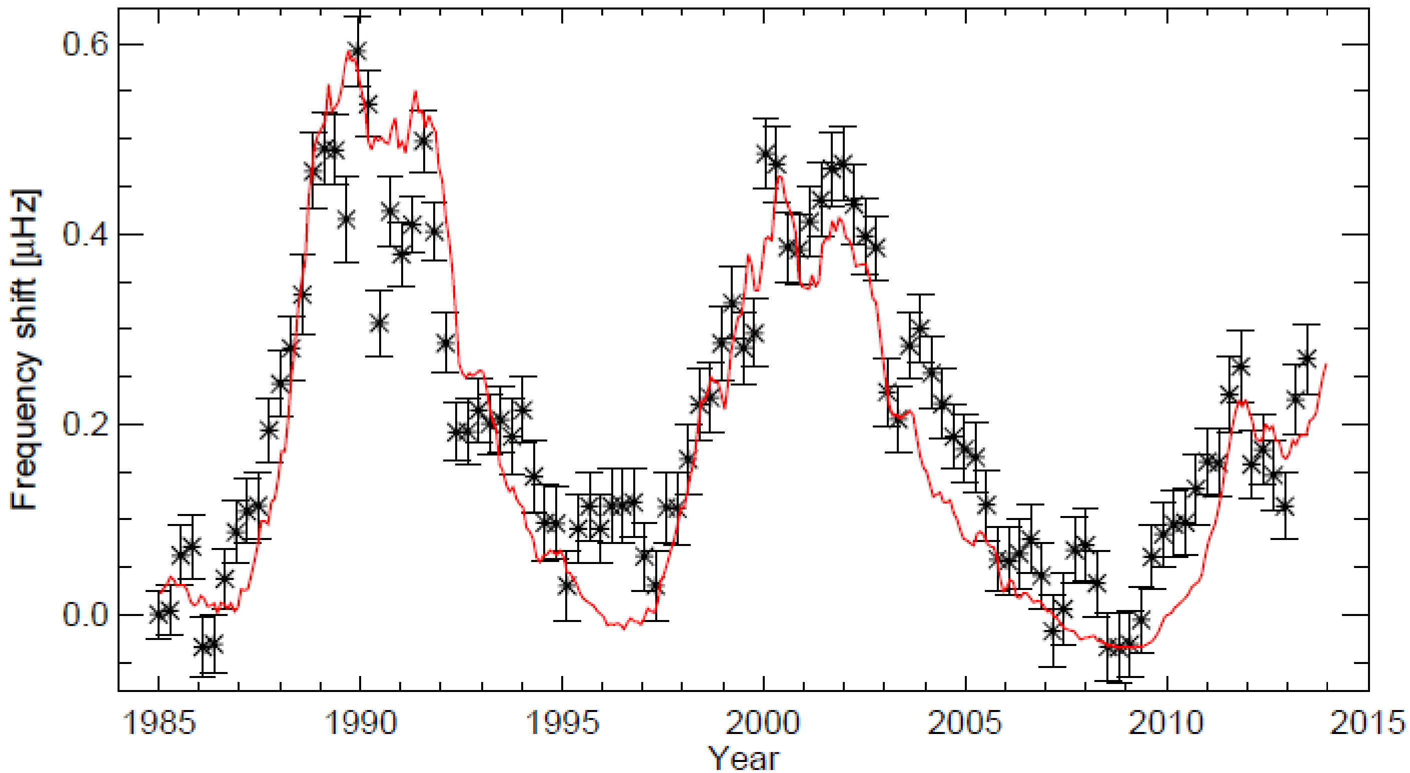
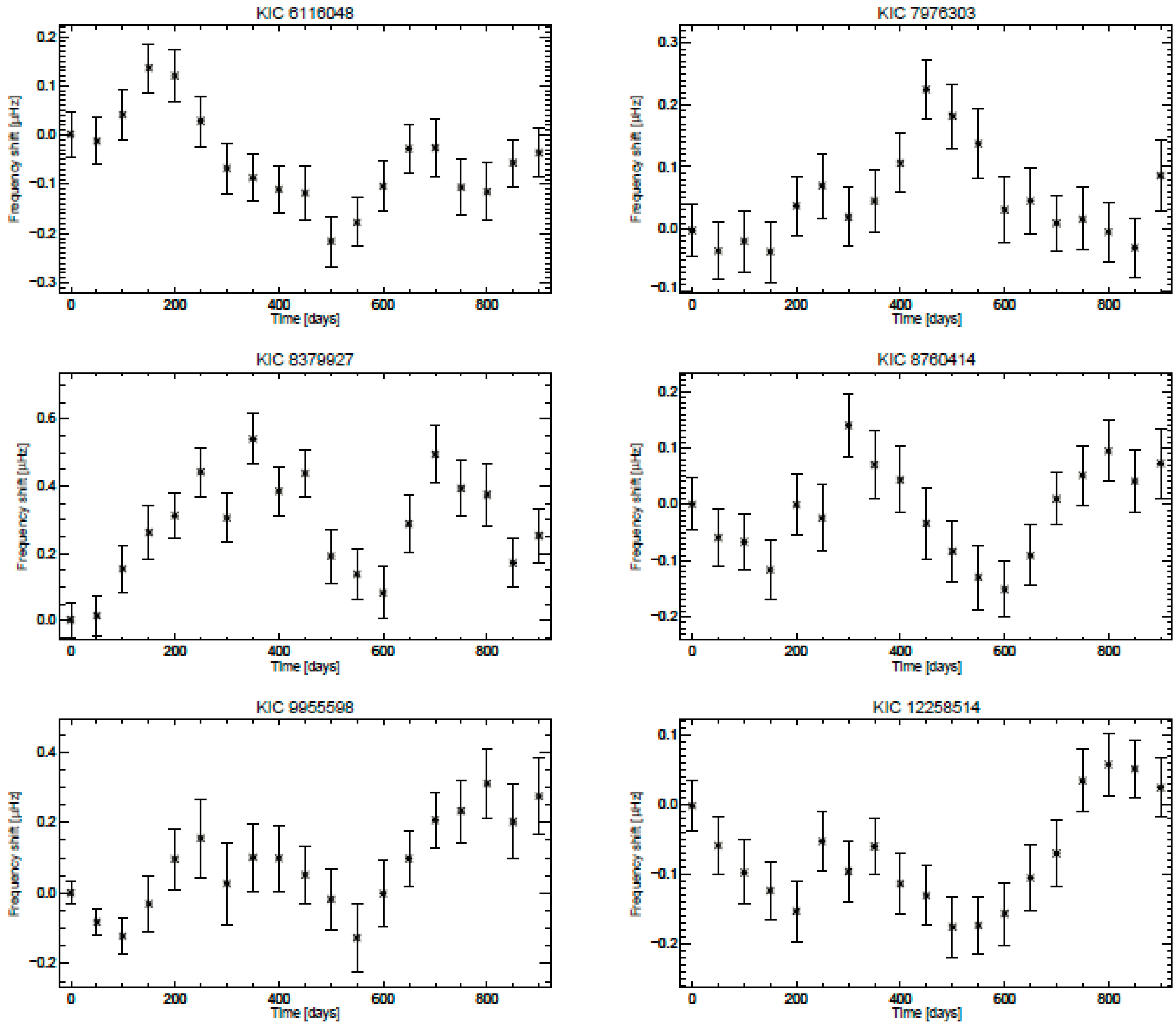


Fig. 3. Frequency shifts of solar p modes from BiSON data (black). The seven month boxcar smoothed monthly sunspot number is shown in red. Source of the sunspot number: WDC-SILSO, Royal Observatory of Belgium, Brussels

Application to Kepler Time Series

25 stars investigated: Many stars seem to have a cycle



Application to Kepler Time Series

KIC	A (10^{-7}Hz)	σ_A (10^{-7}Hz)	A/σ_A
3632418	3.79	0.92	4.11
3656476	0.98	0.54	1.81
4914923	2.13	0.66	3.22
5184732	3.22	0.86	3.74
6106415	1.87	0.56	3.34
6116048	3.52	0.72	4.89
6603624	1.84	0.83	2.22
6933899	2.04	0.63	3.24
7680114	1.88	0.65	2.89
7976303	2.16	0.57	3.79
8006161	7.59	0.87	8.73

KIC	A (10^{-7}Hz)	σ_A (10^{-7}Hz)	A/σ_A
8228742	5.67	1.23	4.61
8379927	4.45	0.78	5.71
8760414	2.42	0.66	3.66
9025370	6.66	2.00	3.33
9955598	4.37	1.38	3.16
10018963	4.15	1.19	3.48
10516096	3.39	0.98	3.46
10644253	8.07	2.01	4.02
10963065	2.45	1.46	1.68
11244118	3.06	0.82	3.73
11295426	2.65	1.10	2.41
11395018	3.96	3.27	1.21
12009504	5.12	1.86	2.75
12258514	2.33	0.62	3.76

- All frequency shifts seem to be significant

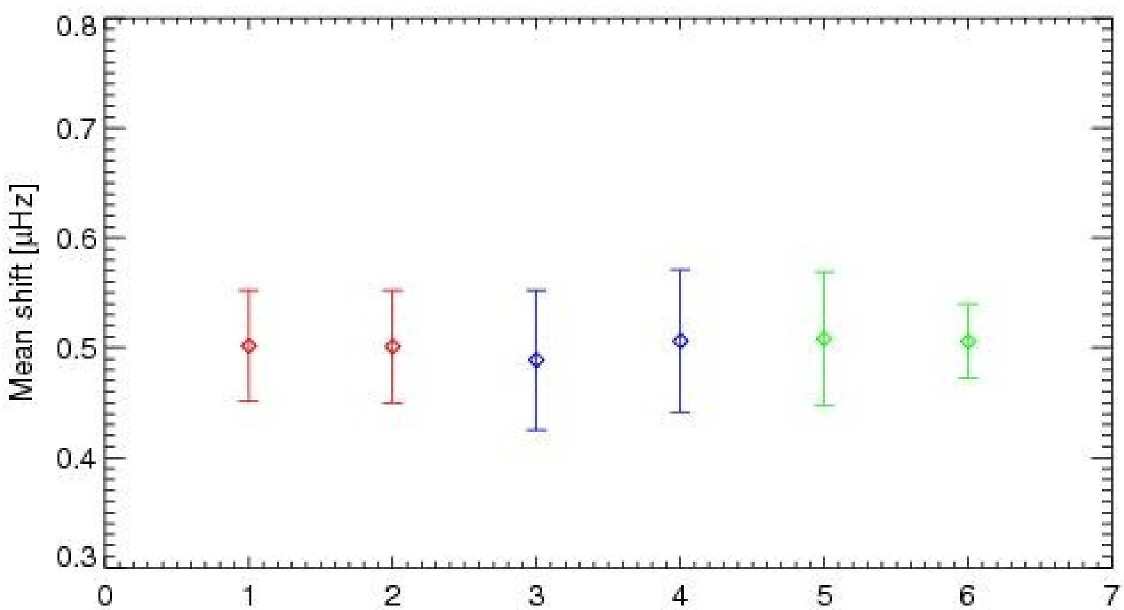
But: Correlation in the Periodogram

Problem: Oversampling

- Estimation of Lomb-Scargle periodogram with higher frequency resolution as length of time series would allow
- This creates correlations in the frequency shifts, which need to be considered
- Not taking this into account results in an underestimation of the error on the frequency shift.

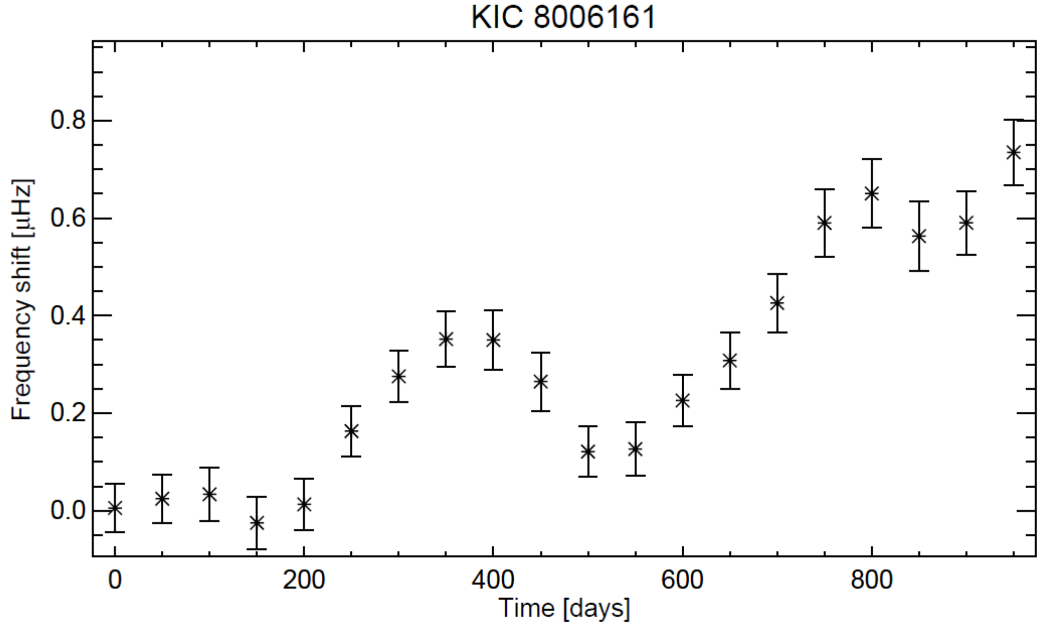
Testing Sensitivity to Oversampling

- Simulating time series of damped harmonic oscillators with frequency shift of 0.5 µHz
- Several methods (with and without 4 times oversampling):
  - Drawing new time series (error estimate is good)
  - Drawing new Fourier transform (error estimate is good)
  - Drawing new periodograms **(underestimated error!)**



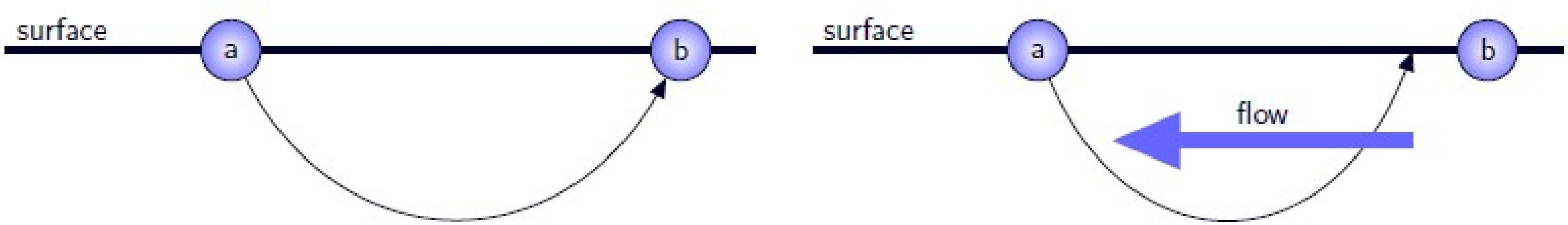
Conclusions

- Oversampling in the Lomb-Scargle periodogram leads to an underestimation of the error on the frequency shift
- Not all observed cycles are significant!
- We find one significant cycle in KIC 8006161



Introduction

- Presence of large scale flows leads to coupling of oscillation modes through advection, which shifts their frequencies
- First calculations for the Sun by Roth & Stix (1999) based on theoretical work by Lavely & Ritzwoller (1992)
- For the Sun, shifts are very small ~ nHz (Chatterjee & Antia, 2009)
- Subgiants likely to have bigger convection cells, higher flow velocities
  ? measurable shifts?
- Rotation inversions for subgiant stars (Deheuvels et al., 2014)
  ? (How) do flows affect the rotational splitting?
- Estimate frequency shifts of dipole modes to first order with perturbation theory



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How to calculate Frequency Shifts?

- **Perturbation Theory**
  - Quasi-degenerate perturbation theory applied to stars and their oscillations (Lavely & Ritzwoller, 1992):

$$H = H_0 + V$$

  - Solutions to H_0 are the (unperturbed) oscillation modes of the star
  - Perturbation V = advection caused by the flow
  - Treat a range of close frequencies as fully degenerate ($\rightarrow$ limits precision)

$$\Delta\omega = \omega_i - \omega_{\rm ref} \leq 120\mu{\rm Hz}$$

- **Additional ingredients:**
  - Stellar model + oscillation modes
  - Model for convection

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- Additional ingredients:
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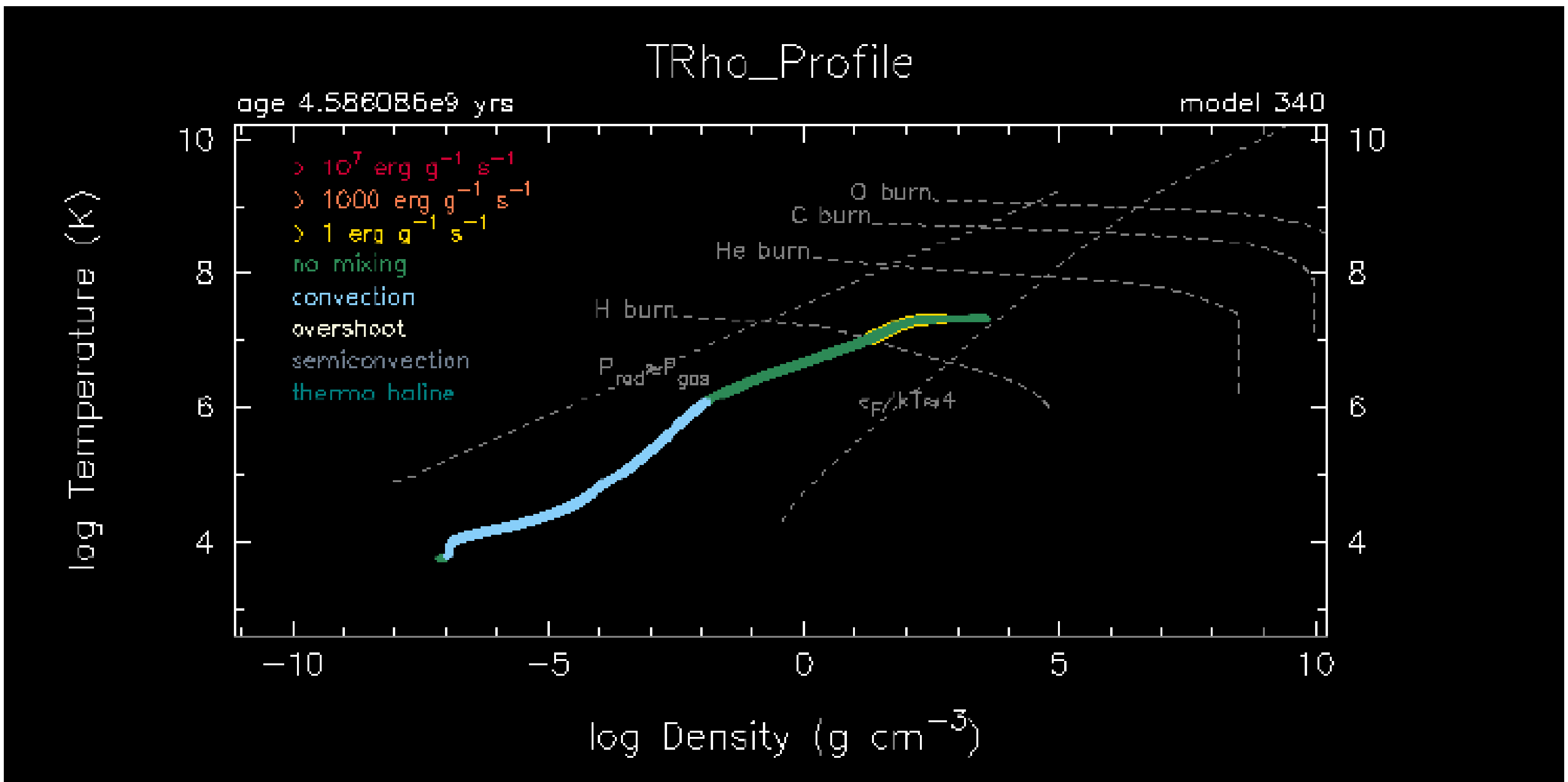
Stellar Model

- MESA stellar evolution code (Paxton et al., 20111)
- **Subgiant stage:** He core not ignited, surroundes by H burning shell
- Mass M=1.25 $M_?$ initial metallicity Z=0.02, age ~4.6 Gyr
- Convection zone extends through outer 28% of the stellar radius



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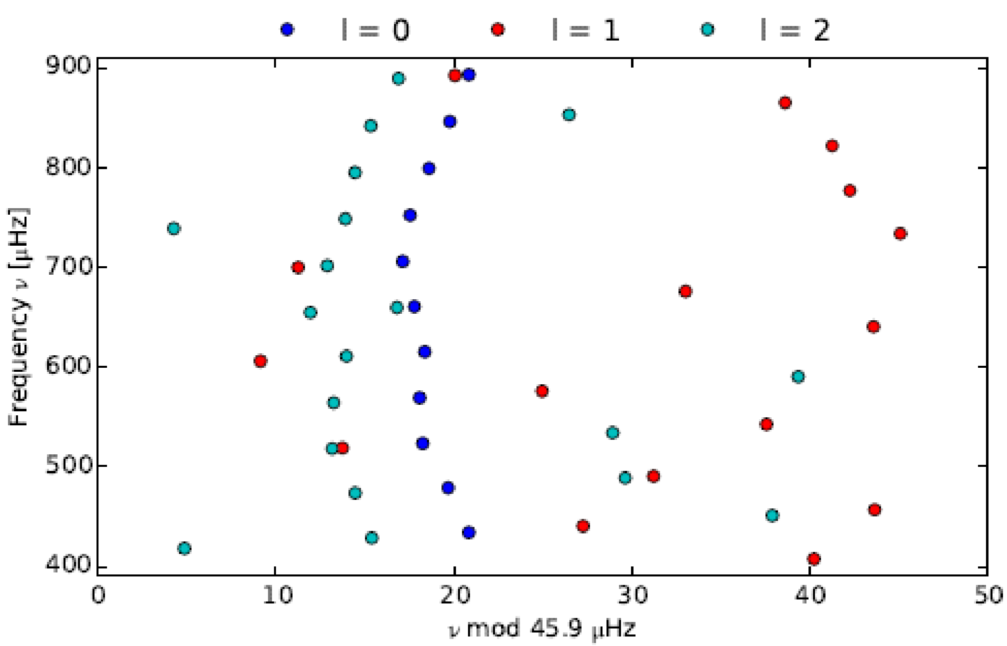
Oscillation modes of sub-giant model

- Mixed modes, p-g character
- Computed set of modes from 400 – 900 µHz (observable range for this model) up to l=20
- Frequency scan with GYRE (Townsend & Teitler, 2013)
- Actual mode calculation with ADIPLS (Christensen-Dalsgaard, 2008)



Echelle diagram containing modes of degree l=0,1,2

Typical l=0,2 ridges

l=1 modes: strong mixed behaviour

Model for Convection Cells

- Simple approximation for large-scale convective motions
- Velocity field expanded in terms of spherical harmonics (degree: s, azimuthal order: t)
- Spherical harmonic determines the flow „pattern“

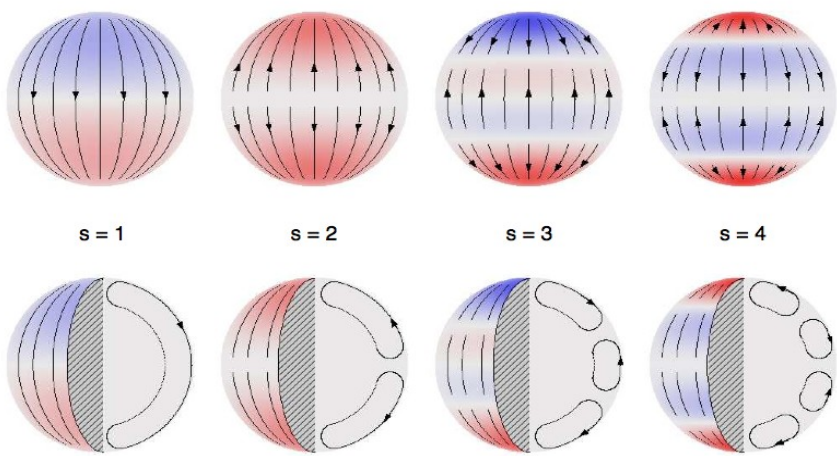
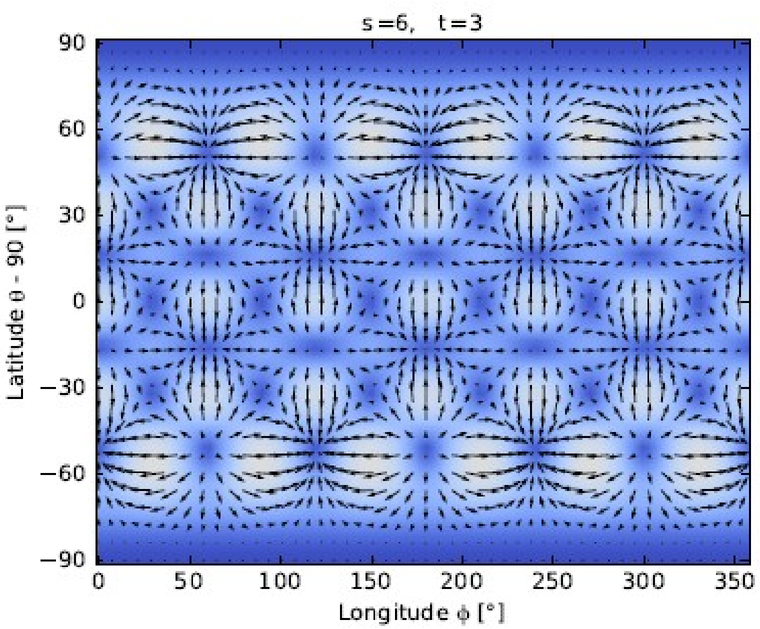


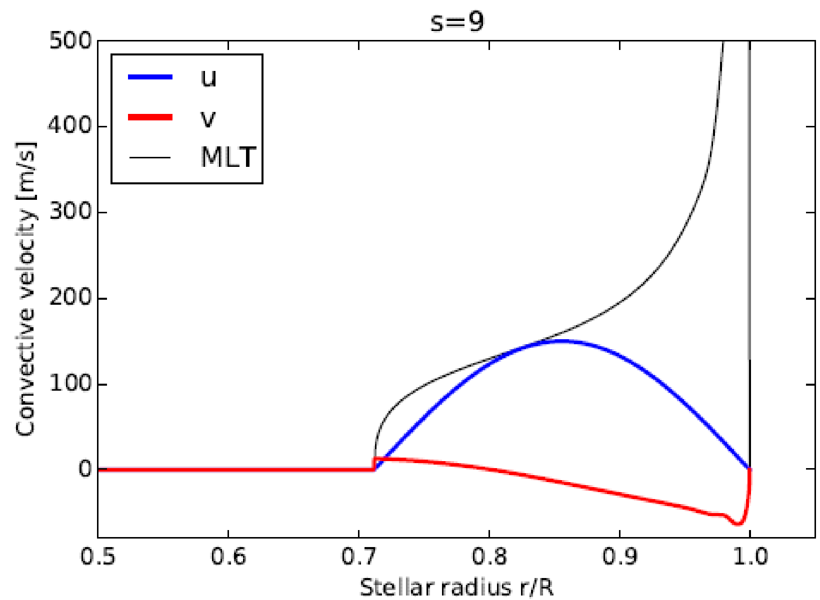
Image: Hathaway/NASA/MSFC



Model for Convection Cells

Spherical harmonic representation of velocity field u:

$$\vec{u}(r) = \underbrace{u_s^t(r) Y_s^t(\theta, \phi)\vec{e}_r}_{\text{radial component}} + \underbrace{v_s^t(r)\nabla_\mathrm{h} Y_s^t(\theta, \phi)}_{\text{horizontal component}}$$



Conservation of mass:

$$\frac{\partial}{\partial r} r^2 \rho_0 \, u_s^t(r) = \rho_0 r s(s+1) v_s^t(r)$$

Shifts proportional to $(u_s^t)^2$ main modifier of the effect

Chosen parametrization:

- Sinusoidal u_s^t,
- Mixing Length Theory (MLT) velocity as upper limit
- v_s^t below surface 60 m/s

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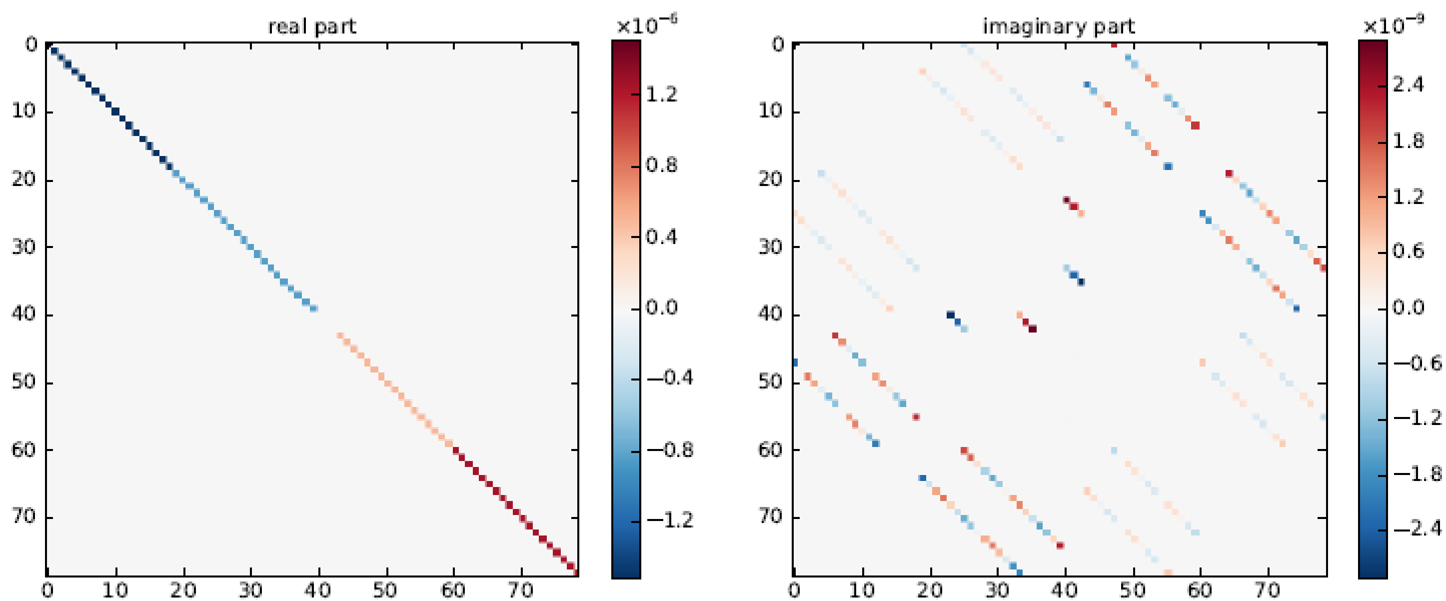
Quasi-degenerate perturbation theory

- Construct the coupling matrix Z (in state space):
  Matrix representation of the advection operator in the basis of unperturbed eigenstates
- Solve eigenvalue problem for matrix Z
  - Eigenvalues are the frequency shifts
  - Eigenvectors are the perturbed new eigenstates, i.e. linear combinations of unperurbed modes **?mode mixing**



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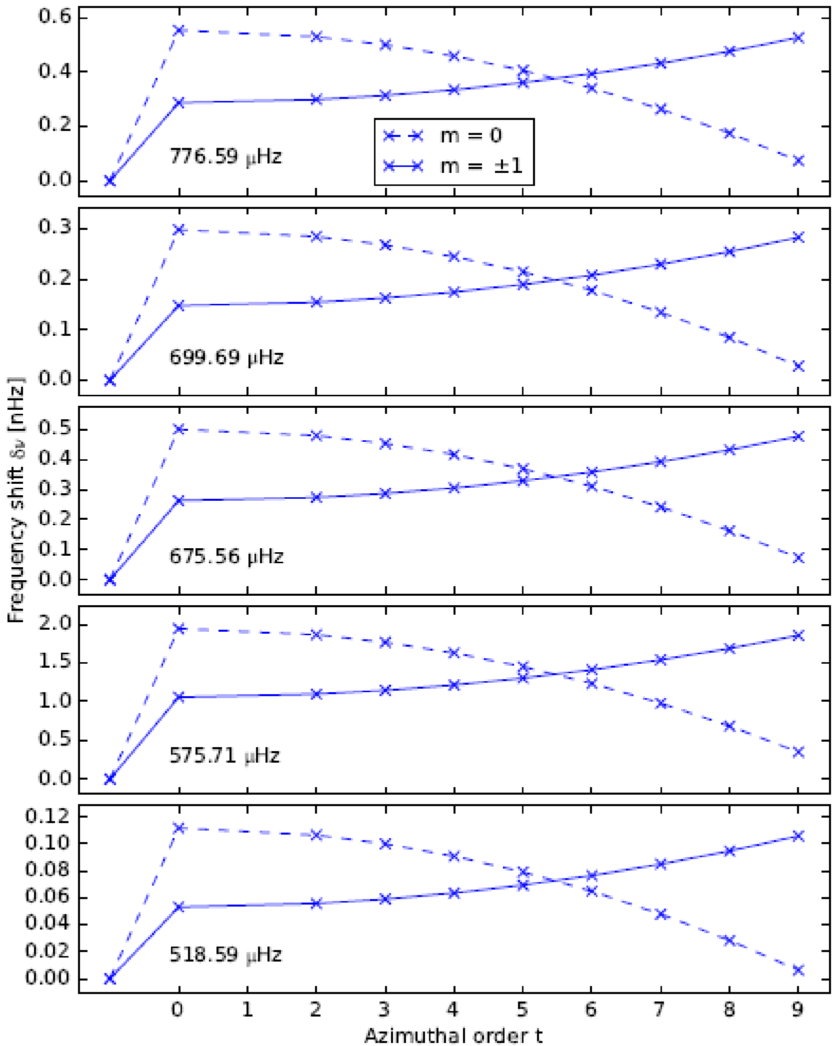
Frequency Shifts of Dipole Modes



- l = 1 multiplets at 5 different frequencies
- Flow configuration: s = 9, all corresponding t values
- Degenerate triplets split into 2 components
- Distinct pattern depending on t: crossing of m = 0 and m = 1 component, for all triplets
- Shifts are two orders of magnitude lower than typical frequency errors (Kepler data)
- Shifts measurable for realistic flow velocities?
- Magnitude varies for different frequencies

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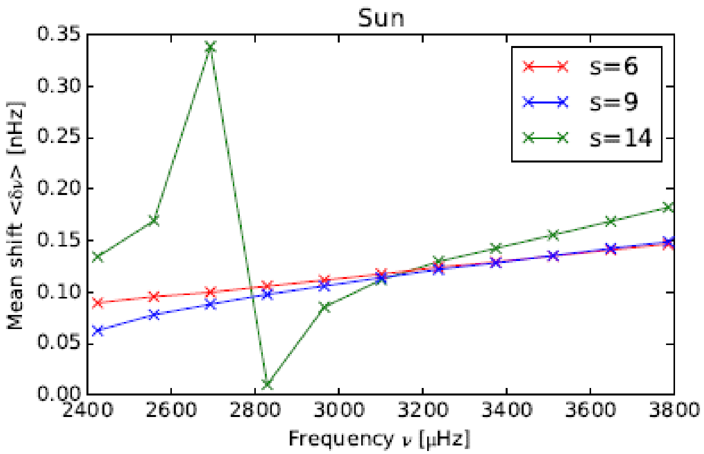
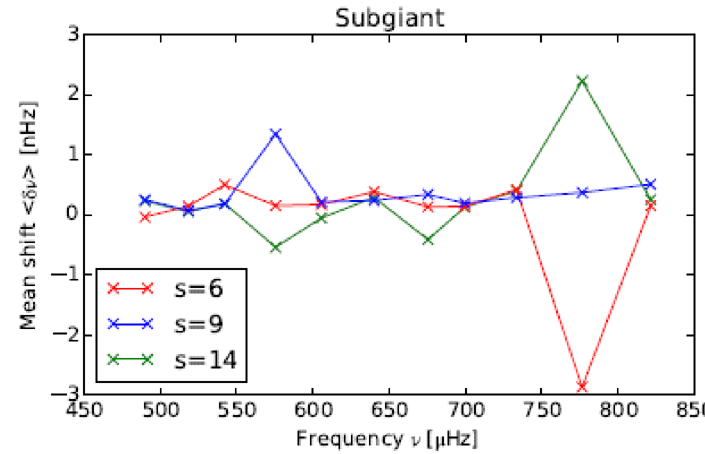
Mean Shift of Triplets, Frequency Dependence





Sun

- Clear trend with frequency
- Slightly different for different s
- Smooth frequency dependence due to regular frequency separations of p-modes
- Exception: special positioning of coupling modes (e.g. $s=14$)

Subgiant

- Shifts dominated by positioning of coupling modes $\sim (\omega_{\rm ref}^2 - \omega_{k'}^2)^{-1}$
- Mixed modes: irregular frequency separations, vicinity of $\omega_{\rm ref}$ differs
- Jumps up to an order of magnitude when coupling modes are close

- Shifts dominated by positioning of coupling modes
- Mixed modes: irregular frequency separations, vicinity of ref differs
- Jumps up to an order of magnitude when coupling modes are close

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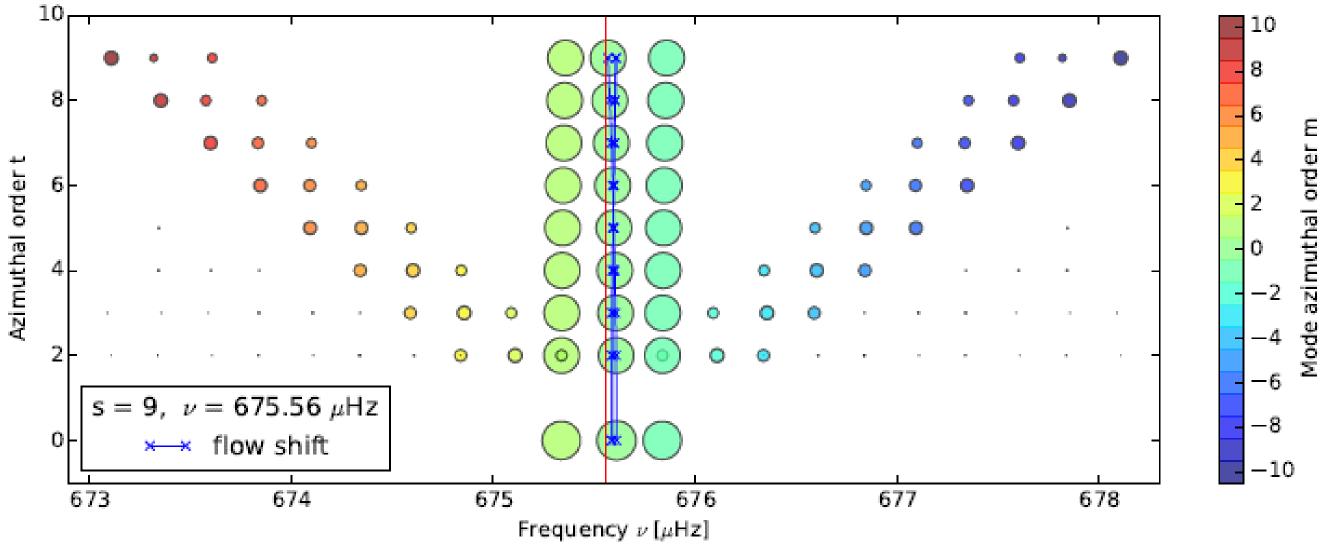
Transformation into Observer's Inertial Frame (Rotating Star)



- Flow effect at one ω_{ref} for star that rotates with $\Omega/(2\pi) = 250$ nHz
- Flow velocity $u_s{}^t$ increased by factor of 10 (for better visibility)
- Dots: theoretical frequency peaks in power spectrum
- Size: amplitude relative to peaks of main triplet
- Main triplet is shifted slightly and shows asymmetry
- Mixing of modes leads to additional splittings in observers inertial frame
- Additional peaks belong to modes with $l = 8 \ldots 10$ $\rightarrow$ not observable for unresolved stellar disks

Flow effect at one ref for star that rotates with nHz
Flow velocity increased by factor of 10 (for better visibility)
Dots: theoretical frequency peaks in power spectrum
Size: amplitude relative to peaks of main triplet
Main triplet is shifted slightly and shows asymmetry
Mixing of modes leads to additional splittings in observers inertial frame
Additional peaks belong to modes with
? not observable for unresolved stellar disks

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Summary

- **Effects of the flow (non-rotating star / co-rotating frame)**
  - Degenerate $l = 1$ triplets split up into two components: $m = 0$ and $m = \pm 1$.
  - The two components switch places after $t \sim [s/2]$
- **Transformation into observer's inertial frame (rotating star)**
  - General shift and asymmetry of a rotationally split $l = 1$ triplet
  - Additional splitting components due to mode mixing

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Summary

- **Measurability of the frequency shift**
  - For the Sun, the effect is very small due to low flow velocities
  - For a subgiant, the effect is slightly bigger, but still well below the detection threshold Δ nHz (for simple flow model)
  - Rotational splitting:
    Even for a strong effect (visible asymmetry of triplet), the rotational splitting between $m = \pm 1$ modes is not altered, since poloidal flows shift $m = \pm 1$ components equally.

- **Outlook**
  - Include differential rotation → affects calculation?
  - Explore different flow setups