Mode coupling by convection as possible contribution to the surface effect

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Advection reduces the frequencies

Brown (1984) investigated the effect of **stationary** velocity perturbations on high-frequency p-modes:

The inhomogeneous velocity in the convection zone modifies the wave propagation

- Wave scattering
- Slowing of mean wave front

Happens wherever there are flows; not only near the surface.

Near-surface flows have highest amplitudes

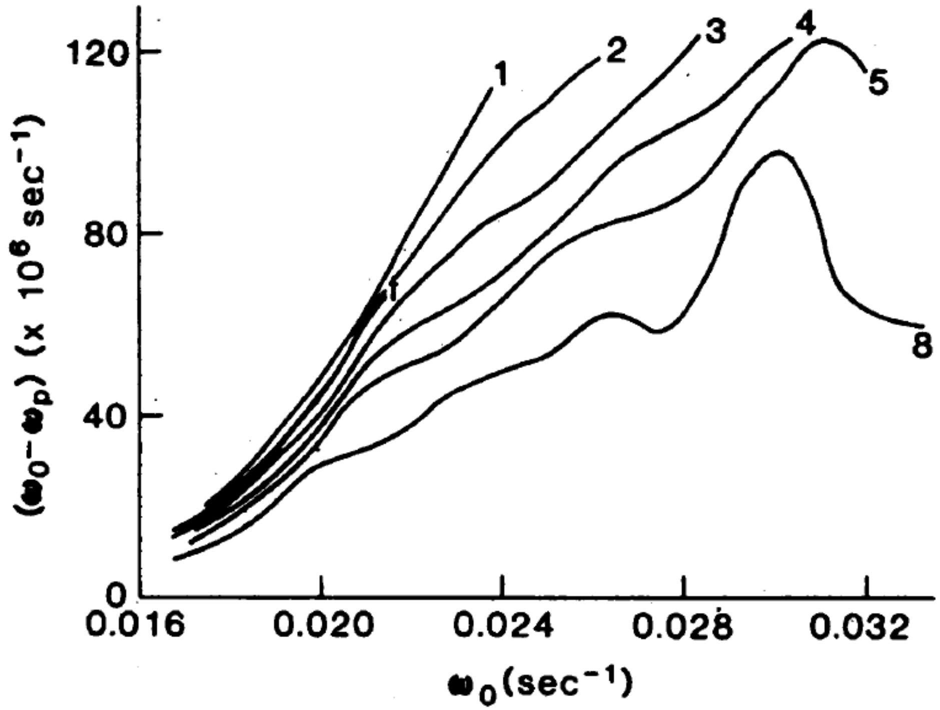

Fig. 1. Difference between eigenfrequencies ω_0 and ω_p computed, respectively, without and with vertical turbulent velocities, plotted as a function of ω_0. The frequencies with turbulent motions are always smaller than those without, leading to positive differences.

Further Simple Models

Stix & Zhugzhda (1994):

Structured atomsphere with constant pressure

Flow velocity, temperature and density are functions of the horizontal coordinate x

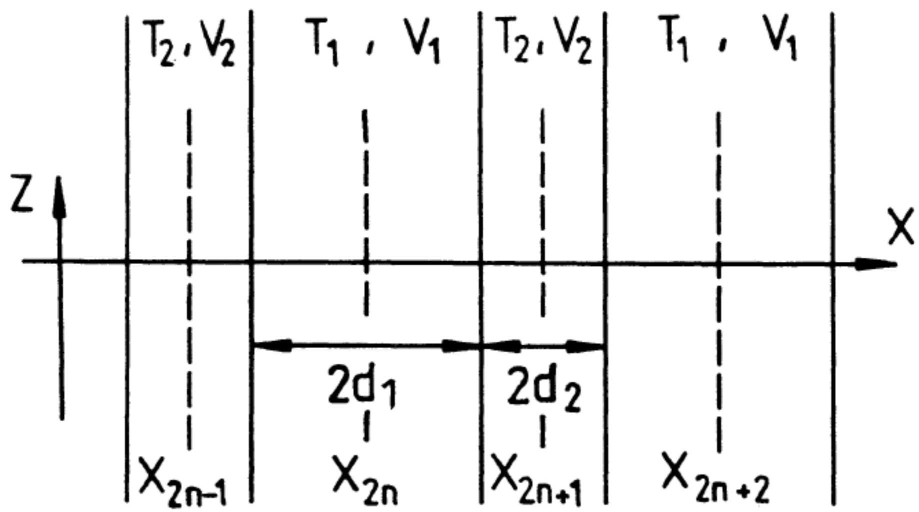

Fig. 1. The model of alternating vertical layers

Calculations

Equations for a vertical wave of frequency ω and wavenumber k_z

the hydrodynamical equations are:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho_0}\nabla \delta p, \qquad (1)$$

$$\frac{\partial \delta\rho}{\partial t} + \rho_0 \mathrm{div}\,\mathbf{v} + \mathbf{V} \cdot \nabla \delta\rho + \mathbf{v} \cdot \nabla \rho_0 = 0, \qquad (2)$$

$$\frac{\partial \delta T}{\partial t} + \mathbf{v} \cdot \nabla T_0 + \mathbf{V} \cdot \nabla \delta T + (\gamma - 1) T_0 \mathrm{div}\,\mathbf{v} = 0, \qquad (3)$$

$$\rho_0 T_0 \delta p = \rho_0 p_0 \delta T + p_0 T_0 \delta\rho \qquad (4)$$

Can be reduced to a wave equation:

$$C_0^2 \frac{d^2\psi}{dr^2} + \left[\ldots \frac{d}{dr} + \frac{\ldots}{V_{\perp} - V}\frac{dV}{dr} \right] \frac{d\psi}{dr} + k_z^2 \left[(V_{ph} - V)^2 - C_0^2 \right] \psi = 0, \qquad (5)$$

With solution

[illegible]

(Zhugzhda 1997, SCORe Proceedings)

Results

- Brown (1984); Stix & Zhugzhda (1994):
  "In the … medium the mean wave phase speed is not the same as the phase speed of the mean medium"

$$\frac{1}{\bar{V}_{ph}} = \frac{1}{2}\left(\frac{1}{V_{ph+}} - \frac{1}{V_{ph-}}\right)$$

- Mean phase speed defines the eigenfrequencies of a structured atmosphere

$$\frac{\Delta\omega}{\omega} = \frac{\int c_{mod}^{-1} dr}{\int \bar{V}_{ph}^{-1} dr} - 1\ .$$

Frequency Shifts for Radial Modes

Effect could be strong enough to correct the frequencies.

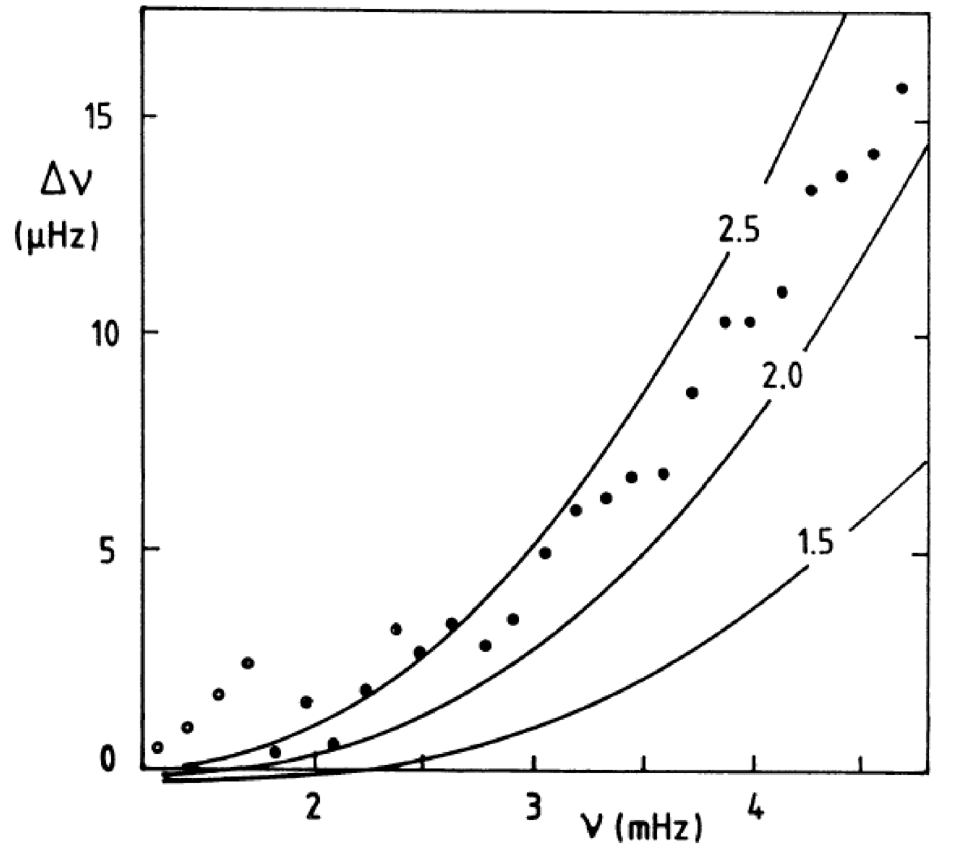

Fig. 6. Frequency difference between calculated and observed radial solar p modes (*dots and circles*), and (negative) frequency corrections for $\lambda = 0.7$ and three values of the size factor, f_S (*solid curves*)

(Stix & Zhugzhda 1994)

Convection Rolls

- Stix & Zhugzhda (2004)

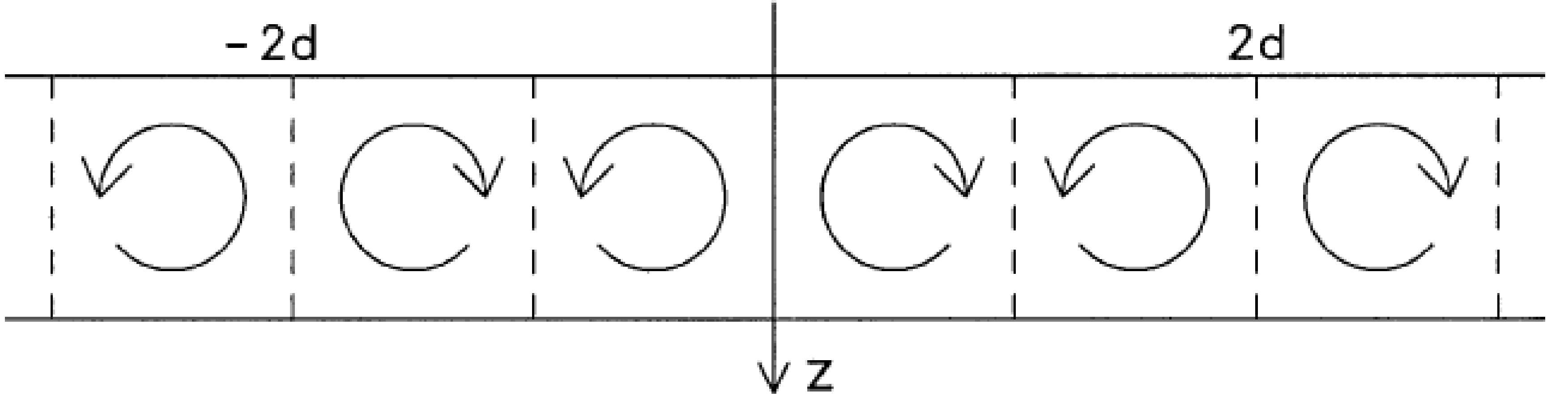

Fig. 1. The model of a layer with convection rolls. The horizontal period is used to define the unit of length, $2d/\pi$.

$$\frac{\Delta\omega}{\omega} = \frac{\int c_0^{-1}\,\mathrm{d}r}{\int [c_0(1 - Ma(r)^2]^{-1}\,\mathrm{d}r} - 1$$

Frequency Shifts for Non-Radial Modes

- Remark in the Conclusions of their paper: change of granulation size to explain cycle-dependent frequency shift?

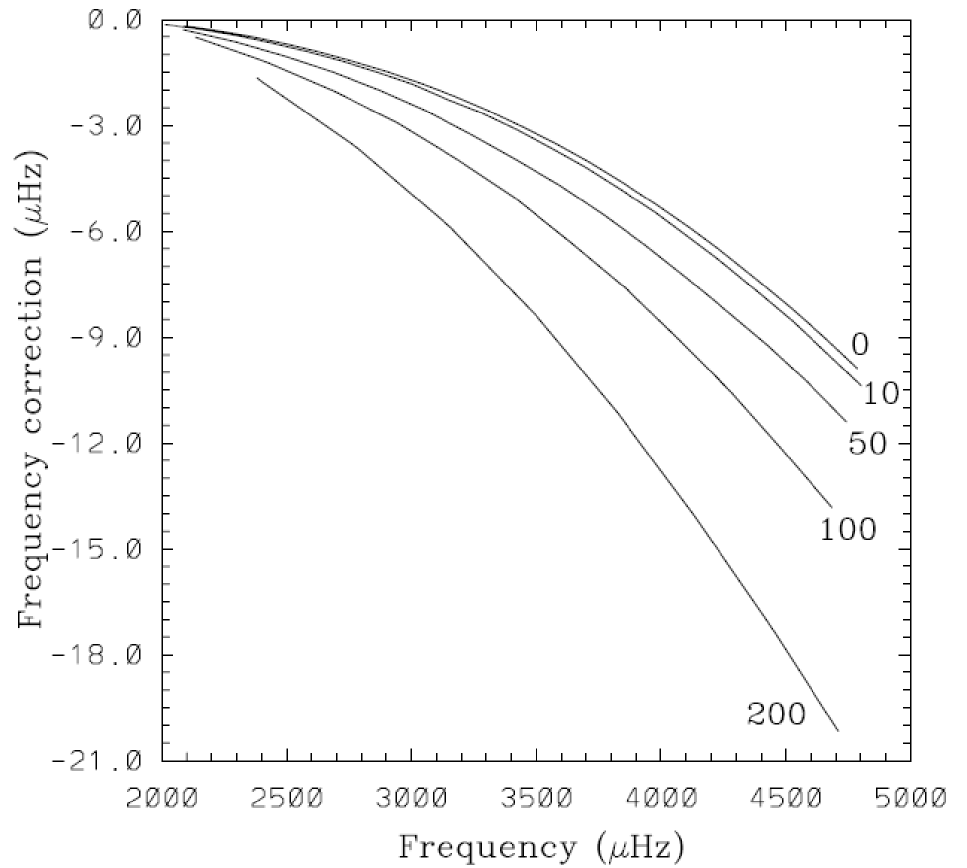

Fig. 3. Frequency correction for solar p modes, calculated with the harmonic model. The label is the degree l of the mode; the size parameter is $f_S = 2.5$.

(Stix & Zhugzhda, 1998)

Another Approach: Perturbation Theory

Following the approach described by Lavely & Ritzwoller (1992)

Perturbing the equilibrium model with a **slow** flow, i.e. **small perturbation**

$$-\omega_k^2\rho_0\xi_k = -\nabla p_1 + \rho_0\mathbf{g}_1 + \rho_1\mathbf{g}_0$$

$$-\omega_k^2\rho_0\xi_k - 2\mathrm{i}\omega_k\rho_0(\mathbf{v}\cdot\nabla)\xi_k = -\nabla p_1 + \rho_0\mathbf{g}_1 + \rho_1\mathbf{g}_0$$

$$-\rho_0\omega_k^2\xi_k = H_0(\xi_k) + \varepsilon H_1\xi_k$$

Mode Coupling: Perturbation matrix elements for calculation of new eigenvalues

$$H_{k'k} = \langle\xi_k|2\mathrm{i}\omega_k\rho_0(\mathbf{v}\cdot\nabla)|\xi_{k'}\rangle$$
$$= 2\mathrm{i}\omega_k\int\rho_0\xi_k(\mathbf{v}\cdot\nabla)\xi_{k'}\,d\mathbf{r}$$

Eigenvalues of perturbation matrix are frequency corrections:

$$\tilde{\omega}_k^2 = \omega_k^2 + \delta\omega_k^2$$

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Flow Modelling

Decomposition into a
- toroidal flow (includes differential rotation) and
- poloidal flow (includes meridional flow, giant cells, supergranulation, granules(?))

$$\mathbf{v}(\mathbf{r}) = \sum_{s=0}^{\infty} \sum_{t=-s}^{s} \mathbf{T}_s^t(r,\theta,\phi) + \mathbf{P}_s^t(r,\theta,\phi)$$

where components are expanded in terms of spherical harmonics

$$\mathbf{T}_s^t(r,\theta,\phi) = -w_s^t(r)\,\mathbf{e}_r \times \nabla_h Y_s^t(\theta,\phi)$$

$$\mathbf{P}_s^t(r,\theta,\phi) = u_s^t(r)\,Y_s^t(\theta,\phi)\,\mathbf{e}_r + v_s^t(r)\,\nabla_h Y_s^t(\theta,\phi)$$

Mass conservation:

$$\rho_0 r s(s+1) v_s^t = \partial_r (r^2 \rho_0 u_s^t)$$

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Differential Rotation: Frequency Splitting

Toroidal Flow: $$\mathbf{T}(\mathbf{r}) = \sum_{s} \sum_{t=-s}^{s} -w_s^t(r)\mathbf{e}_r \times \nabla_h Y_s^t(\theta,\phi)$$

Differential rotation (s odd, t=0) : $\omega_{k}(m)=\omega_{k}(m=0)+\delta\omega(m)$

with $\delta\omega(m)=\sum_{s=1,3,5,\ldots} c_{nl,s}\gamma_{nl,s}(m)$

where $c_{nl,s}=\int_0^R w_s(r)\, K_{nl,s}(r)\, r^2\, dr$

and $\gamma_{nl,s}$ orthogonal functions (Clebsch-Gordon coefficients)

! **Inversion problem for $w_s(r)$**

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Differential Rotation: Frequency Splitting

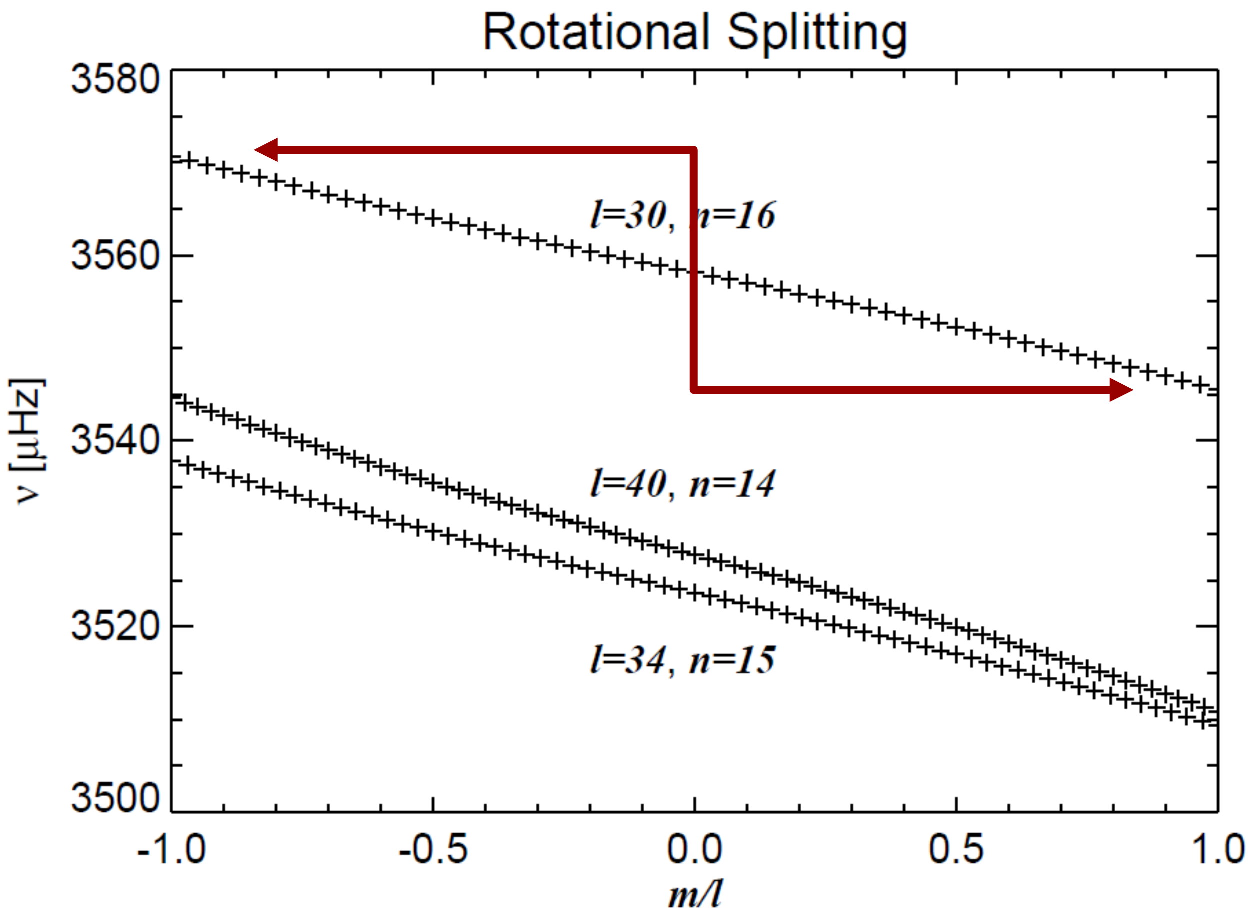
Lifting of degeneracies

! "Self-coupling" within multiplets, i.e.

n′=n & l′=l

Central frequency of a multiplet is **not** shifted!

Antisymmetric around m=0 (in first order perturb. theory)



Poloidal Flows: Additional Frequency Shifts

Effect of a poloidal flow on the eigenmodes

$$\mathbf{P}(\mathbf{r}) = \sum_{s,t} u_s^t(r) Y_s^t(\theta,\phi)\mathbf{e}_r + v_s^t(r)\nabla_h Y_s^t(\theta,\phi)$$

Coupling Matrix:

$$H_{kk'} = \langle \xi_k | \overbrace{2\mathrm{i}\omega_k\rho_0(\mathbf{v}_s^t\cdot\nabla)}^{H_1} | \xi_{k'} \rangle, \qquad k \neq k'$$

$$H_{kk} = 0$$

$$H_{kk'} = \sum_{s,t}\int_0^R u_s^t(r)\, K_{st,kk'}(r)\, dr\, \mathcal{P}(\underbrace{s,l,l'|s,t,m,m'}_{\text{Clebsch-Gordon coefficients}})$$

Frequency Corrections (two mode coupling):

$$\delta\omega_k^2 = \frac{|H_{kk'}|^2}{\omega_k^2 - \omega_{k'}^2}, \qquad k \neq k'$$ **Second-order effect!**

(Roth & Stix 1999, A&A)

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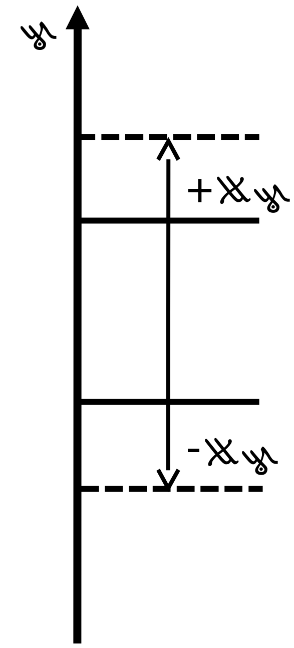
Frequency Shifts by Mode Coupling

Additional to rotational effect:

Poloidal flows leave signatures in oscillation data as additional frequency shifts

$$\delta\omega_k^2 = \frac{|H_{kk'}|^2}{\omega_k^2 - \omega_{k'}^2}, \qquad k \neq k'$$

$$|H_{kk'}|^2 \propto (u_s^t)^2$$ **Second-order effect!**



- Two coupling modes are shifted with opposite signs
- (approx.) nearest matching neighbor in frequency has strongest influence

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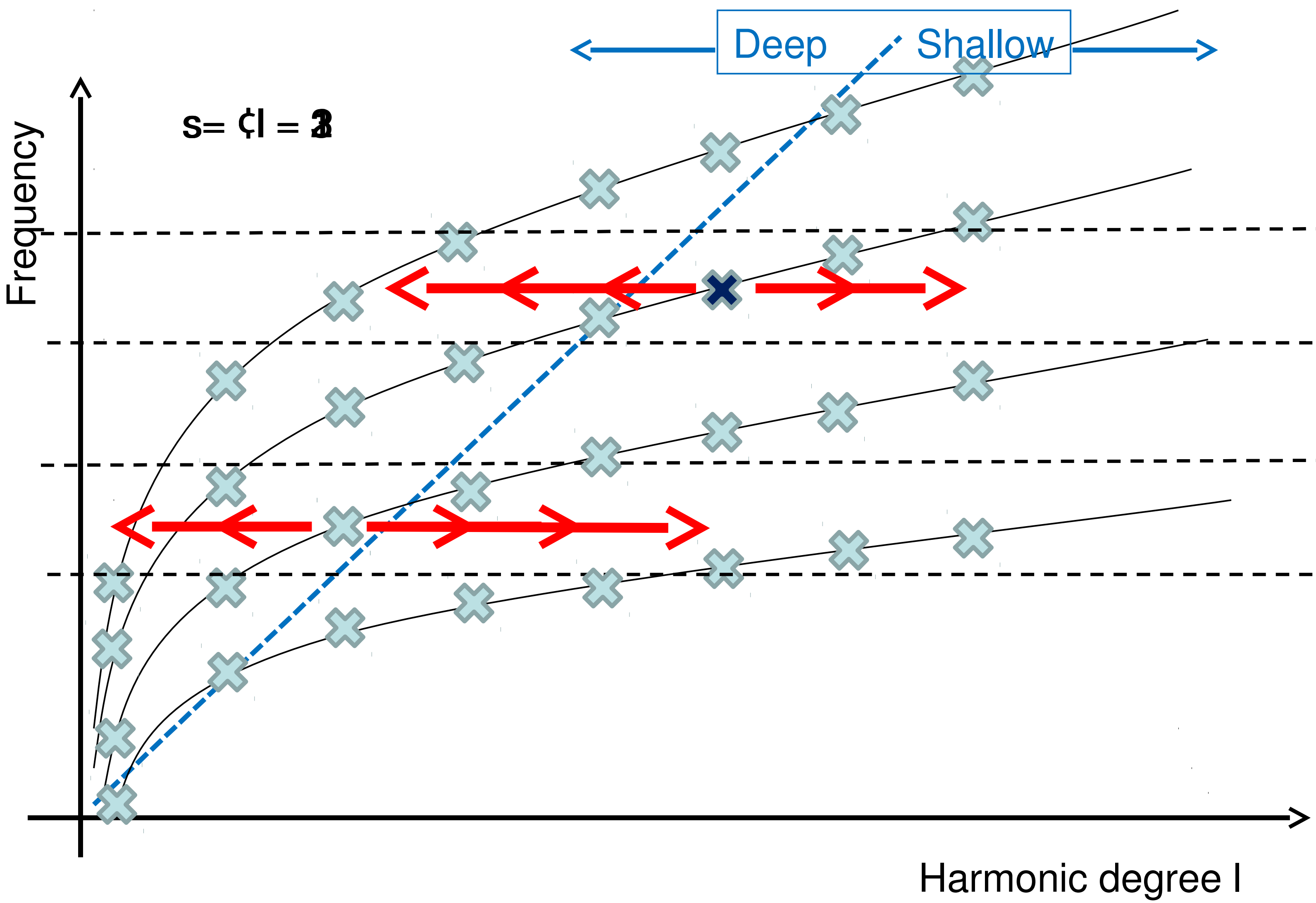
Spatial scale of flow & form of l-nu-diagram defines coupling range



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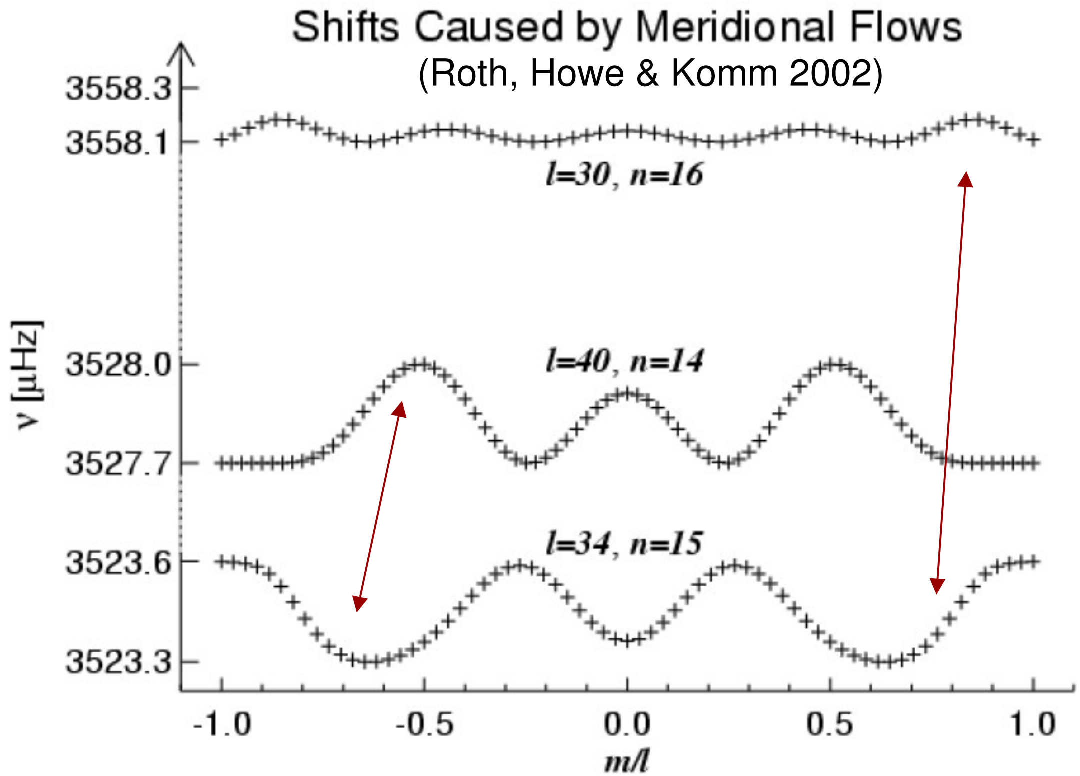
Poloidal Flows: Additional Frequency Shifts

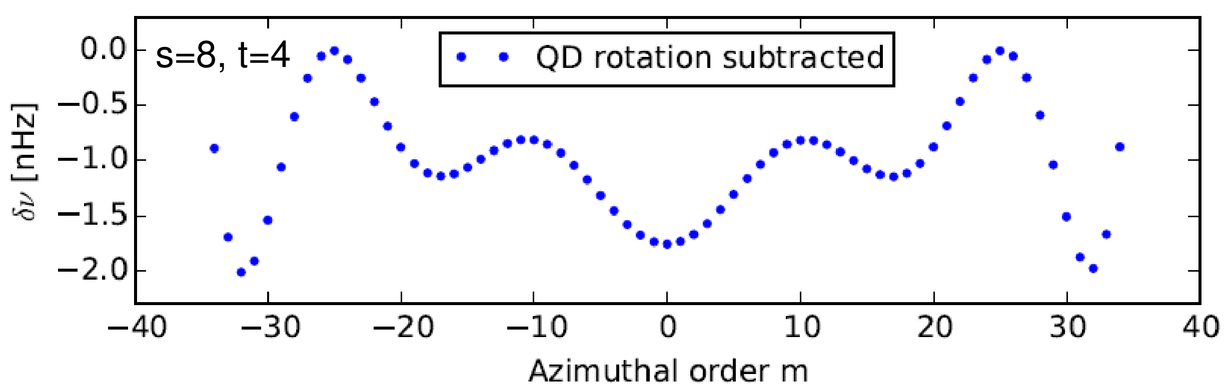
- $V_{max=500\ m/s}$
- s=8, t=0

Central frequency of a multiplet is affected, i.e. whole multiplet is shifted

Symmetric to the mode m=0 (even function)

Real effect is very small in comparison to rotational splitting (Chatterjee & Antia 2009).





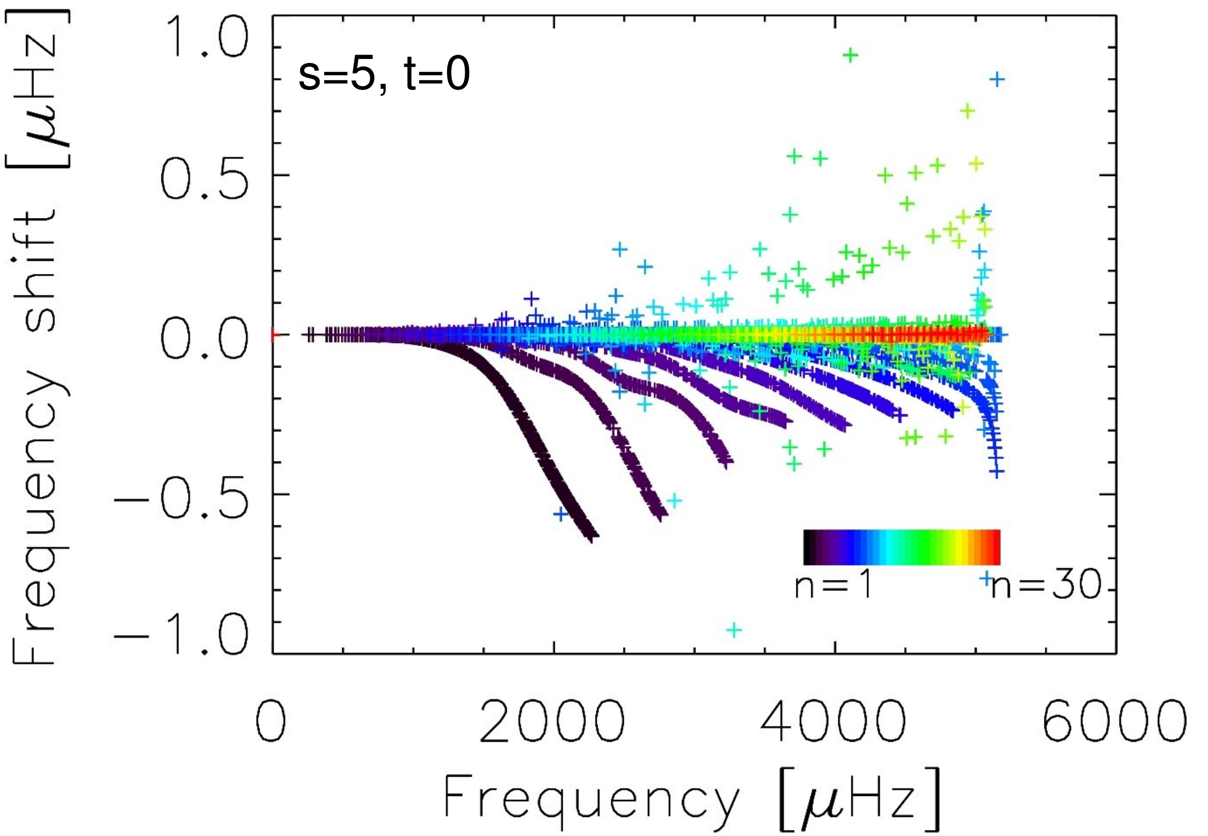
Reduced frequencies due to large-scale flows

Example (not a surface effect):
Amplified meridional flow
reaching deep

Frequency reduction is small

Color: radial order

All large-scale poloidal flows should result in a similar effect



(Roth & Stix 2008)

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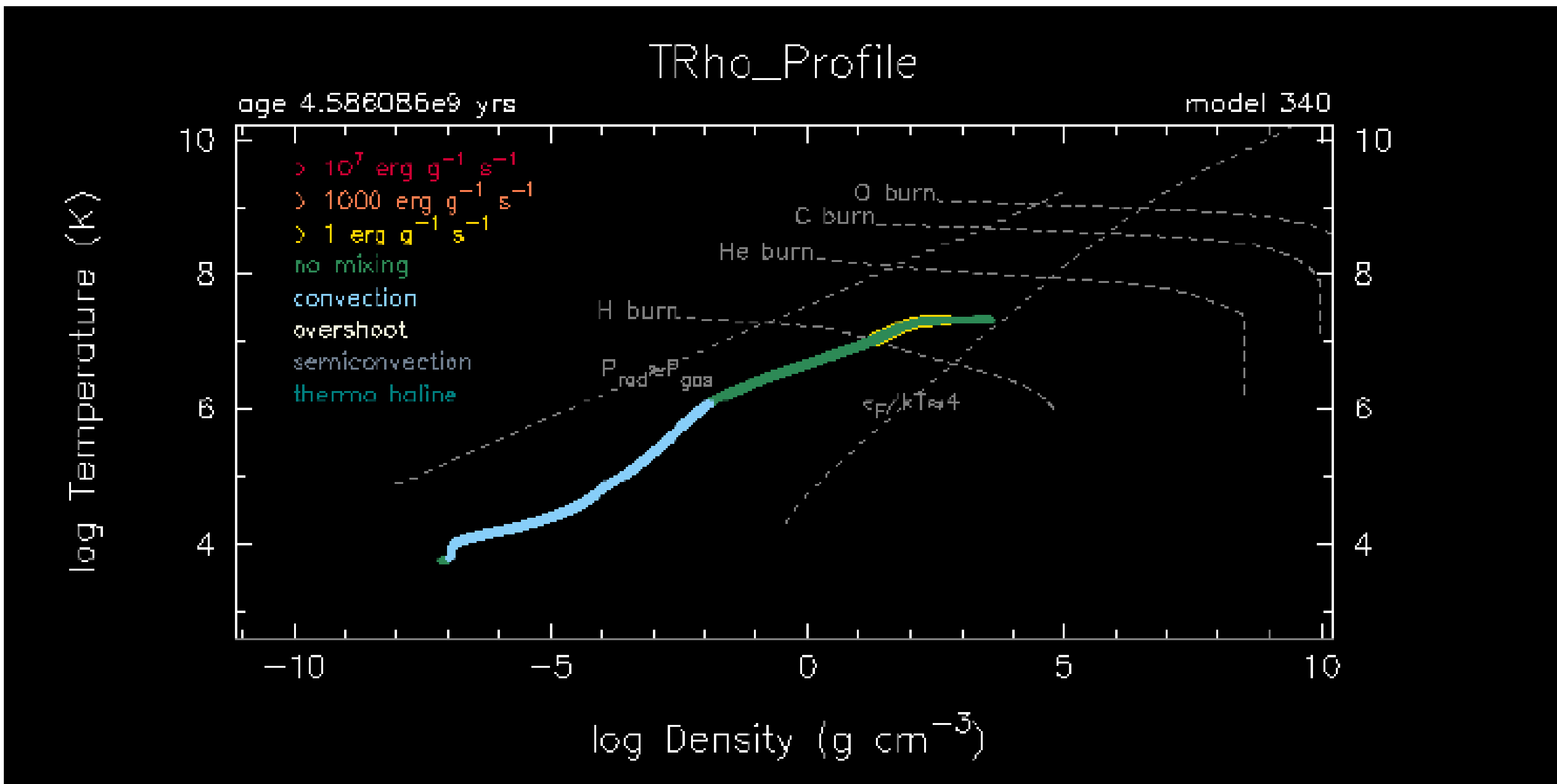
Mode Coupling in a Sub-Giant

- MESA stellar evolution code (Paxton et al., 2011)
- **Subgiant stage:** He core not ignited, surrounded by H burning shell
- Mass M=1.25 M? initial metallicity Z=0.02, age ~4.6 Gyr
- Convection zone extends through outer 28% of the stellar radius



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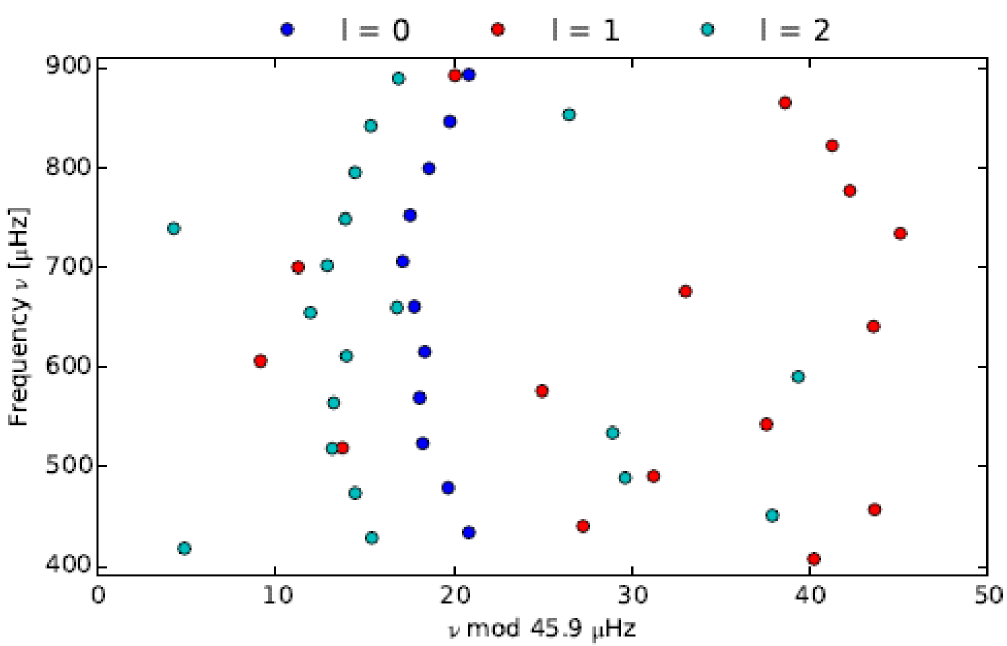
Oscillation modes of sub-giant model

- Mixed modes, p-g character
- Computed set of modes from 400 – 900 μHz (observable range for this model) up to l=20
- Frequency scan with GYRE (Townsend & Teitler, 2013)
- Actual mode calculation with ADIPLS (Christensen-Dalsgaard, 2008)



Echelle diagram containing modes of degree l=0,1,2

Typical l=0,2 ridges

l=1 modes: strong mixed behaviour

(Herzberg & Roth, in prep.)

Model for Convection Cells

- Simple approximation for large-scale convective motions
- Velocity field expanded in terms of spherical harmonics (degree: s, azimuthal order: t)
- Spherical harmonic determines the flow „pattern"

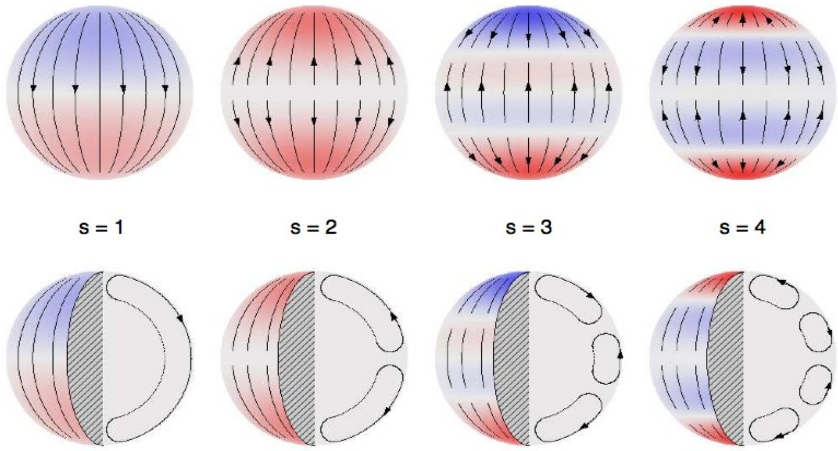
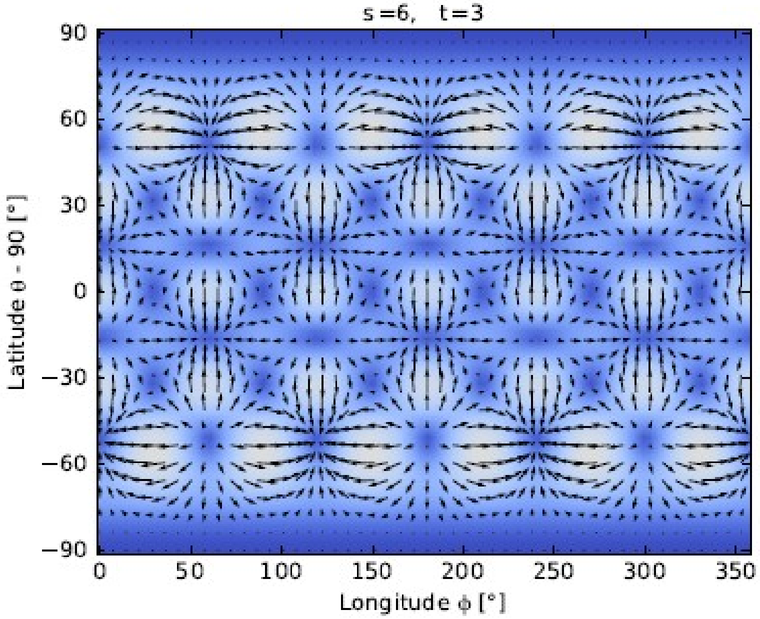


Image: Hathaway/NASA/MSFC



(Herzberg & Roth, in prep.)

Model for Convection Cells

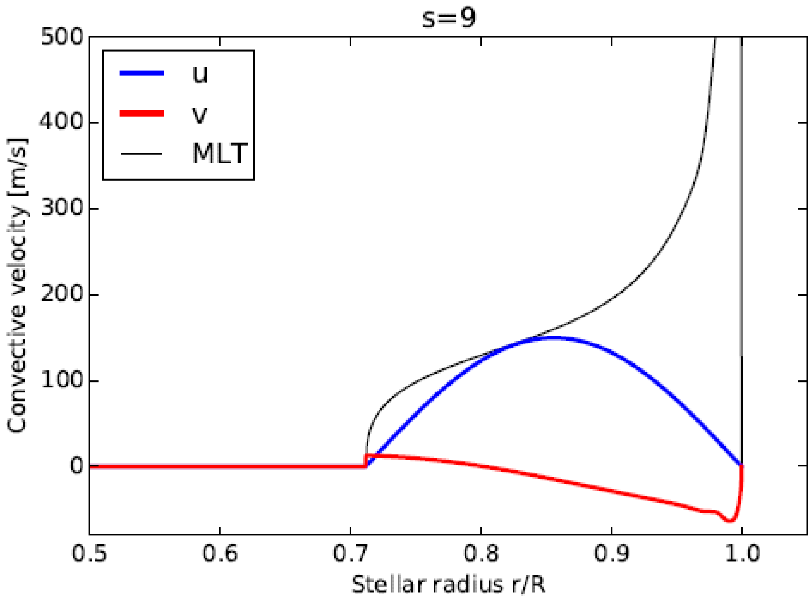
Spherical harmonic representation of velocity field u:

$$\vec{u}(r) = \underbrace{u_s^t(r) Y_s^t(\theta,\phi)\vec{e}_r}_{\text{radial component}} + \underbrace{v_s^t(r)\nabla_h Y_s^t(\theta,\phi)}_{\text{horizontal component}}$$

Conservation of mass:

$$\frac{\partial}{\partial r} r^2 \rho_0\, u_s^t(r) = \rho_0 r s(s+1) v_s^t(r)$$

Shifts proportional to $(u_s^t)^2$ main modifier of the effect



Chosen parametrization:

- Sinusoidal u_s^t,
- Mixing Length Theory (MLT) velocity as upper limit
- v_s^t below surface 60 m/s
- Gradients can create high horizontal flow components

  (not necessary realistic)

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(Herzberg & Roth, in prep.)

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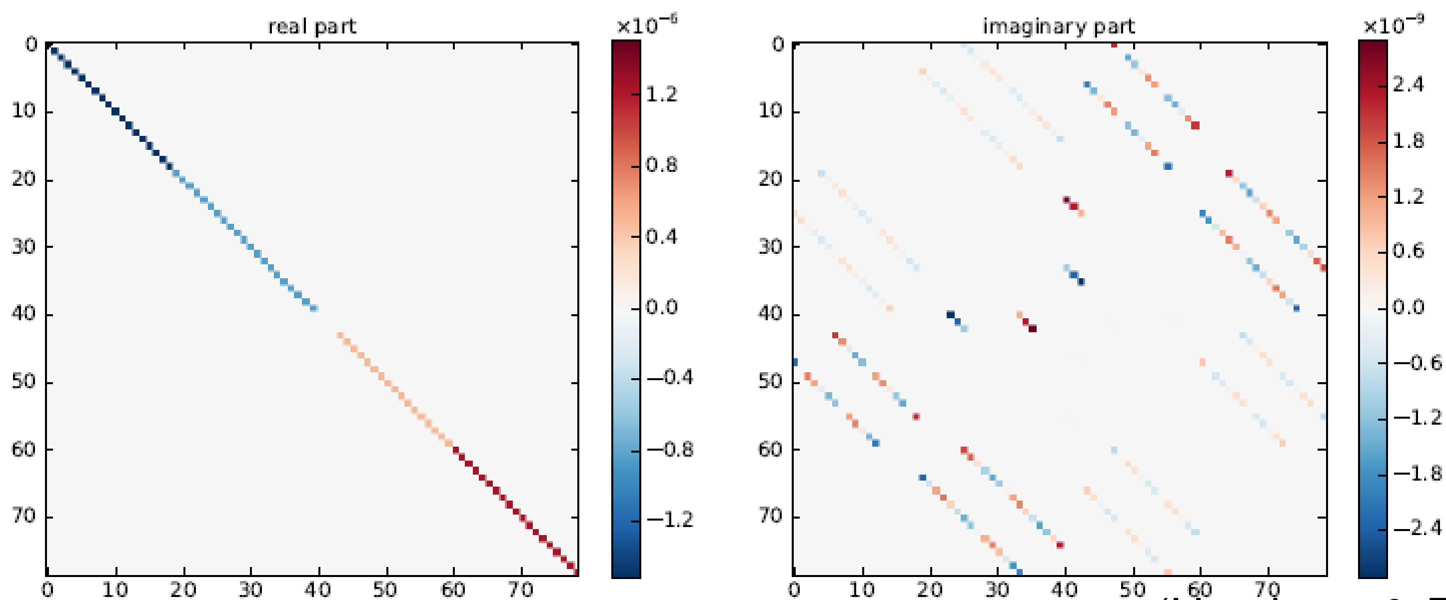
Quasi-degenerate perturbation theory

- Construct the coupling matrix Z (in state space):
  Matrix representation of the advection operator in the basis of unperturbed eigenstates
- Solve eigenvalue problem for matrix Z
  - Eigenvalues are the frequency shifts
  - Eigenvectors are the perturbed new eigenstates, i.e. linear combinations of unperturbed modes **?mode mixing**



(Herzberg & Roth, in prep.)

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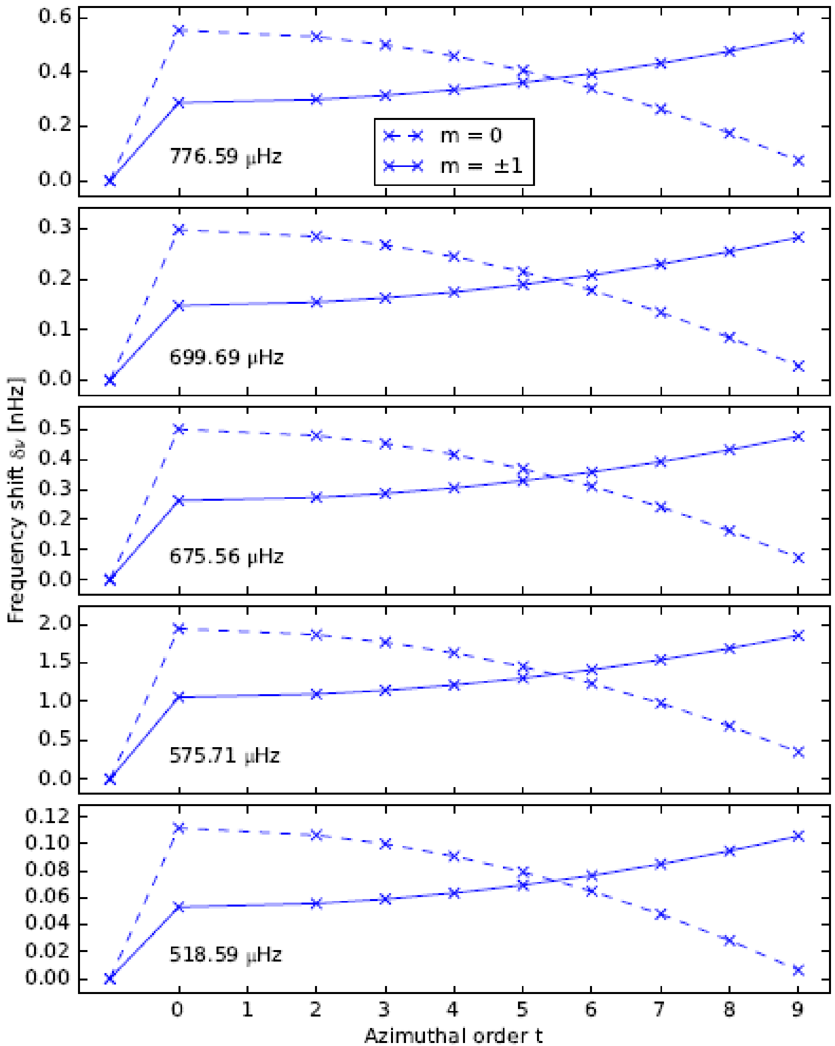
Frequency Shifts of Dipole Modes



- l = 1 multiplets at 5 different frequencies
- Flow configuration: s = 9, all corresponding t values
- Degenerate triplets split into 2 components
- Distinct pattern depending on t: crossing of m = 0 and m = 1 component, for all triplets
- Shifts are two orders of magnitude lower than typical frequency errors (Kepler data)
- Shifts measurable for realistic flow velocities?
- Magnitude varies for different frequencies

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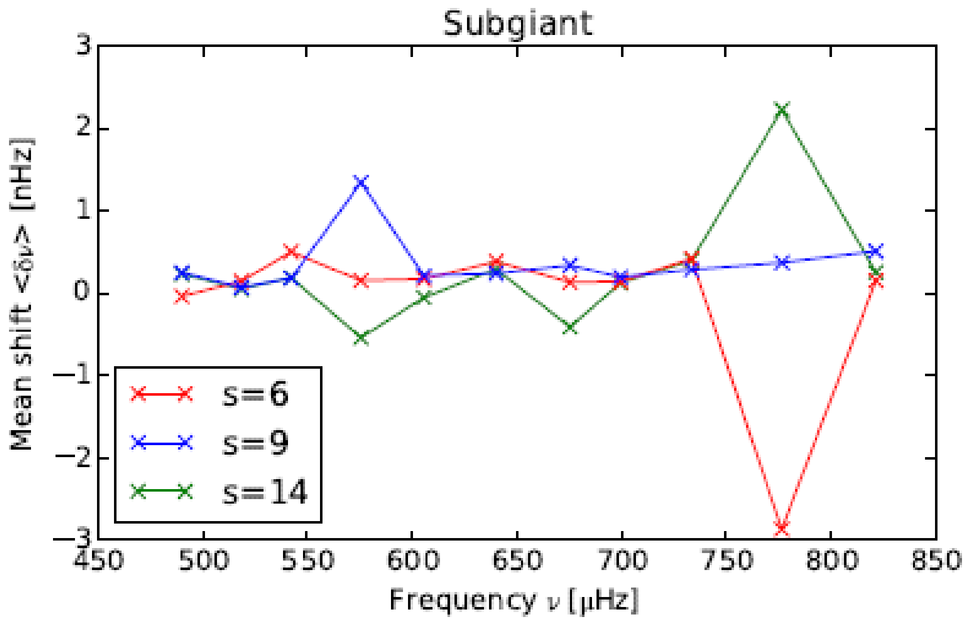
(Herzberg & Roth, in prep.)

Mean Shift of Triplets, Frequency Dependence



- Shifts dominated by positioning of coupling modes $\sim (\omega_{\mathrm{ref}}^2 - \omega_{k'}^2)^{-1}$
- Mixed modes: irregular frequency separations, vicinity around a mode is different compared to the Sun.
- Jumps up to an order of magnitude when coupling modes are close

(Herzberg & Roth, in prep.)

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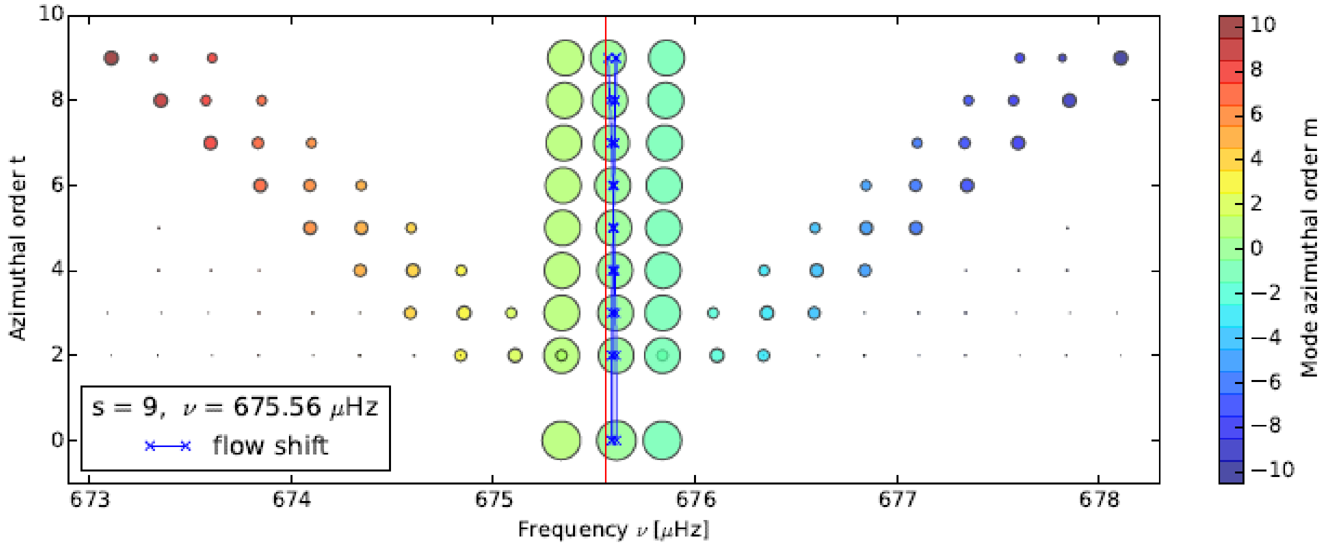
Transformation into Observer's Inertial Frame (Rotating Star)



- Flow effect for one mode for a star that rotates with $\Omega/(2\pi) = 250$ nHz
- Flow velocity u_t increased by factor of 10 (for better visibility)
- Dots: theoretical frequency peaks in power spectrum
- Size: amplitude relative to peaks of main triplet
- Main triplet is shifted slightly and shows asymmetry
- Mixing of modes leads to additional splittings in observers inertial frame
- Additional peaks belong to modes with $l = 8 \dots 10$
→ not observable for unresolved stellar disks

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- Flow velocity increased by factor of 10 (for better visibility)
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(Herzberg & Roth, in prep.)

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How to make a surface effect with mode coupling

- ***Sun (& solar-like stars):***
  calculations for smaller spatical scale of flow components
  - s~o(100) for supergranulation
  - s~o(1000) for granulation
- ***Evolved stars:***
  large convective amplitudes possible

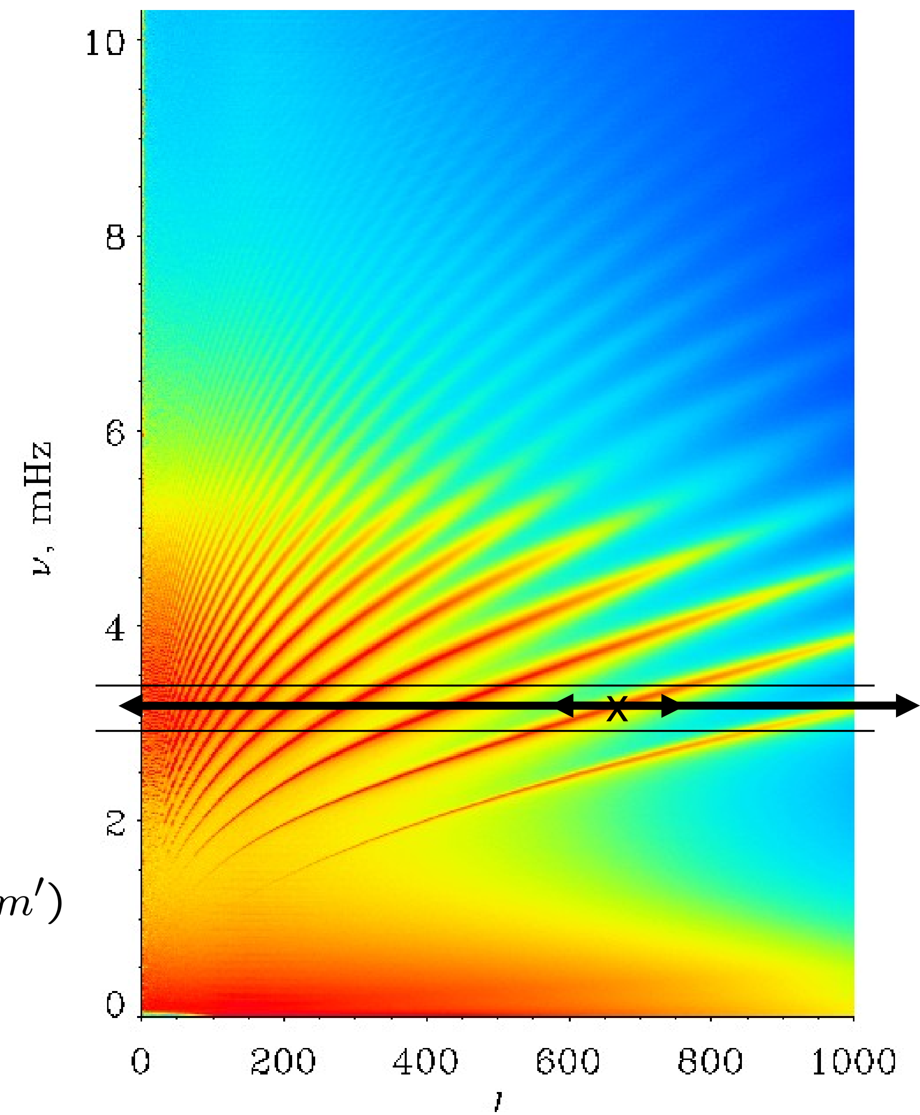
Caution: amplitudes must be small to apply perturbation theory
(gradients of radial component to determine horizontal component)

Possible solution:
No expansion in spherical harmonics

$$H_{kk'} = H_{kk'} = 2i\omega_k \int_0^R \rho_0 \xi_k (\mathbf{v}\cdot\nabla)\xi_{k'}\, d\mathbf{r}$$

3D models

(Courtesy to SOHO Image Gallery)



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