Validation of Helioseismic Fourier-Legendre Analysis

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Verification of the Helioseismic Fourier-Legendre Analysis for Meridional Flow Measurements

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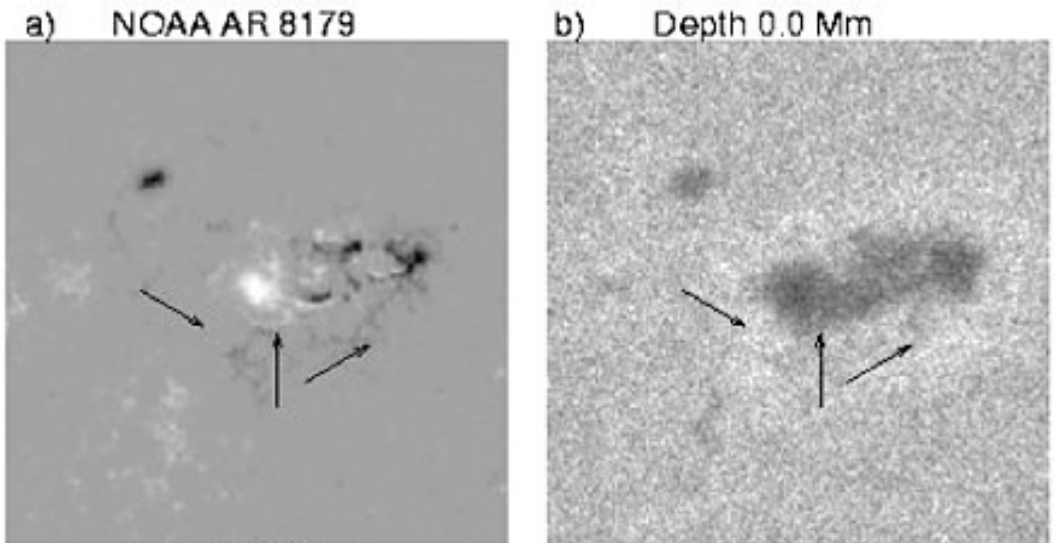
ABSTRACT

Context. Measuring the Sun's internal meridional flow is one of the key issues of helioseismology. The Fourier-Legendre Analysis is a technique for addressing this problem.
Aims. We validate this technique with the help of artificial helioseismic data.
Methods. The analysed data set was obtained by numerically simulating the effect of the meridional flow on the seismic wave field in the full volume of the Sun. In this way a 51.2 hours long time series was generated. The resulting surface velocity field is then analyzed in various settings: Two $360° \times 90°$ halfspheres, two $120° \times 60°$ patches on the front and **farside** of the Sun (North and South, resp.) and two $120° \times 60°$ patches on the northern and southern frontside only. We compare two possible measurement setups: observations from Earth and from an additional spacecraft on the solar **farside**, and observations from Earth only, with the case where the full information of the global solar oscillation wave field was available.
Results. We find that with decreasing observing area the accessible depth range decreases: the $360° \times 90°$ view allows probing the meridional flow almost to the bottom of the convection zone, while the $120° \times 60°$ view allows only the outer layers to be probed.
Conclusions. These results confirm the validity of the Fourier-Legendre analysis technique for helioseismology of the meridional flow. Furthermore they are of special interest for missions like Solar Orbiter that promise complementing helioseismic data obtained

Fourier-Hankel Decomposition

Braun (1988) demonstrated that waves are absorbed in sunspots. Technique could potentially be used to infer flow amplitudes and solar structure below the spot.

Absorption of oscillations by sunspots



Egression power map of active region (Braun & Lindsey 1999, ApJ)

Fourier-Legendre Analysis

Basic Idea:
Decomposition of the wave signal in ***incoming and outgoing*** waves

$$\Phi(R_\odot, \theta, \phi, t) = \sum_{L,m,\nu} \mathrm{e}^{i(m\phi - \nu t)} [A_{L,m,\nu} H_m(L\theta) + B_{L,m,\nu} H_m^*(L\theta)]$$

$H_m(L\theta)$

$L^2 = l(l+1)$

Hankel functions in plane geometry or in spherical geometry:
AssociatedTravelling-wave Legendre functions
(Nussenzveig 1965)

$$H_m(L\theta) \propto P_l^m(\theta) \pm \frac{2i}{\pi} Q_l^m(\theta)$$

P_l^m Legendre function of 1st kind

Q_l^m Legendre function of 2nd kind

Fourier-Legendre Decomposition (FLD)

Under some constraints, Hankel functions are orthogonal.
Wave field can be decomposed into ingoing and outgoing field:

Decomposition of the wave field:

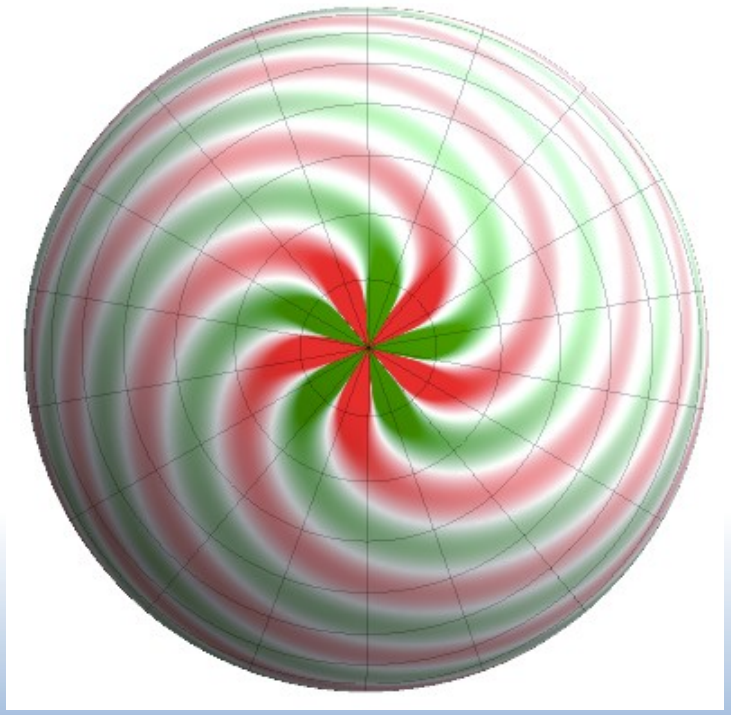
$$A_m(L,\nu) \propto \int_0^T dt \int_0^{2\pi} d\phi \int_{\theta_{min}}^{\theta_{max}} \theta d\theta \, \Phi(\theta,\phi,t) H_m^*(L\theta) e^{-i(m\phi+2\pi\nu t)}$$

$$B_m(L,\nu) \propto \int_0^T dt \int_0^{2\pi} d\phi \int_{\theta_{min}}^{\theta_{max}} \theta d\theta \, \Phi(\theta,\phi,t) H_m(L\theta) e^{-i(m\phi+2\pi\nu t)}$$

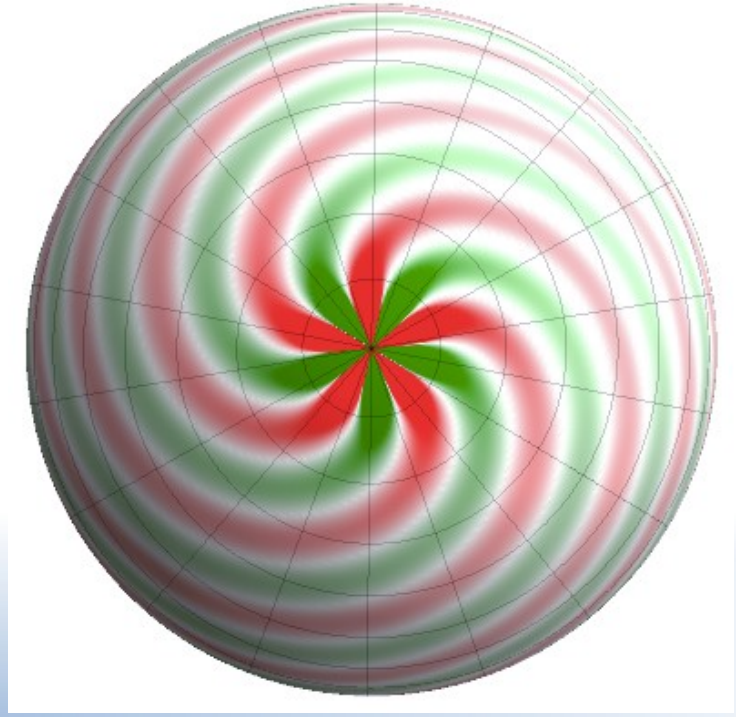
Two steps: Hankel decomposition and Fourier transform

Fourier-Hankel/Legendre Decomposition

inward

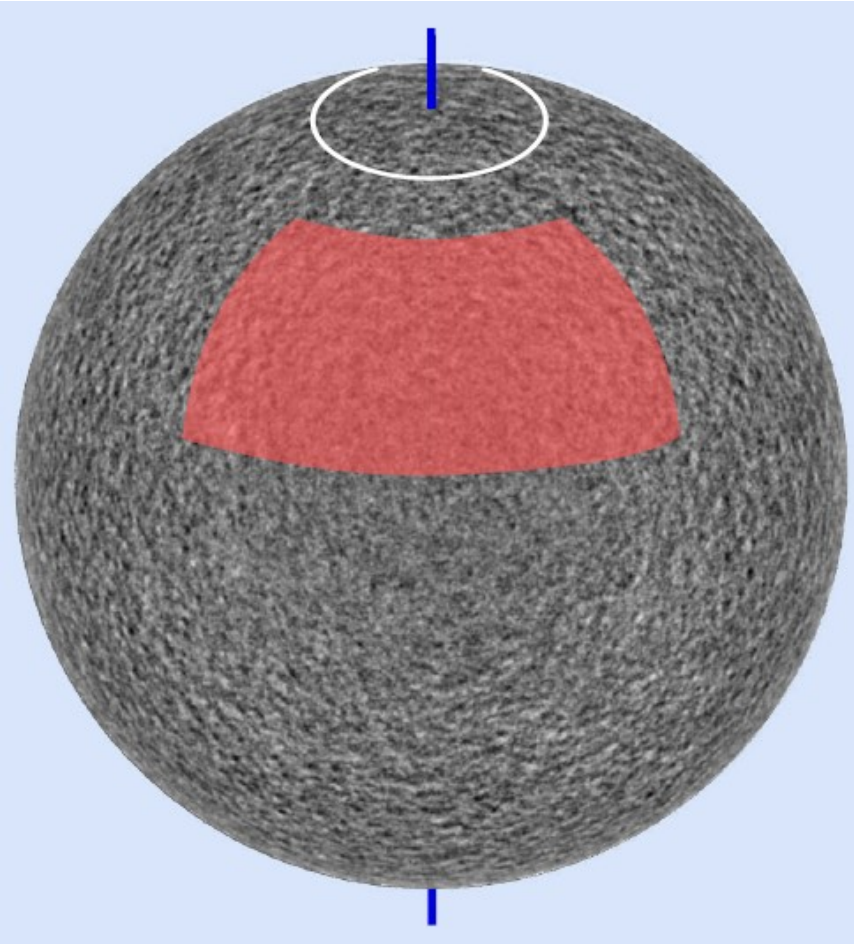


outward



(Source: Vigeesh Gangadaharan)

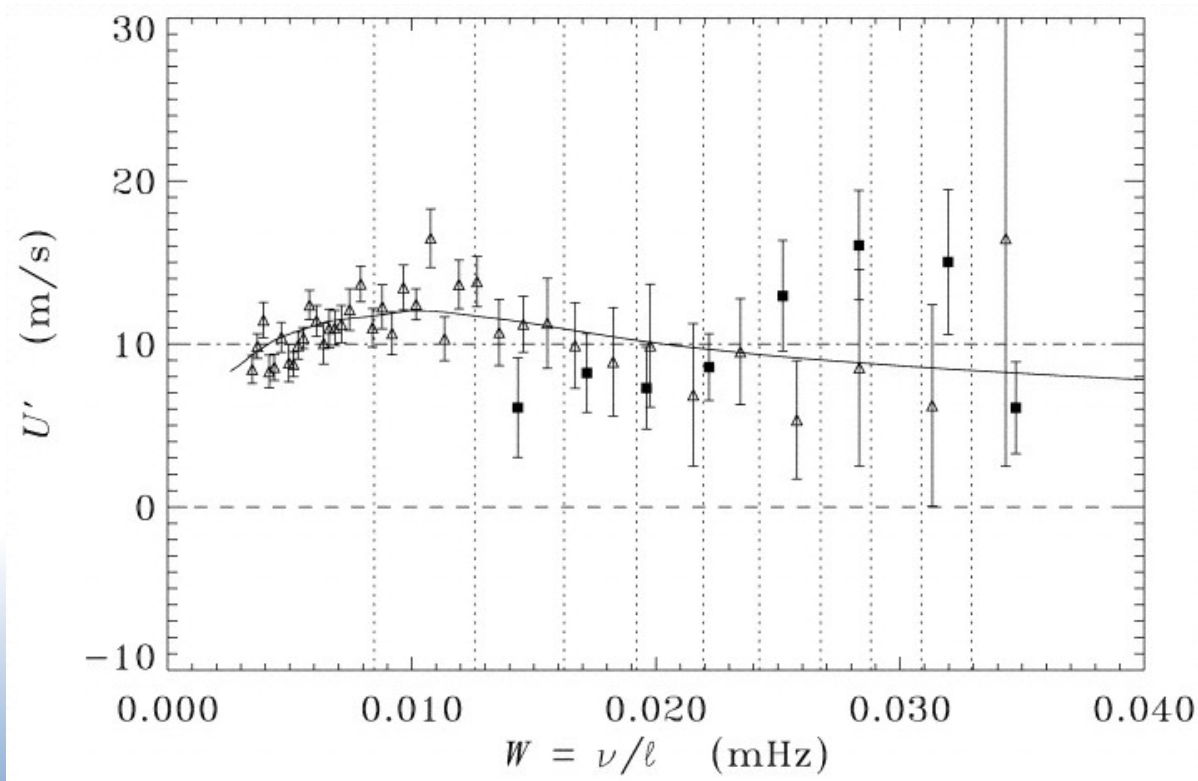
Fourier-Legendre Decomposition for Meridional Flow Measurements



Meridional Flow seen by Fourier-Hankel Decomposition

Braun & Fan, 1998:

Result: Close to the surface poleward flow with ¼ 5 – 15 m/s



mode turning point

Gough & Toomre (1983)
Large-scale flow results in frequency shift:

$$\Delta\nu_{nl} = \frac{l}{\pi R_\odot} \int \bar{U}_{\mathrm{mer}}(r) K_{nl}(r)\, \mathrm{d}r$$

Flow amplitude estimated by:

$$\bar{U}_{\mathrm{mer}} = \pi R_\odot \Delta\nu_{nl}/l$$

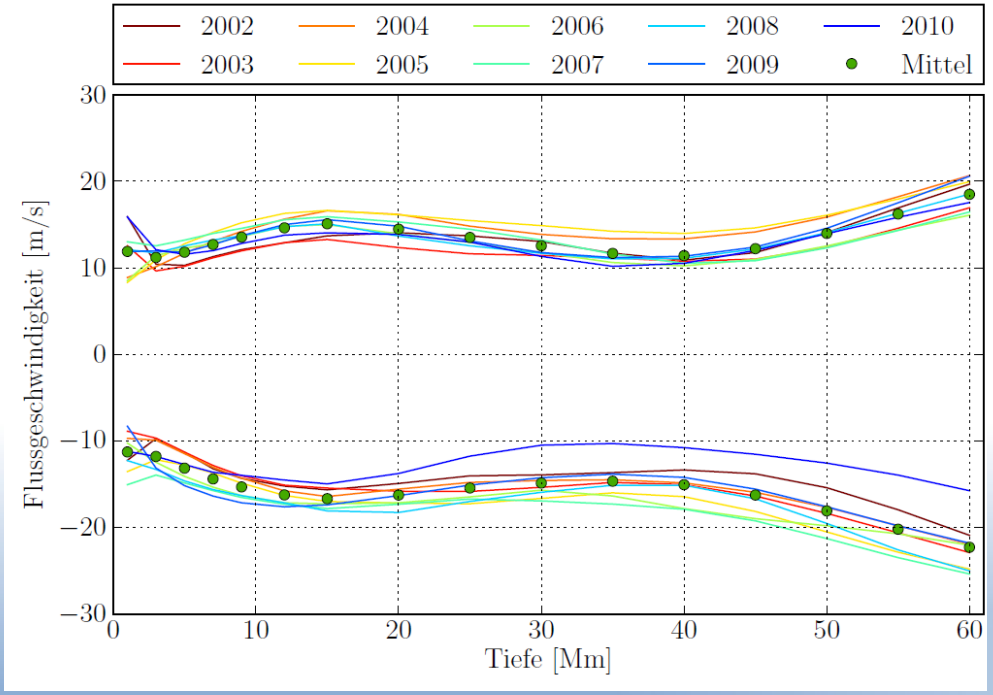
No real inversion!
Acceptable only for shallow layers

Not correct for meridional flow (see Roth & Stix, 2008):
no frequency shift in first order perturbation theory

Extension of the method to probe greater depths

Application of inversion methods

- **GONG 2002 – 2010 (K. Glogowski)**



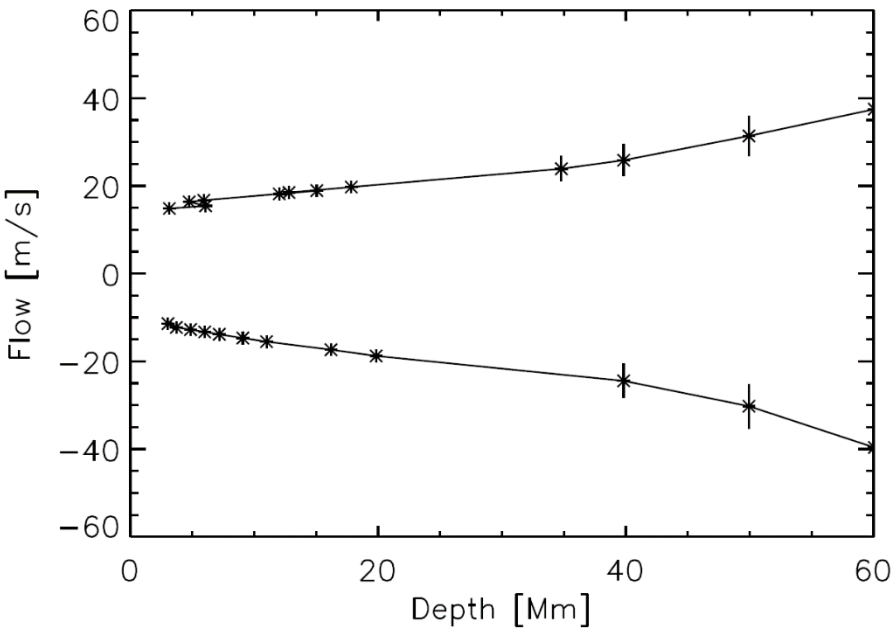
$$\Delta\nu_{nl} = \frac{l}{\pi R_\odot} \int \bar{U}_{\mathrm{mer}}(r) K_{nl}(r)\, \mathrm{d}r$$

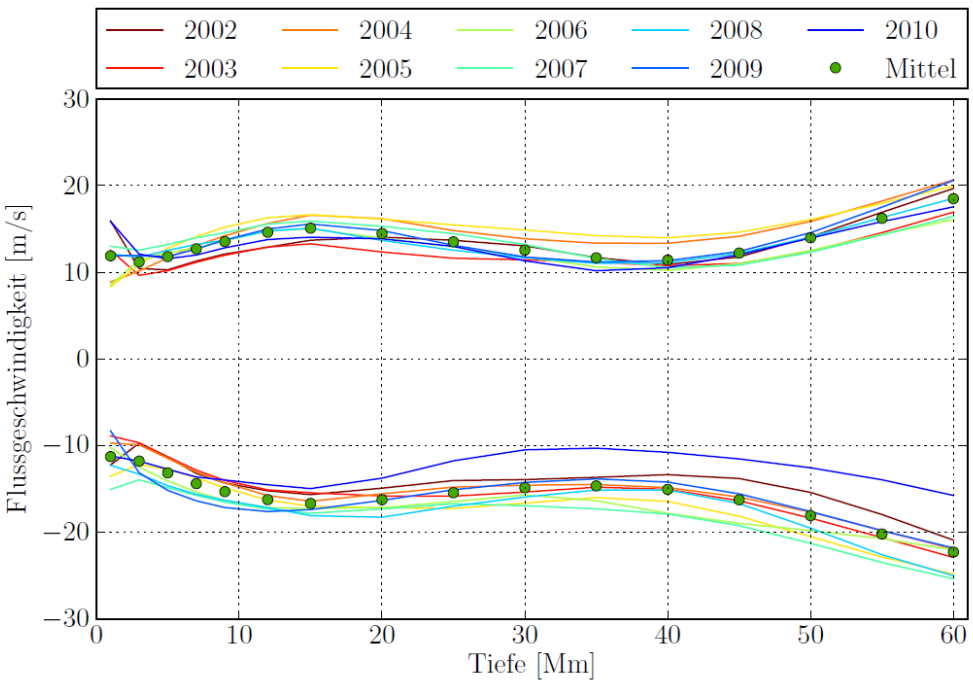
Approxmation:
Kernels $K_{nl}(r)$ as mode kinetic energy density

Meridional Flow Measurements – Frontside View

GONG 2002 – 2010 (K.

Simulation

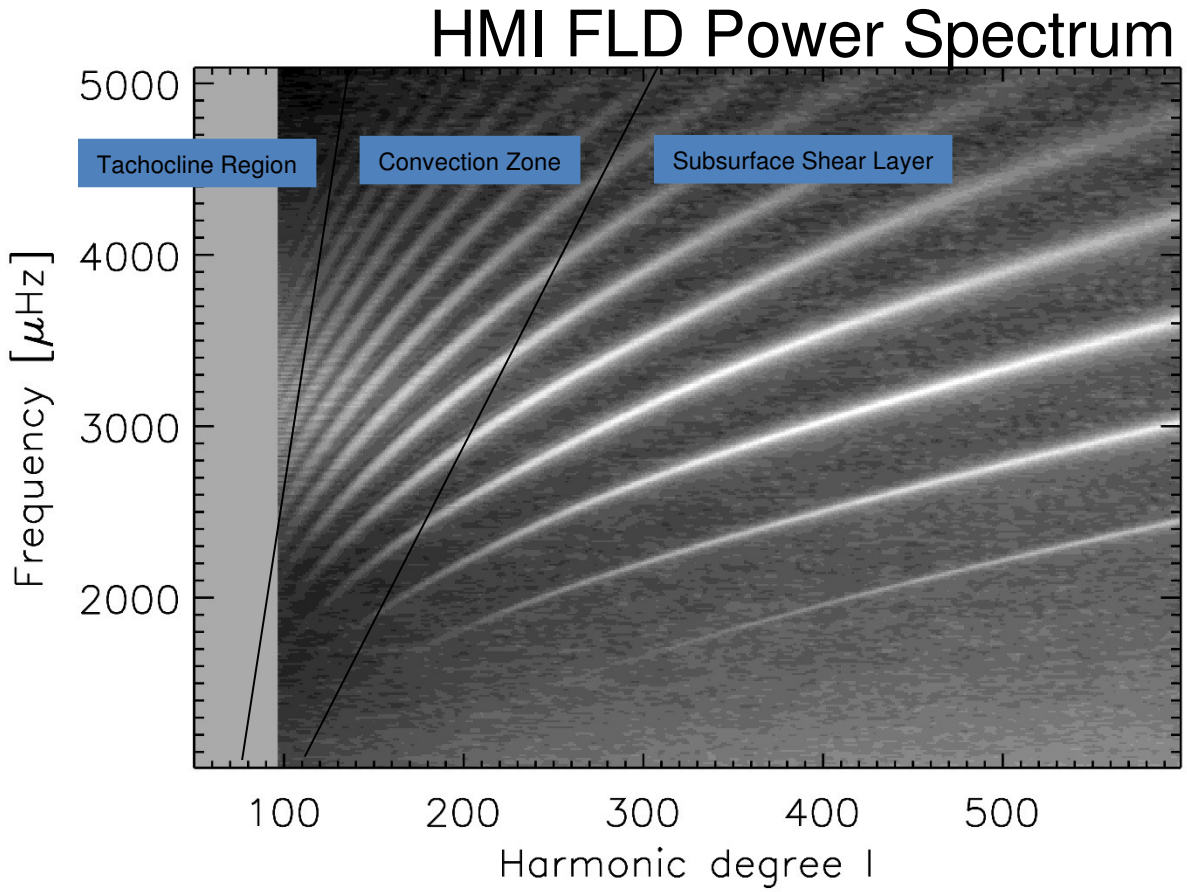




Depth down to 60 Mm can be probed – increase in amplitude w
greater depths seems to be an systematic effect of the method

Depth Information of Acoustic Waves

Power Spectra of Fourier-Legendre decomposed Dopplergrams

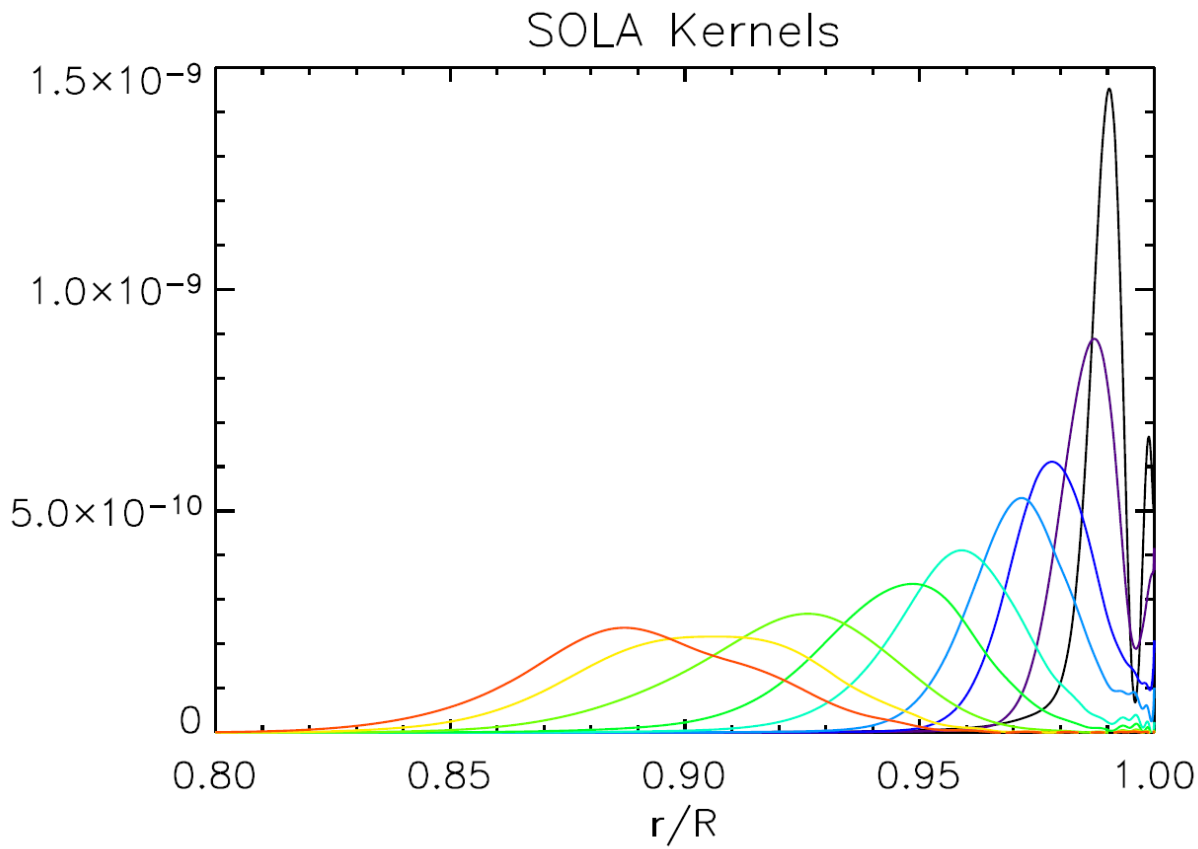


2 weeks of HMI data

m-average:
for m = -25 ⋯ +25

Sensitivity Kernels for the Meridional Flow

- **Fourier–Legendre Decomposition (code from HP Doerr)**
  - with 2 weeks of HMI data
  - Potential to probe to the base of the convection zone w



(Doerr & Roth, 2011)

Ways to Simulate Solar Oscillations

Realistic, compressible MHD Simulations:

- Conservation equations for mass, momentum, energy; plus equations for magnetic field, and radiation
- Allows studying excitation and propagation of acoustic waves, as well as convective flows, magnetic structures, etc.
- High numerical cost; limited to relatively small domains (e.g. 3D box with size of maybe 100 Mm horizontally near solar surface)

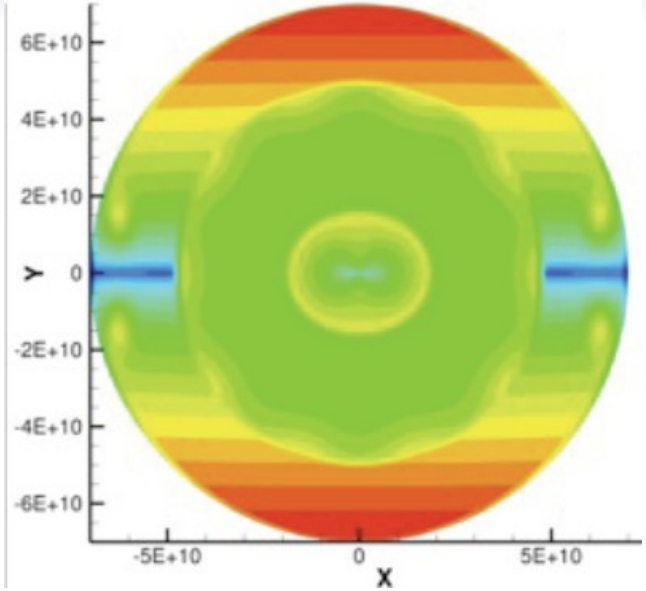
Simulations of Linearized Wave Propagation:

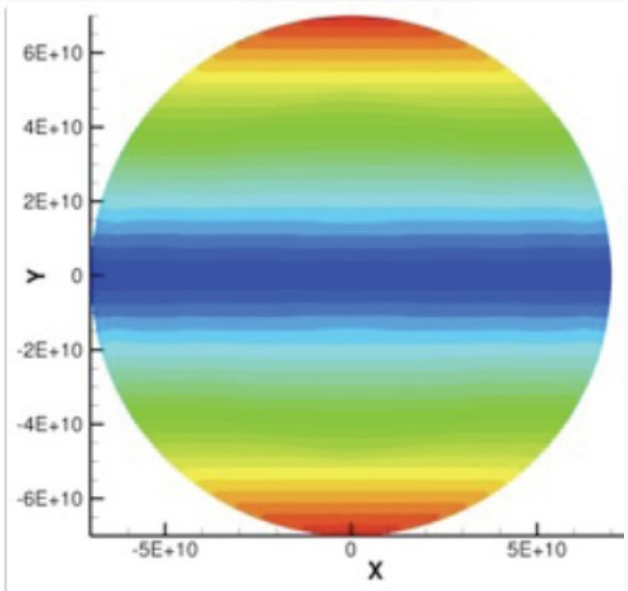
- Linearized equations for propagation of waves; in most of the Sun, waves can be considered small perturbations from a quasi-steady background state
- Artificial structures such as models of sunspots or flows and their effect on acoustic waves can be studied
- Numerical cost is low enough for 3D simulations of the whole Sun

! carried out by Thomas Hartlep at Stanford University

Start-up Project: Measuring Flows

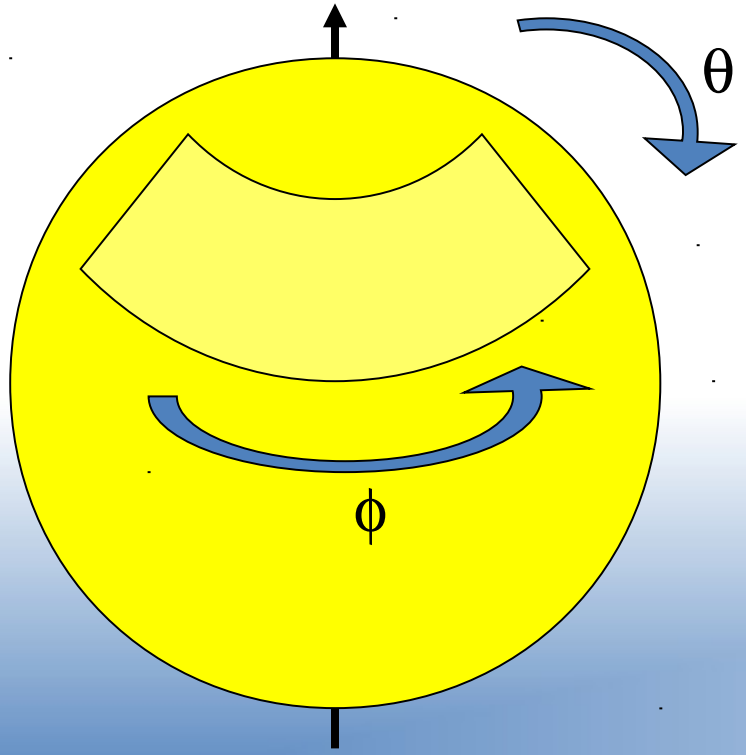
- Simulation with artificially prescribed models of differential rotation and meridional flow
- Relatively low seismic resolution: harmonic degree $0 \leq l \leq 170$
- 51 hours of simulation data with high flow amplitude of 500 m/s to mimic S/N ratio of observations of 3.65 ye





Validation of Meridional Flow Measurements with FLD

- **Center of annulus located at the poles**
  - Only a section of annulus is observed
  - As larger the area as deeper one can probe



Three setups:

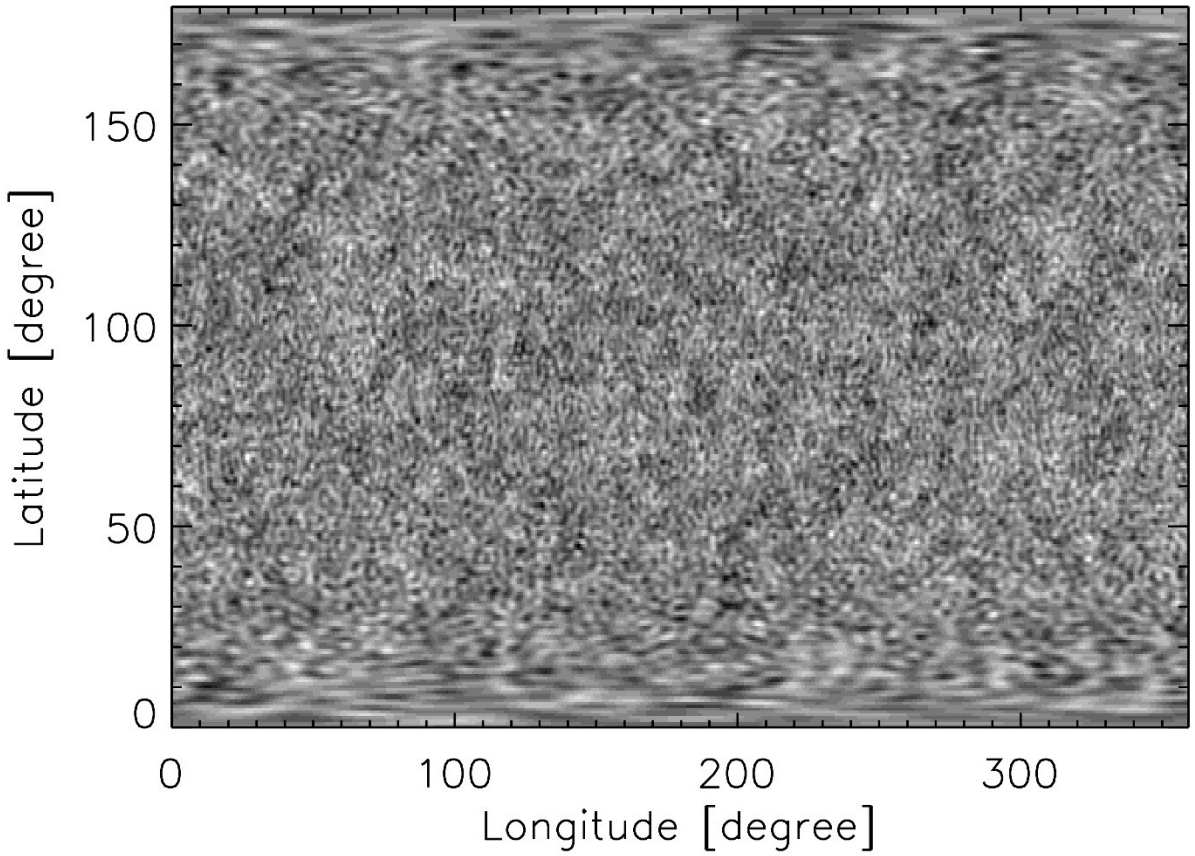
1.Ideal Observations of northern and southern hemispheres:
Two patches with angular dimensions of 360° x 90° (lon x lat)

2.Combination of front- and backside:
Four patches with the dimensions 120° x 60°.
Two on frontside and two on the farside at ±35° latitude.

3.Standard helioseismology, frontside view only:
Two patches on the frontside located at ±35° latitude.

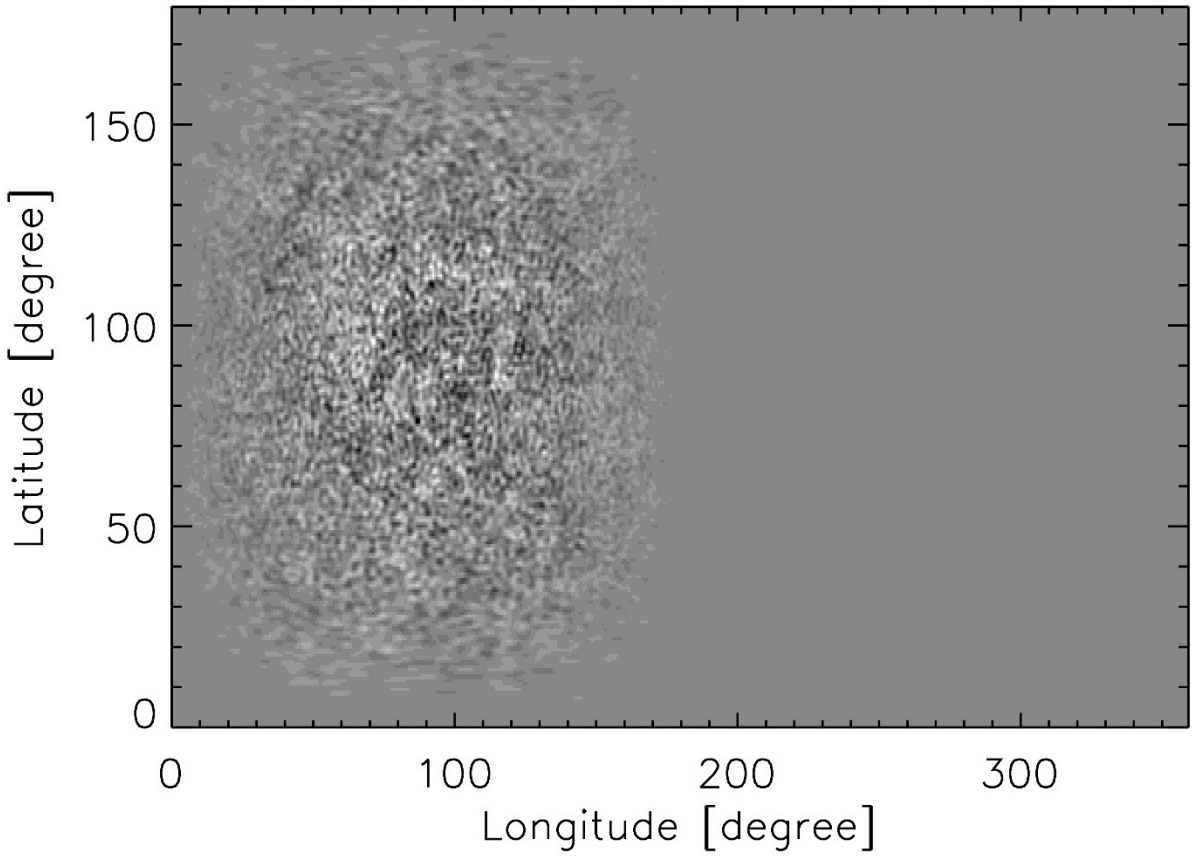
Artificial Data

- **Optimal case observation of the full sun**



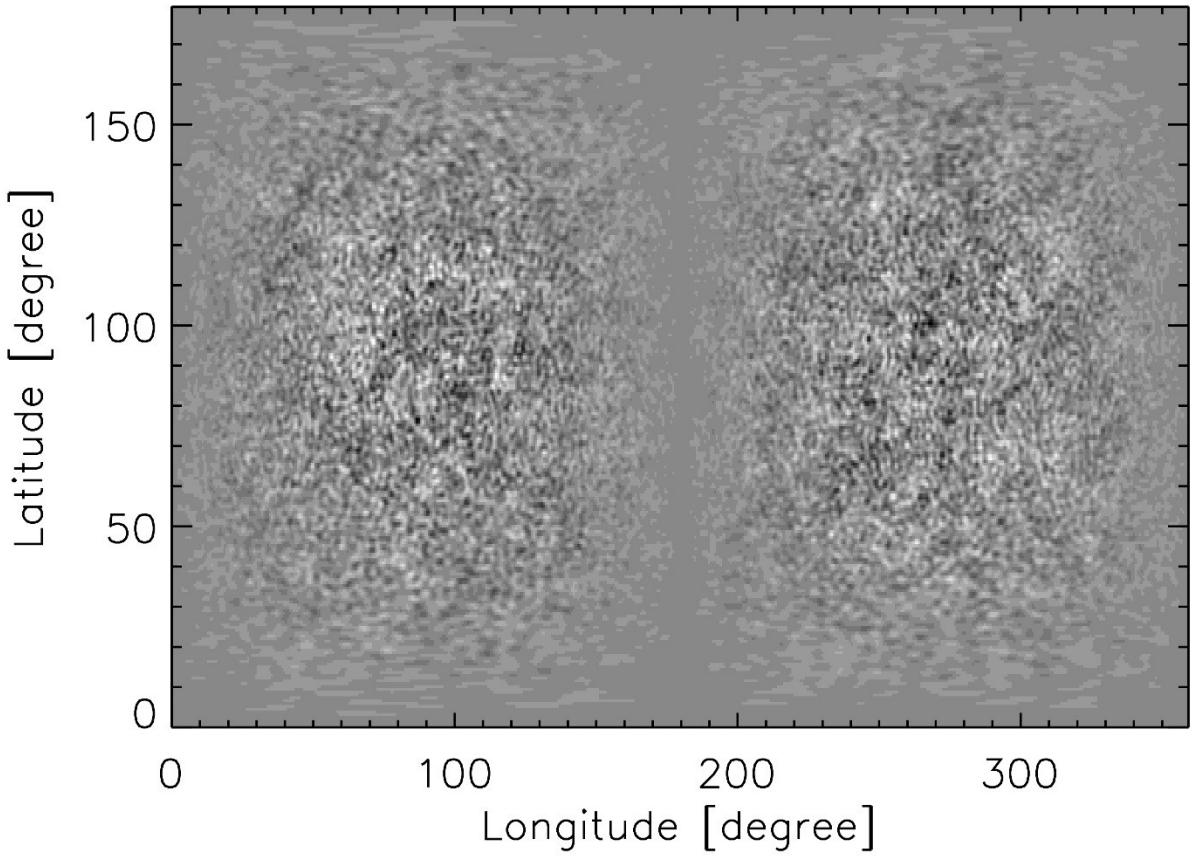
Artificial Data

- **Current Situation – Front side only**



Artificial Data

- **Frontside Data + Solar Orbiter Data (in ecliptic)**



Result: Power Spectra

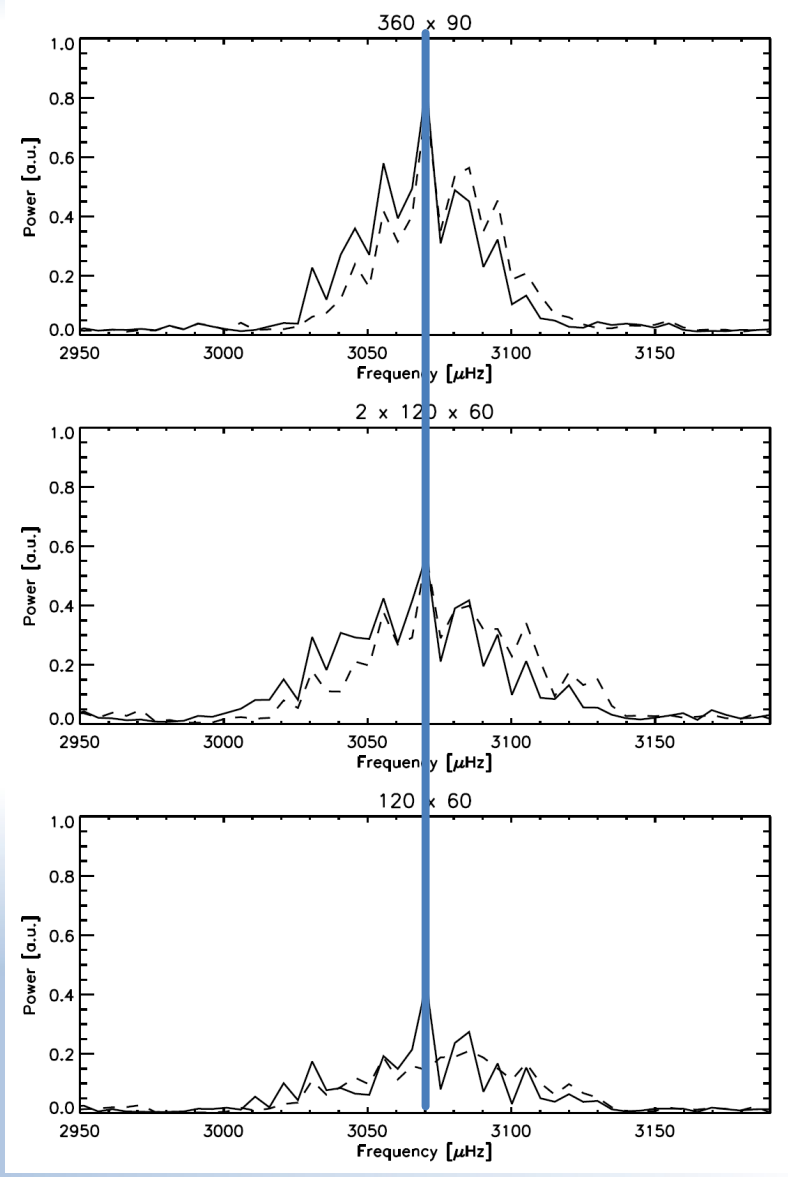
Power of one particular mode (l=90, n=7) for the three setups.

Visible effect of leakage, i.e. incomplete observations cause side lobes.

Redistribution of power becomes worse with smaller observation area.

Central peak is not shifted, meridional flow causes *asymmetric power redistribution*

- This could be interpreted as frequency shift.



Result: Meridional Flow Measurements

Inversions for the meridional flow for the three observational setups

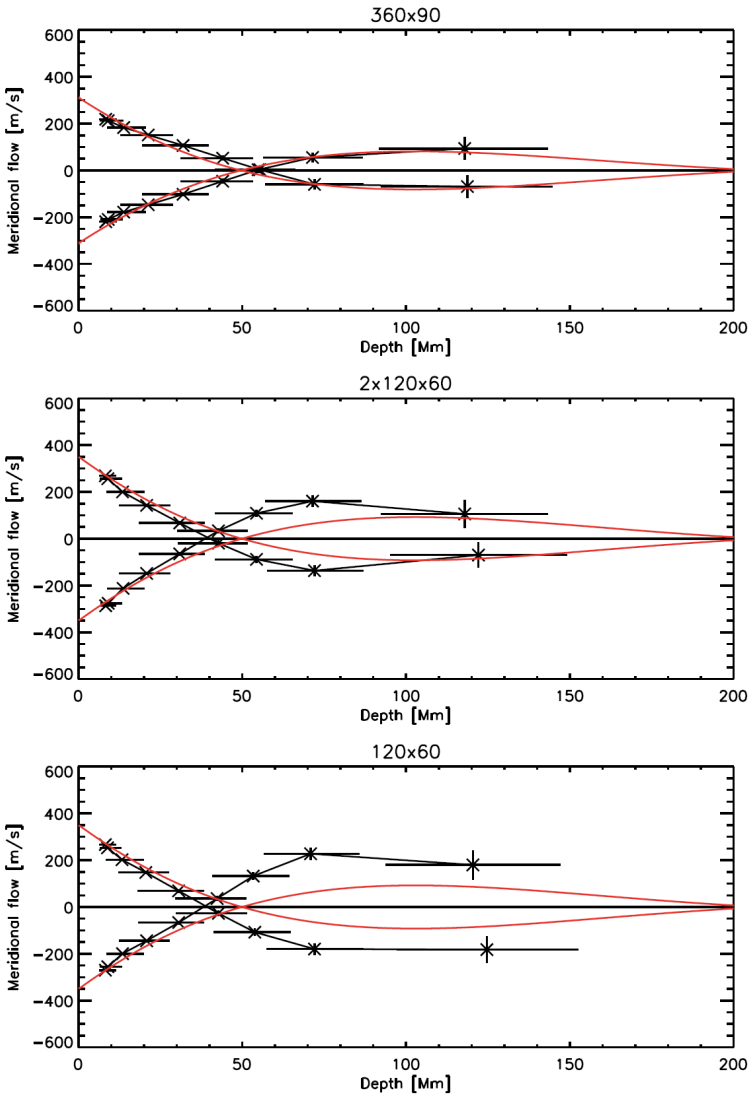
Red line: true input flow

Asteriks with error bars: inversion results based on the given procedure

With smaller observation area measurements in the deeper layers get worse.

Agreement between inversion and actual flow is good down to ~30Mm.

Disagreement worst ~50–80 Mm due to missing side views



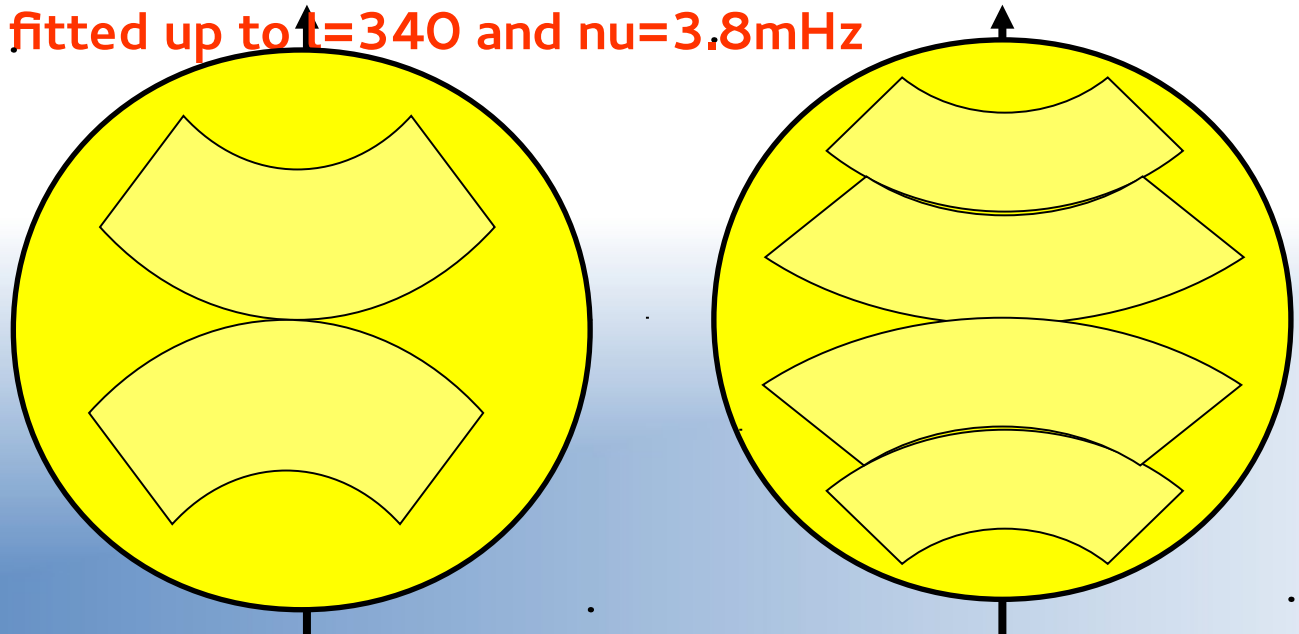
New Simulation Data

Analysis of data (a,b,c) available from Thomas Hartlep's webpage:

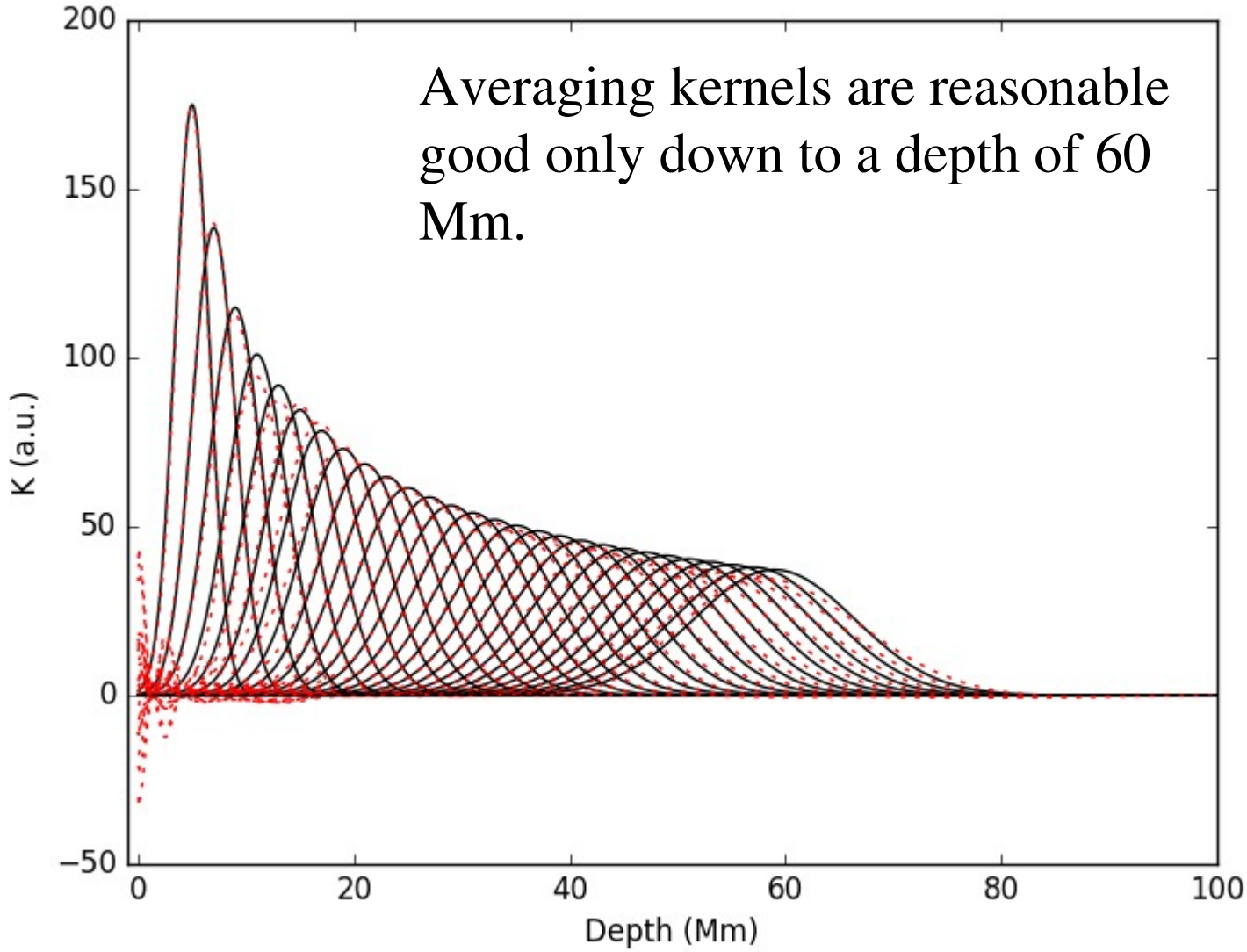
- 2x120°x60°degree patches centered ±30° and on opposite sides of the Sun (180° degree separated in longitude)

- 2x 2x 120°x40° degree patches at ±20° & ±60° and on opposite sides of the Sun

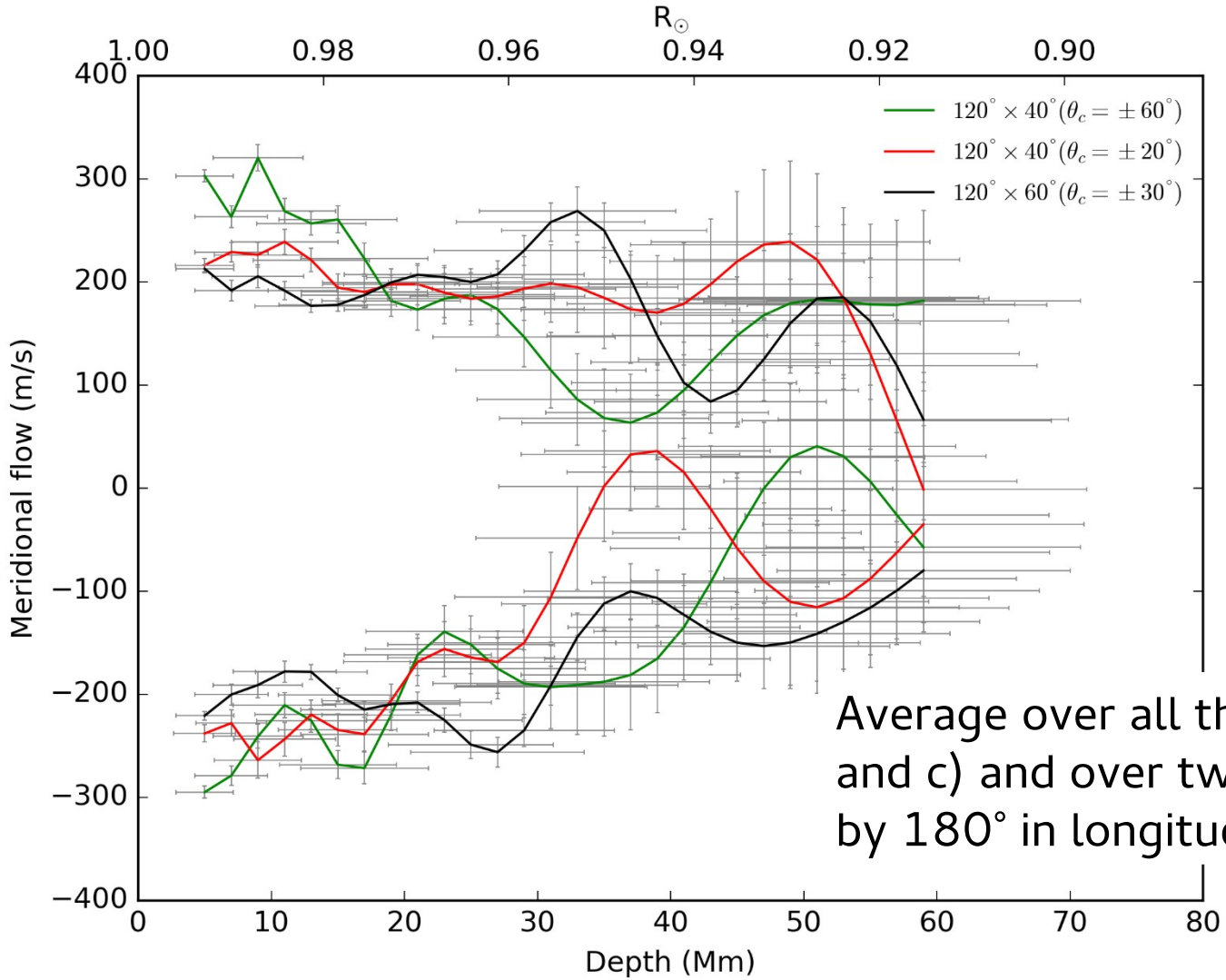
- Modes fitted up to l=340 and nu=3.8mHz



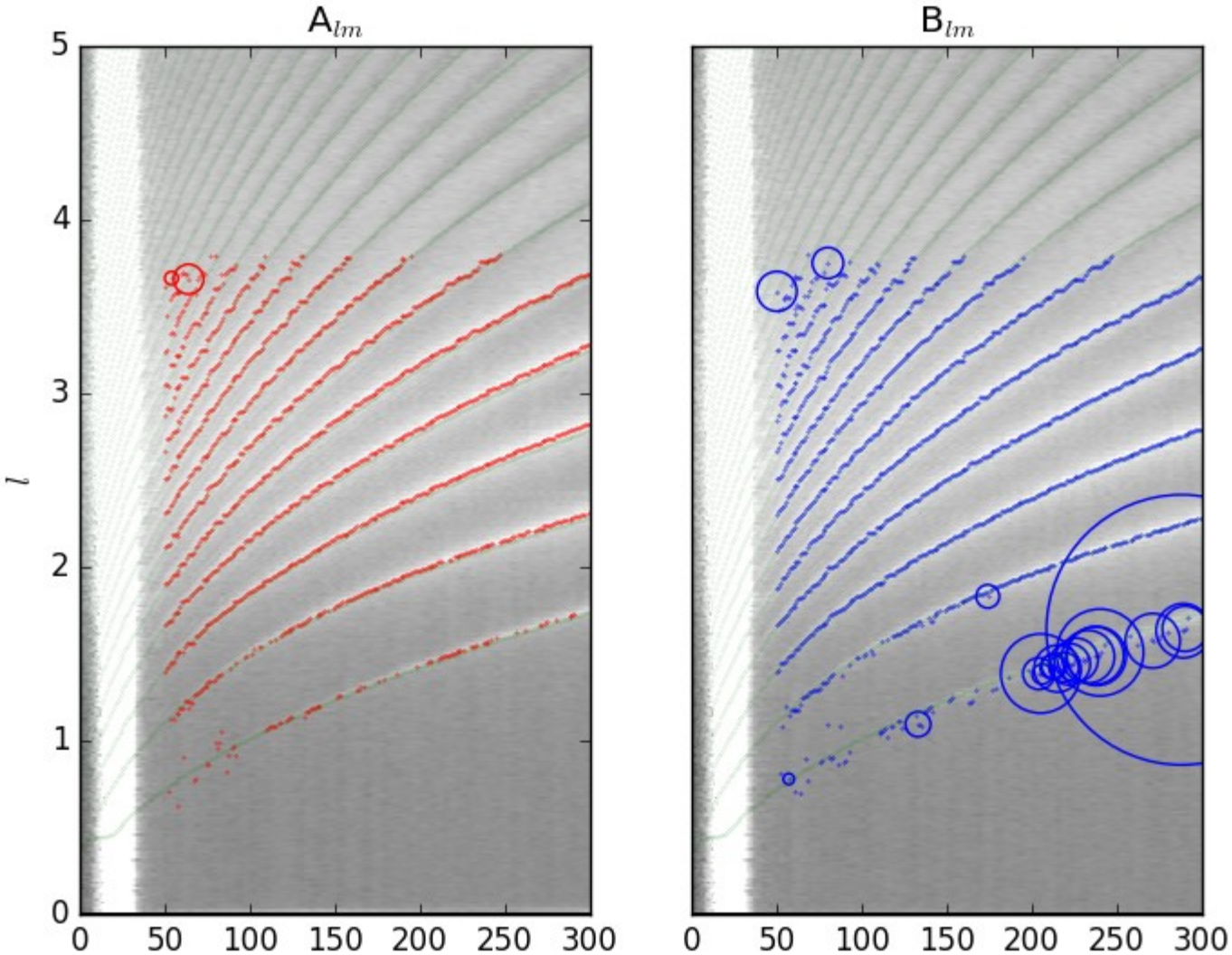
Averaging Kernels



Inversion Results



Average over all three data sets (a,b, and c) and over two patches separated by 180° in longitude


A_{lm}
B_{lm}
l
0
1
2
3
4
5
0
50
100
150
200
250
300

Conclusions

- **Helioseismic Fourier-Legende Analyis technique was validated**
- **Flow could be recovered if large observing areas are available**
  - Techniques to do so exist today, c.f. Solar Orbiter
- **Required observing time: 3.65 years**
  - based on this simple numerical simulation
  - More advanced simulations are needed
- **Leakage effect needs to be smoothed out to generate a spurious frequency shift**
  - Currently, better methods to measure meridional flow in greater depths exist, i.e. time-distance and eigenfunction perturbation analysis
- **Next Step of Improvement:**
  Correct calculations of Kernels for power redistribution
  - Taking leakage effect into consideration