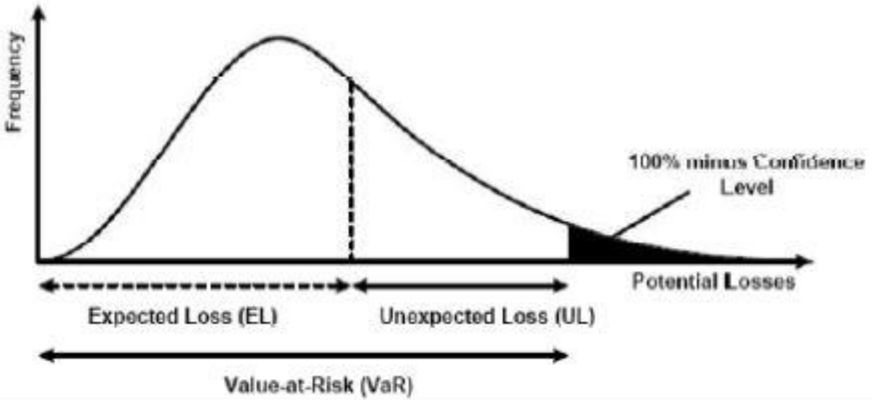
VaR Introduction I: Parametric VaR

Summary

- VaR Definition
- VaR Roles
- VaR Pros and Cons
- VaR Approaches
- Parametric VaR
- Parametric VaR Methodology
- Parametric VaR Implementation
- VaR Scaling
- VaR Backtest

Value at Risk (VaR) Definition

- The maximum likely loss on a portfolio for a given probability defined as *x%* confidence level over N days
- Pr(*Loss* > *VaR*(*x%*)) < 1- *x%*



VaR Roles

- Risk measurement
- Risk management
- Risk control
- Financial reporting
- Regulatory and economic capital

VaR Pros & Cons

- Pros
  - Regulatory measurement for market risk
  - Objective assessment
  - Intuition and clear interpretation
  - Consistent and flexible measurement
- Cons
  - Doesn’t measure risk beyond the confidence level: tail risk
  - Non sub-additive

Three VaR Approaches

- Parametric VaR
- Historical VaR
- Monte Carlo VaR

The presentation focuses on parametric VaR.

Parametric VaR

- Assumption
  - Asset returns follow normal distribution
- Pros
  - Fast and simple calculation
  - Intuitive
- Cons
  - Poor accuracy for non-linear products
  - Second order approximation
  - Hard to incorporate stress test

Parametric VaR Methodology

- Assuming an asset return/valueChange follows normal distribution, the quantile of 99% confidence level is 2.326σ where σ is standard derivation
- If absolute return $X_1 - X_0$ is normally distributed, the 99% worse change of X is $X_1 - X_0 = 2.326\sigma$
- The VaR is given by $VaR = \frac{\partial F}{\partial X}\Delta X = \frac{\partial F}{\partial X} \times 2.326 \times \sigma$ where $\frac{\partial F}{\partial X}$ is the delta
- Similarly for a relative return $\frac{X_1 - X_0}{X_0}$, the VaR can be expressed as

$$VaR = \frac{\partial F}{\partial X}\Delta X = \frac{\partial F}{\partial X}(X_1 - X_0) = \frac{\partial F}{\partial X} \times X_0 \times 2.326\sigma$$

Parametric VaR Implementation

- For each asset/instrument/riskFactor, calibrate volatility σ_i based on daily return
- For each risk factor pair, calibrate correlation ρ_{ij}
- Calculate the variance of a portfolio value change

$$V_p^2 = \lfloor \Delta(P_1)\sigma_1 \quad \dots \quad \Delta(P_n)\sigma_n \rfloor \begin{bmatrix} \rho_{11} & \cdots & \rho_{1n} \\ \vdots & & \vdots \\ \rho_{n1} & \cdots & \rho_{nn} \end{bmatrix} \begin{bmatrix} \Delta(P_1)\sigma_1 \\ \vdots \\ \Delta(P_n)\sigma_n \end{bmatrix}$$

- The portfolio VaR is $2.326\sqrt{V_p^2}$

VaR Scaling

- Normally firms compute 1-day 99% VaR
- Regulators require 10-day 99% VaR
- Under IID assumption, 10-day VaR = $\sqrt{10} * VaR_{1-day}$

VaR Backtest

- The only way to verify a VaR system is to backtest
- At a certain day, compute hypothetic P&L. If (hypothetic P&L > VaR) ➔ breach, otherwise, ok
- Hypothetic P&L is computed by holding valuation date and portfolio unchanged
- In one year period,
  - If number of breaches is 0-4, the VaR system is in Green zone
  - If number of breaches is 5-9, the VaR system is in Yellow zone
  - If number of breaches is 10 or more, the VaR system is in Red zone

Thanks!

You can find more online presentations at

https://finpricing.com/lib/CdCreditSpreadCurve.html