Interest Rate Capped Swap Valuation and Risk
Capped Swap

Summary

- Capped Swap Definition
- Floored Swap Definition
- Valuation
- A real world example
Capped Swap Definition

- A capped swap is an interest rate swap with a cap where the floating rate of the swap is capped at a certain level.
- It limits the risk of the floating rate payer to adverse movements in interest rates.
- Given the optionality, an up-front fee or premium has to be paid by the floating rate payer.
- A capped swap can be decomposed as an interest rate swap plus an interest rate cap.
A floored swap is an interest rate swap with a floor where the floating rate of the swap is floored at a certain level.

It limits the risk of the floating rate receiver to adverse movements in interest rates.

Given the optionality, an up-front fee or premium has to be paid by the floating rate receiver.

A floored swap can be decomposed as an interest rate swap plus an interest rate floor.
There are four types of capped or floored swaps.

- Capped payer swap
- Capped receiver swap
- Floored payer swap
- Floored receiver swap

The present value of a capped payer swap is given by:

$$PV_{\text{CappedPayerSwap}}(t) = PV_{\text{float}}(t) - PV_{\text{fixed}}(t) - PV_{\text{cap}}(t)$$

where

- $PV_{\text{float}}$ is the present value of the floating leg of the underlying swap;
- $PV_{\text{fixed}}$ is the present value of the fixed leg of the underlying swap;
- $PV_{\text{cap}}$ is the present value of the embedded cap.
The present value of a capped receiver swap can be expressed as

\[ PV_{\text{CappedReceiverSwap}}(t) = PV_{\text{fixed}}(t) - PV_{\text{float}}(t) + PV_{\text{cap}}(t) \]

The present value of a floored payer swap can be represented as

\[ PV_{\text{FlooredPayerSwap}}(t) = PV_{\text{float}}(t) - PV_{\text{fixed}}(t) + PV_{\text{floor}}(t) \]

Where \( PV_{\text{floor}} \) is the present value of the embedded floor.

The present value of a floored receiver swap can be computed as

\[ PV_{\text{FlooredReceiverSwap}}(t) = PV_{\text{fixed}}(t) - PV_{\text{float}}(t) - PV_{\text{floor}}(t) \]
The present value of the fixed leg is given by

\[ PV_{\text{fixed}}(t) = RN \sum_{i=1}^{n} \tau_i D_i \]

where \( R \) – the fixed rate; \( N \) – the notional; \( \tau_i \) – the day count fraction for period \( [T_{i-1}, T_i] \); \( D_i = D(t, T_i) \) – the discount factor.

The present value of the floating leg is given by

\[ PV_{\text{float}}(t) = N \sum_{i=1}^{n} (F_i + s)\tau_i D_i \]

where \( s \) – the floating spread; \( F_i = F(t; T_{i-1}, T_i) = \frac{1}{\tau_i} \left( \frac{D_{i-1}}{D_i} - 1 \right) \) – the simply compounded forward rate.
The present value of the cap is given by

\[ PV_{\text{cap}}(t) = N \sum_{i=1}^{n} \tau_i D_i (F_i \Phi(d_1) - K \Phi(d_2)) \]

where \( d_{1,2} = \left( \ln \left( \frac{F_i}{K} \right) \pm 0.5 \sigma_i^2 T_i \right) / (\sigma_i \sqrt{T_i}) \) and \( \Phi \) – the cumulative normal distribution function.

The present value of the floor is given by

\[ PV_{\text{cap}}(t) = N \sum_{i=1}^{n} \tau_i D_i \left( K \Phi(-d_2) - F_i \Phi(-d_1) \right) \]
# Capped Swap

## A real world example

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<thead>
<tr>
<th>Cap/Floor specification</th>
<th>Underlying swap specification</th>
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<tbody>
<tr>
<td><strong>Buy Sell</strong></td>
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Thanks!

Reference:
https://finpricing.com/lib/EqLookback.html