Pricing Quanto Himalayan Option

We present analytics for pricing quanto European-style Himalayan options on equity (stock or index), where the single best return is locked in each fixing period. Specifically, we considered the impact of the quanto adjustment term on calibration and the computation of option premium and hedge ratios.

The equity price model is based on a discrete dividend treatment and results in shifted lognormal distributions for the equity price. A calibration step is required to obtain the required shifted-lognormal volatility parameters from the Black's term implied volatility inputs. Monte Carlo technique is employed to compute the price and the hedge ratios.

We assume that the capital gains processes associated with the individual equities follow respective risk-neutral geometric Brownian motions with time dependent volatility. Discrete dividend payments are modelled directly as a component of the capital gains process.

Like other exotic option, Quanto Himalayan may also have cap floor constraints and barrier conditions. The barrier option may knock out the trade before maturity, referred to as https://finpricing.com/lib/EqBarrier.html

For a set of d equities (stocks or equity indexes), let their respective prices at time t be Y1(t), . . . , Yd(t). Given a collection of fixing dates, t0, t1, . . . , td, let the equity returns Ri,j , for i = 1, . . . , d, j = 1, . . . , d be defined as

$$R_{i,j} = \frac{Y_i(t_j)}{Y_i(t_0)}.$$

Moreover, let the integer-valued function k : {1, . . . , d} 7! {1, . . . , d} be defined so as to satisfy the following condition, for i = 1, . . . , d

$$R_{k(i),i} = \max_{j \in \{1,\ldots,d\} \setminus \bigcup_{l=1}^{i} \{k(l)\}} R_{j,i}.$$

In other words, k(i) is the index of the equity with the highest return, at the fixing date ti, among those equities, whose returns at the previous fixing dates were not the highest2. Given a strike level, K, the Himalayan option payoff is

$$\max\left(\sum_{i=1}^{d} (R_{k(i),i} - K), 0\right).$$

The delta hedge ratio for i-th equity is defined as

$$\frac{\partial V}{\partial \mathbf{Y}_i(t)},$$

where V is the option price and Yi(t) is the price of the i-th equity on valuation date. The cross-gamma hedge ratio for i-th and j-th equities is defined as

$$\frac{1}{\mathbf{Y}_i(t)} \frac{\partial V^2}{\partial \mathbf{Y}_i(t) \partial \mathbf{Y}_j(t)}.$$

The vega hedge ratio for i-th equity is defined as

$$\frac{1}{100} \sum_{j=1}^{d} \frac{\partial V}{\partial v_{B,i}(t_j)},$$

where vB,i(tj) is the tj-term Black's volatility for equity i.

This definition is consistent with the one in the case of domestic equities, that is, in the absence of a quanto adjustment. In the quanto case, the effect of the change in the quanto adjustment resulting from a change in the volatility is ignored.

The rho hedge ratio for i-th equity is defined as

$$\frac{1}{10000}\sum_{j=1}^{d}\frac{\partial V}{\partial R_i(t_j)},$$

where Ri(tj) is the tj-term spot interest rate for the currency of equity i.

The theta hedge ratio is defined as

$$V(t+1) - V,$$

where V (t + 1) is the option premium computed with the valuation date, t, shifted forward by one day.

With the exception of the theta, the hedge ratios are computed using Malliavin weights. The presence of the volatility calibration step requires an adjustment in the calculated Malliavin hedge ratios, according to the chain rule.

Let X(_) be the d-dimensional vector of logarithms of the d gains processes. The vector process X(_) will be assumed to follow the SDE

$$
\begin{aligned}
d\mathbf{X}(\tau) &= \left(\mathbf{r}(\tau) - \mathbf{q}(\tau) - \frac{1}{2}\boldsymbol{\sigma}^2(\tau)\right)d\tau + \mathbf{D}(\tau)\mathbf{L}\,d\mathbf{B}(\tau),\ t < \tau,\\
\mathbf{X}(t) &= \mathbf{x}_t,\ \text{a.s.,}
\end{aligned}
$$

Let the known dividend schedule for equity i consist of payments _i,1, . . . , _i,mi made at times _i,1, . . . , _i,mi , all of which lie between the valuation date t and the last relevant fixing tn. Furthermore, let the discount curve in the native currency of equity i be di, that is, for any time s greater than the valuation time t, let the discount factor be di(s − t).For times t _ s1, t _ s2, let us define di(s1, s2) = di(s2)/di(s1). We shall define

$$
\begin{aligned}
\tilde{D}_i(s) &= \sum_{j=1}^{m_i} \kappa_{i,j} d_i(s, \tau_{i,j}) \mathbf{I}_{[t,\tau_{i,j}]}(s),\ \text{and}\\
\tilde{\mathbf{D}}(s) &= (\tilde{D}_1(s), \ldots, \tilde{D}_d(s))'.
\end{aligned}
$$

We employ the definitions of the respective hedge ratios, as stated in the section on the WM pricing method. With the exception of the theta, calculated through the finite difference technique, all hedge ratios are computed using the Malliavin weight approach. Additional considerations arise from the impact of the calibration procedure on the sensitivity ratios, as describe in the next section.

Moreover, as can be seen from the test results below, the relative error in the gamma hedge ratios can be quite large. These quantities are generally very small, making the relative error a poor indicator of agreement. Moreover, it appears that the Monte Carlo sampling error is quite large for the gamma hedge ratio.

For a large number of paths, this ratio should be approximately normally distributed with mean zero and unit variance, if the means in the two models are equal. We believe that this is a more meaningful way of comparing results whose Monte Carlo sampling error is large relative to the size of their expectation.