Local Volatility Model for Quanto Options

We present a model for calculating the price of European call and put options in the domestic currency on an underlying foreign equity with tenor up to 7 years. The calculation includes option price, Delta, Gamma, Hedge Rho, Discount Rho, Vega, Theta.

We employed three calibration schemes for valuation. One scheme determines a constant exchange rate correlation parameter by matching with Balck's forward equity price dynamics. The function implements a local volatility based pricing method.

The payoff in the domestic currency of a European option at maturity T is given by

$$\left(\psi\left(S_T - K\right)\right)^+ ,$$

where

- ψ is +1 for a call option and -1 for a put option,
- S_T is the foreign spot equity level at time T, and
- K is a strike level.

The equity price process satisfies a risk-neutral stochastic differential equation (SDE) when where are no dividend payments. Let St denote the equity price at time t. We assume that the process satisfies a SDE of the form under the domestic risk-neutral probability measure:

$$dS_t = S_t\left[\left(r_t + \hat{q}_t\right)dt + \sigma(S, t; S_0)dW_t\right],$$

where

- r_t is a deterministic short-interest rate,
- $\hat{q}_t$ is a deterministic quanto-adjustment level,
- $\sigma(S, t; S_0)$ is the equity price local volatility, and
- W_t is a standard Brownian motion.

We note that the volatility σ depends only on time and on the instantaneous value of the state variable S, but does not explicitly depend on W.

The quanto-adjustment q is calculated by

$$\int_0^t \hat{q}_s ds = \rho_{fx}\sigma_{fx}\sigma_B(S_0, t)t,$$

where

- ρ_{fx} is an equity-exchange rate correlation parameter,
- σ_{fx} is a particular exchange rate volatility parameter, and
- $\sigma_B(S_0, t)$ is a Black's implied volatility associated with the at-the-money (ATM) strike level, S_0, and tenor, t, such that market call and put prices c and p satisfy,

  $$c = S_0N(d_1) - e^{-\int_0^T r_s ds}KN(d_2) \quad \text{and} \quad p = e^{-\int_0^T r_s ds}KN(-d_2) - S_0N(-d_1),$$

  respectively, where

  $$d_1 = \frac{\ln(S_0/K) + \sigma_B(S_0, t)^2T/2}{\sigma_B(S_0, t)\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma_B(S_0, t)\sqrt{T}.$$

Note that we also implement a second quanto-adjustment technique that is of the form

$$\int_0^t \hat{q}_s ds = \rho_{fx}\sigma_{fx} \int \sigma_B(K,t)\omega_K dK,$$

where w_k is a weight.

Consider the calibration of the local volatility function based on market option prices or, equivalently, market Black's implied volatilities. If there exists a smooth surface of either option price or implied volatility as a function of option strike and maturity, then this surface uniquely determines the local volatility function.

Moreover, an explicit expressions for local volatility is provided. For example, if C denotes the price of a call option on a non-dividend paying stock, with a constant risk-free interest rate, then

$$\sigma(K,T;S_0) = 2\frac{\frac{\partial C}{\partial T} + Kr\frac{\partial C}{\partial K}}{K^2\frac{\partial^2 C}{\partial K^2}}.$$

Local volatility model can also be applied to value callable exotic notes. Callable option is more volatile as callable events make the remaining part of the trade potentially be cancelled as a result of a trigger condition or an exercise option. You can find a good illustration on the topic of callable notes at https://finpricing.com/lib/EqCallable.html