Pricing Amortizing Floor Option

An amortizing floor option consists of 12 floorlets, or put options, on the arithmetic average of the daily 12-month Pibor rate fixings over respective windows of approximately 30 calendar days. Furthermore the notional amount corresponding to each floorlet is specified by an amortization schedule.

We consider a floor option consisting of a series of floorlets as follows. Here each floorlet is specified by

- settlement time, T,
- set of Pibor fixing times, $\{\tau_i\}$, such that $0 < \tau_1 < ... < \tau_m < T$,
- payoff at T of the form

$$N \times \Delta \times \left(X - \frac{1}{m} \sum_{i}^{m} L_i \right)^+$$

  where

  - L_i denotes the δ-period Pibor rate that sets at τ_i,
  - N is an FRF notional amount,
  - Δ is an accrual period.

For each period in the tables above, a party is short a floorlet specified by

- strike, 3.55%,

- underlying interest rate equal to the arithmetic average of the 12 month Pibor-rate daily fixings in the averaging window, $\frac{1}{m}\sum_{i}^{m} L_i$,
- payment at settlement date of the form

$$N \times \Delta \times \left(3.55\% - \frac{1}{m}\sum_{i}^{m} L_i\right)^{+}$$

where

  - N is a corresponding notional amount,
  - m is the number of daily fixings in the averaging window,
  - Δ is the interval of time from the accrual start date to the settlement date calculated according to the ACT/360 day-counting convention.

Consider a floorlet with associated Pibor rate averaging window, and let T denote the corresponding settlement date. We assume that, for each reset in this window, the associated 12-month forward Pibor rate, L, satisfies an SDE, of the form

$$dL = L\sigma \, dW ,$$

under the T-forward probability measure, where

- σ is a constant volatility parameter,
- W is a standard Brownian motion.

We note that, mathematically, the Pibor rates above cannot *simultaneously* be martingales under the common T-forward probability measure; moreover, in order to simultaneously express the Pibor rates above under this same measure, the SDE above requires a drift correction term.

Let $Z = \frac{1}{m}\sum_{i}^{m} L_i$ be the arithmetic average of the 12-month Pibor rates over the daily fixings in the averaging window above. Since each Pibor rate is log-normal, Z cannot also be log-normal. For computational speed, then, we approximate Z by a random variable,

$$\hat{Z} = \left(\frac{1}{m}\sum_{i}^{m} L_i^0\right) e^{-\frac{\sigma^2}{2}\hat{T} + \sigma \hat{W}_{\hat{T}}},$$

where

- $\hat{T} = \frac{1}{m}\sum_{i}^{m} T_i$,
- $\sigma = \sqrt{\frac{1}{\hat{T}}\left(\frac{1}{m}\sum_{i}^{m} \sigma_i^2 T_i\right)}$,
- $\hat{W}$ is a standard Brownian motion.

Here, with respect to the i^{th} fixing point in the averaging window,

- T_i denotes the reset time,
- σ_i is the forward Pibor rate volatility,
- L_i^0 denotes the forward Pibor rate.

From the above (see Appendix A) we see that $E(Z) = E(\hat{Z})$ and $E(Z^2) \approx E(\hat{Z}^2)$.

The random variable Z can then be viewed as a moment matching approximation to $\hat{Z}$.

We now approximates the price of a floorlet by

$$N \times \Delta \times d_T \times E^T\left[\left(3.55\% - \hat{Z}\right)^+\right]$$

where

- Δ is the interval from the accrual start date to the settlement date,
- d_T is the discount factor to the settlement date, and
- N is a Euro notional amount.

We compute, over a specified averaging window,

- the average Pibor fixing time, $\hat{T} = \frac{1}{m}\sum_i^m T_i$,
- the average forward Pibor rate, $\frac{1}{m}\sum_i^m L_i^0$,
- an average volatility, $\sqrt{\frac{1}{\hat{T}}\frac{\sum_i^m \sigma_i^2 T_i}{m}}$.

Here the forward Pibor rate volatility, σ_i, is taken from a corresponding Euro forward swap rate volatility curve. Furthermore Pibor forward rates are calculated from a curve sheet of EURIBOR discount factors (see https://finpricing.com/lib/IrBasisCurve.html).

Consider a European style option of the form,

- maturity, T,
- underlying, L,
- strike, X,
- payoff at maturity, $(X - L_T)^+$.

We calculate, by an analytical formula,

$$d_T\, E^T\left[(X - L_T)^+\right] \qquad (3.3.3.1)$$

where

- d_T denotes the discount factor to time T,
- $L_T = L_0 e^{-\frac{\sigma^2}{2}T + \sigma W_T}$, under the T-forward measure, with
  - σ a constant volatility parameter, and
  - W a standard Brownian motion.

If the underlying asset, L, is a Δ- period Libor rate, then L is a martingale under the $T + \Delta$**-forward measure.** In this case *black_opt* should be called with a zero interest rate to maturity, so that d_T in (3.3.1) reduces to 1; the resulting option price can then be scaled by the discount factor to $T + \Delta$.

With respect to our deal, Tranche 1, we apply *black_opt* to calculate

$$E^T\left[\left(3.55\% - \hat{Z}\right)^+\right]$$

where $\hat{Z}$ is defined in Section 3.2; here the interest rate to maturity is set to zero.

We note that the *black_opt* addin list of formal parameters includes a string input, “C” or “P”, to respectively denote call or put option calculation. In the WM spreadsheet implementation, however, the option calculation parameter input is not of the required form above. It appears from numerical tests, however, that in this case the addin defaults to a put option calculation. To be consistent with the addin’s formal parameter types, we suggest modifying the spreadsheet’s corresponding input to

If (M5, “C”, “P”).

Let τ be a fixing time for a δ - period Pibor rate, L, in an averaging window (see Figure 4.1 below). We wish to express L under the $T-$forward probability measure. In Appendix A we show that

$$dL = L\left(-\lambda\,\gamma\, dt + \lambda\, dW\right)$$

where $\gamma = \dfrac{\upsilon \sigma l}{1+\upsilon l}$. Here l denotes the forward Pibor rate that sets at $\tau + \delta$ for the accrual period $\upsilon = T - (\tau + \Delta)$; furthermore, under the $T-$forward measure,

$$d\,l = l\,\sigma\,dW\,.$$

Figure 4.1. Pibor fixing points.

Using the technique above, we now simultaneously express all the Pibor rate fixings in the averaging window under the common $T-$forward probability measure. We then evaluate

$$N \times E^{T}\left[\left(3.55\% - \frac{1}{m}\sum_{i}^{m} L_i\right)^{+}\right] \qquad (4.1)$$

based on an Euler discretization scheme. Next we scale the numerical value for Formula (4.1) by $\Delta\, d_T$.