Pricing Partial Payoff Swap

Partial payoff swap pays periodically, the payoff from a particular European style put option on the spread between respective ten and two-year CMS rates. Moreover, this payoff is algebraically equivalent to the sum of the spread above and the payoff from a related European style put option.

Let

- S_{10} denote a swap rate for a swap specified by
  - ten year maturity,
  - 6-month JPY Libor paid semi-annually, in arrears,
  - a fixed rate paid semi-annually,
- S_2 represent a swap rate for a swap specified by
  - two year maturity,
  - 6-month JPY Libor paid semi-annually, in arrears,
  - a fixed rate paid semi-annually.

Here one party must pay, semi-annually,

$$N \times \Delta \times \max\left(S_{10}(T) - S_2(T) + 2.15\%, 0\right),$$

at time $T + \Delta$, where

- N is a 1,000,000,000 JPY notional amount,
- Δ is an accrual period,

In addition the party receives period payments based on JPY Libor.

Let $X = 2.15\%$ be a strike level. Recall that one party must pay, periodically,

$$N \times \Delta \times \max\left(S_{10} - S_2 + X, 0\right).$$

Moreover,

$$\max\left(S_{10} - S_2 + X,\ 0\right) = \left(S_{10} - S_2 + X\right) + \max\left(-X - \left(S_{10} - S_2\right),\ 0\right). \tag{3.1}$$

We note that the price is

$$N \times \Delta \times \max\left(-X - \left(S_{10} - S_2\right), 0\right),$$

which can be viewed as the payoff from a European style put option specified by

- strike, $-X$,
- underlying security, $S_{10} - S_2$.

The remaining term, $N \times \Delta \times \left(S_{10} - S_2 + X\right)$, is valued.

Let

- T denote a reset time,
- $T + \Delta$ be the corresponding payment time.

We assume that the forward swap rate process, $\left\{S_{10}(t) \middle| \; 0 < t \le T\right\}$, satisfies under the T-forward probability measure an SDE, of the form

$$dS_{10} = S_{10}\sigma dW \, , \qquad (3.1.1)$$

where

- σ is a constant volatility parameter,
- W is a standard Brownian motion.

Here $S_{10}(0)$ is a timing and convexity adjusted, forward swap rate; the forward swap rate, convexity and timing adjustments are respectively computed.

Note that the forward swap rate process above may be assumed to satisfy an SDE of the form (3.1.1) under a corresponding *forward swap measure*; moreover, the forward swap rate will then not be log-normally distributed under the T-forward probability measure.

Let

- T denote a reset time,
- $T + \Delta$ be the corresponding payment time,
- $P(t,T)$ represent the price at time t of a zero coupon bond that matures at T.

Then

$$\begin{aligned} &P(0,T+\Delta)E^{T+\Delta}\left[\max\left(-X - \left(S_{10}(T) - S_2(T)\right), 0\right)\right] \\ &\quad = P(0,T)E^{T}\left[\frac{\max\left(-X - \left(S_{10}(T) - S_2(T)\right), 0\right)}{1 + \Delta L(T;T,T+\Delta)}\right], \end{aligned} \qquad (4.1.1)$$

where

- E^{T} and $E^{T+\Delta}$ respectively denote expectation under the T and $T+\Delta$-forward probability measure,
- $L(t;T,T+\Delta)=\frac{1}{\Delta}\left(\frac{P(t,T)}{P(t,T+\Delta)}-1\right)$ is a forward JPY Libor rate that sets at T for the accrual period Δ.

Assume that, under the $T+\Delta$-forward measure, W_1, W_2 and W_3 are independent, standard Brownian motions. Moreover, assume that

$$d\,L(t;T,T+\Delta)=L(t;T,T+\Delta)\sigma\,dW_1(t)$$

where σ is a constant volatility parameter. Furthermore let

$$\gamma(t)=\frac{\Delta L(t;T,T+\Delta)}{1+\Delta L(t;T,T+\Delta)}\sigma.$$

From the above, under the T-forward probability measure,

$$\tilde{W}_1(t)=W_1(t)-\int_0^t \gamma(s)ds,\ W_2(t) \text{ and } W_3(t)$$

are independent, standard Brownian motions.

Assume that, under $T+\Delta$-forward measure,

$$\left\{W_t^1 \middle| 0\le t\le T+\Delta\right\}$$

and

$$\left\{ W_t^2 \middle| 0 \le t \le T + \Delta \right\}$$

are independent, standard Brownian motions. Observe that

$$\left.\frac{dQ^T}{dQ^{T+\Delta}}\right|_t = \frac{P(0,T+\Delta)}{P(0,T)}\frac{P(t,T)}{P(t,T+\Delta)}.$$

Let

$$L(t;T,T+\Delta) = \frac{1}{\Delta}\left(\frac{P(t,T)}{P(t,T+\Delta)} - 1\right),$$

and assume that

$$d\,L(t;T,T+\Delta) = L(t;T,T+\Delta)\sigma dW_t^1 .$$

Let $X_t = \dfrac{P(t,T)}{P(t,T+\Delta)}$; then

$$dX_t = X_t \gamma(t) dW_t^1$$

where

$$\gamma(t) = \frac{\Delta L(t;T,T+\Delta)}{1 + \Delta L(t;T,T+\Delta)}\sigma,$$

$$= \frac{\Delta L(0;T,T+\Delta)e^{-\frac{\sigma^2}{2}t + \sigma dW_t^1}}{1 + \Delta L(0;T,T+\Delta)e^{-\frac{\sigma^2}{2}t + \sigma dW_t^1}}\sigma.$$

From the above, under T-forward probability measure,

$$\tilde{W}_t^1 = W_t^1 - \int_0^t \gamma(s) ds$$

and

$$W_t^2$$

are independent, standard Brownian motions. Let

- S_t^{10} denote a forward swap rate, which sets at time T with respect to an underlying 10 year swap,
- S_t^{2} denote a forward swap rate, which sets at time T with respect to an underlying 2 year swap.

Assume that, under T-forward probability measure,

$$d\log S_t^{10} = -\frac{\nu^2}{2}dt + \nu d\tilde{W}_t^1,$$
$$d\log S_t^{2} = -\frac{\omega^2}{2}dt + \omega\left(\sqrt{1-\rho^2}dW_t^2 + \rho d\tilde{W}_t^1\right)$$

Then, under $T+\Delta$-forward measure,

$$d\log S_t^{10} = -\left(\frac{\nu^2}{2} + \nu\gamma(t)\right)dt + \nu dW_t^1,$$
$$d\log S_t^{2} = -\left(\frac{\omega^2}{2} + \rho\omega\gamma(t)\right)dt + \omega\left(\sqrt{1-\rho^2}dW_t^2 + \rho dW_t^1\right)$$

References:

https://finpricing.com/lib/EqCallable.html