Pricing GIC Option

The GIC price is the sum of the price of closed GIC and the price of a put option with time-varying strike. We assume that the GIC holder receives deterministic payments at the specified payment dates and observe how the redemption option price changes due to changes in the number of the HW tree time slices.

We note that the option price depends critically on the HW volatility level. We develop a technique to calibrate the HW volatility for GIC pricing. The idea is to associate a European swaption specification to the particular GIC specification. The HW volatility can then be determined by matching the HW model price for the swaption to the swaption's market price. We note that this technique may be highly sensitive to the selection of the associated swaption; moreover, this selection must reflect the hedging strategy for the GIC embedded option.

Consider a GIC specified by

- maturity, T ,
- set of future payment times, $\{t_i\}_{i=1}^{N}$, where $t_1 < ... < t_N = T$.

Let

- c_{Cst} be the annualized customer coupon rate,
- c_{Tr} be the transfer coupon rate,
- f_c be the coupon rate compounding frequency,
- f_p be the coupon payment frequency (see table[1]),

We define an "equivalent simple annualized rate", which we denote EAR:

$$EAR_{Cst} = f_p \left[\left(1 + \frac{c_{Cst}}{f_c} \right)^{\frac{f_c}{f_p}} - 1 \right], \tag{1a}$$

$$EAR_{Tr} = f_p \left[\left(1 + \frac{c_{Tr}}{f_c} \right)^{\frac{f_c}{f_p}} - 1 \right] \tag{1b}$$

The payment P_i at time t_i is then

$$P_i = principal \times EAR_{Tr} \times (t_i - t_{i-1}), \tag{2}$$

where the accrual period, $t_i - t_{i-1}$, is calculated using the ACT/365 day counting convention. We have previously reviewed the generation of the payment dates, t_i, and the respective payments, P_i,

Let $d_{Tr}(t)$ denote the price at valuation time of a zero coupon bond, based on the cost of funds rates, with maturity t and unit face value. The closed GIC transfer price, pv_{Tr}, is given by

$$pv_{Tr} = \sum_i d_{Tr}(t_i) P_i + d_{Tr}(t_N) \times principal, \tag{3}$$

where the summation is over the remaining payment dates. To be specific, we first bootstrap a set of Cost of Funds (COF) discount factors $d_{Tr}(\tau_j)$ at the set of fixed times, $\{\tau_j = 0.5j\}_{j=1}^{M}$, where $\tau_1 < ... < \tau_M = T_1$. The discount factor $d_{Tr}(\tau)$, for $\tau_j < \tau < \tau_{j+1}$, where τ_j and τ_{j+1} are consecutive bootsrapping breakpoints, is given by the log-linear interpolation:

$$d_{Tr}(\tau) = \exp\left(-r(\tau)\tau\right),$$

where

$$r(\tau) = r(\tau_j) + \frac{(\tau - \tau_j)\left[r(\tau_{j+1}) - r(\tau_j)\right]}{\tau_{j+1} - \tau_j},$$

and

$$r(\tau_j) = -\frac{\log(d_{Tr}(\tau_j)))}{\tau_j}.$$

To compute the GIC value from the customer's perspective, pv_{cst}, we apply eq. (3) to customer discount factors. The customer discount factor bootstrapping algorithm is analogous to the COF discount factors bootstrapping

Assume that the GIC specified above can be redeemed at time *t* with the call rate r_{call}. We define an equivalent annualized simple call rate by

$$EAR_{call} = f_p\left[\left(1 + \frac{r_{call}}{f_c}\right)^{\frac{f_c}{f_p}} - 1\right]. \qquad (4)$$

Let t_i, where $t_i \leq t$, be the coupon payment date that is immediately prior to time t. The time t redemption value is then

$$R_{Cst}(t) = principal \times \left[1 + (t - t_i) EAR_{Cst} - (EAR_{Cst} - EAR_{call})(t - t_0)\right] \tag{5}$$

where t_0 is the GIC inception time. Note that the term

$$(EAR_{Cst} - EAR_{call})(t - t_0) \tag{6}$$

in eq. (5) represents the penalty interest. For the customer, the intrinsic value of the embedded put (redemption) option is then

$$p_{Cst,\text{int}}(t) = \max\left(R_{Cst}(t) - pv_{Cst}(t), 0\right), \tag{7}$$

where the closed GIC customer price pv_{Cst} is calculated as described in Section 3.1.

Let 1_t be an indicator process such that

$$1_t = \begin{cases} 1, & if\ t\ is\ in\ an\ exercise\ window, \\ 0, & otherwise. \end{cases}$$

Furthermore, let τ be the unique stopping time such that

$$E\left(\left(\frac{1_\tau \left(R_{Cst}(\tau) - pv_{Cst}(\tau)\right)}{e^{\int_t^\tau r_{Cst}(s)ds}}\right)^+ \middle| \Im_t\right) = \sup_{t<\tau<T} E\left(\left(\frac{1_\tau \left(R_{Cst}(\tau) - pv_{Cst}(\tau)\right)}{e^{\int_t^\tau r_{Cst}(s)ds}}\right)^+ \middle| \Im_t\right) \tag{7.a}$$

where

- the expectation is taken under the risk-neutral probability measure, and
- r_{Cst} denotes the customer short rate.

The put option's holding value is then

$$p_{Cst,hld}(t) = E\left(\left(\frac{1_{\tau}\left(R_{Cst}(\tau) - pv_{Cst}(\tau)\right)}{e^{\int_t^{\tau} r_{Cst}(s)ds}}\right)^{+} \middle| \Im_t\right) \tag{8}$$

The put option value at time *t* is

$$p_{Cst}(t) = \max\left(p_{Cst,\text{int}}(t), p_{Cst,hld}(t)\right), \tag{9}$$

where the choice of intrinsic value indicates customer's exercise at the current time.

Next, we define the following indicator process:

$$I_t = \begin{cases} 1, & if\ customer\ execercises\ at\ time\ t, \\ 0, & otherwise. \end{cases}$$

The intrinsic value of the redemption option is

$$p_{Tr,int}(t) = I_t\left(R_{Tr}(t) - pv_{Tr}(t)\right) \tag{10}$$

where

$$R_{Tr}(t) = principal \times \left[1 + (t - t_i) EAR_{Tr}\right] \qquad (11)$$

We note that payoff function (10) is typically discontinuous.

The redemption option cost is then

$$E\left(\frac{p_{Tr,int}(\tau)}{e^{\int_0^{\tau} r_{Tr}(s)ds}} \middle| \Im_0 \right), \qquad (12)$$

where

- r_{Tr} denotes the short rate, and
- τ is the unique stopping time defined by eq. (7a)

We assume that the customer's short interest rates process satisfies a risk-neutral SDE of the Hull-White (HW) form,

$$dr_{cst} = (\theta_{cst}(t) - ar_{cst})dt + \sigma\, dW, \qquad (13)$$

where

- a is a constant mean reversion rate,
- σ is a constant volatility.

We assume that short interest rate, r_{Tr}, satisfies a similar risk-neutral SDE,

$$dr_{Tr} = (\theta_{Tr}(t) - ar_{Tr})dt + \sigma\, dW, \qquad (13.a)$$

where the a and σ parameters and standard Brownian motion W are the same as in

eq. (13). The drift term, $\theta(t)$, for each rate is calibrated to the respective initial interest rate term structure, which is bootsrapped as described in ref. [1]. For this purpose the algorithm requires as inputs

- HW short rate volatility and mean reversion parameter,
- basis yield curve key rates,
- key rate spreads (ref https://finpricing.com/lib/IrBasisCurve.html),
- customer key rate spreads.

We note that equations (13) and (13a) imply that the two short interest rates are perfectly correlated.

We employ the implementation output for customer and Treasury strike levels, given by eq. (5) and (10), respectively. Observe that if the customer and Treasury coupons are set equal, then the two respective strikes obey the following parity relationship:

$$R_{Tr}(t) = R_{Cst}(t) + \left(EAR_{Cst} - EAR_{call}\right)\left(t - t_0\right). \quad (14)$$

We value a Bermudan style put option into-the-tail with a constant strike of $100. The customer payoff from the option upon its exercise is

$$\left(strike - PV\ of\ remaining\ cash\ flows\right)^{+}$$

The Hull-White mean reversion parameter is set to 0.04. Both the benchmark and the Treasury application use 2000 tree time slices.