CAD Government Bond Bootstrapping

An algorithm is presented for bootstrapping a discount factor curve. The bootstrapping procedure uses an input set of instruments with different maturities (i.e., Canadian government money market securities and bonds) to generate successive points on a discount factor curve.

The Canadian zero curves generated will be used to generate particular risk measures, for example DV01's. Moreover, the zero rate curves are not intended for use in pricing (P&L) applications (ref https://finpricing.com/lib/IrInflationCurve.html).

Canadian Government Bonds are traded and quoted based on yield to maturity (YTM). The actual settlement clean price depends on the number of coupons available. For bonds with a single remaining coupon, the bond trades at a pure discount (i.e., like a money market instrument). For bonds with multiple remaining coupons, these are priced with a special formula.

CAD government with a single coupon remaining are quoted using simple interest conventions. In this case, yield is quoted as a money market YTM.

Example: assuming a quoted YTM of 3.0%, a two-day settlement and 120 days to maturity, we compute

$$ {}_2df_{120} = \frac{1}{1 + 0.03\frac{120-2}{365}}, $$

and a dirty price

$$ P_d = {}_2df_{120}\left(100 + C/2\right). $$

The clean price is calculated

$$ P_c = P_d - AccInt. $$

The bond dirty price to trade date, is $({}_0df_2)P_d$.

Consider a CAD government bond with more than one coupon remaining. Let

- N denote the number of remaining coupons,
- E denote the number of days in the first coupon period that includes the settlement date,
- DSC denote the number of days between settlement date and coupon payment, and
- Y denote the semiannual YTM of the bond.

Then, the unadjusted dirty price of the bond (at settlement) is

$$P^*{}_d = \sum_{i=1}^{N} \frac{CF_i}{\left(1+\frac{Y}{2}\right)^{\left(i-1+\frac{DSC}{E}\right)}}.$$

The unadjusted dirty price is used, along with an unadjusted accrued interest, to compute the true clean price. The unadjusted accrued interest is computed

$$AA^* = \frac{E-DSC}{E} \times \frac{C}{2},$$

after which the true clean price is computed

$$P_c = P^*{}_d - AA^*.$$

Finally, the true dirty price may be computed

$$P_d = P_c + AccInt,$$

where $AccInt$ is calculated as described in Equation (A1). Assuming S days to settlement, the bond's dirty price at trade date is $({}_0df_S)P_d$.

To determine discount factors at times intermediate to control points, We apply a particular interpolation technique. There are three available:

- LINEAR
- LOG_LINEAR
- TIME_WEIGHTED_LINEAR

The LINEAR scheme interpolates zero rates linearly between successive control points on the zero curve; that is, if $r(t_1)$ and $r(t_2)$ are bootstrapped continuously compounded zero rates at successive control points, then

$$r(t) = r(t_1) + \left(r(t_2) - r(t_1)\right)\frac{t - t_1}{t_2 - t_1} \text{ where } t_1 \le t \le t_2. \qquad (1)$$

The LOG_LINEAR scheme interpolates linearly between $\ln r(t_1)$ and $\ln r(t_2)$, that is,

$$\ln r(t) = \ln r(t_1) + \left(\ln r(t_2) - \ln r(t_1)\right)\frac{t - t_1}{t_2 - t_1} \quad \text{or}$$

$$r(t) = r(t_1)\left(\frac{r(t_2)}{r(t_1)}\right)^{\left(\frac{t-t_1}{t_2-t_1}\right)}. \qquad (2)$$

The TIME_WEIGHTED_LINEAR scheme interpolates between $r(t_1)t_1$ and $r(t_2)t_2$,

$$r(t) = \frac{1}{t}\left(r(t_1)t_1 + (r(t_2)t_2 - r(t_1)t_1)\frac{t - t_1}{t_2 - t_1}\right), \qquad (3)$$

where $t > 0$ and $t_1 \leq t \leq t_2$. Since $t_2 > t_1$, the TIME_WEIGHTED_LINEAR scheme weights rate information farther in the future more heavily than rate information in the near future.