Cancelable Swap Model

A pricing model is presented for pricing cancelable fixed-for-floating interest rate swap. Here, party A makes regular payments that depend on the average level of a Libor rate over a set of Asian observation points, while party B makes upfront fixed rate payments.

We first describe the general form of the cancelable swap. Let N denote the swap notional, T be the swap maturity, and T_i, for $i = 1,\ldots,n$, denote a reset time; here $0 = T_0 < T_1 < \ldots < T_n = T$. Furthermore let T_j^i, for $j = 1,\ldots, n_i$ and $i = 0,\ldots,n-1$, be points such that

$$T_i = T_1^i < \ldots < T_{n_j}^i < T_{i+1}.$$

Next, let

$$L(\tau,\Delta) = \frac{1}{\Delta}\left(\frac{1}{P(\tau,\tau+\Delta)} - 1\right),$$

where $P(\tau,\tau+\Delta)$ is the price at time τ of a zero-coupon bond with unit face value and maturity $\tau+\Delta$, be the $\Delta-$period Libor rate at time τ. Also, let

$$L_{avg}^i = \frac{1}{n_j}\sum_{j=1}^{n_j} L\left(T_j^i, T_j^i + \Delta\right),$$

for $i = 0,\ldots,n-1$, be the arithmetic average of Libor rates over the set of points, $\{T_j^i\}_{j=1}^{n_j}$.

Party A must pay at T_i, for $i = 1,\ldots,n$,

$$N\left(L_{avg}^{i-1} - c\right)^{+}\Delta_i\,, \tag{2.1a}$$

where c is a cap level and $\Delta_i = T_i - T_{i-1}$. In addition, let R be an annualized fixed rate. Party B must pay to Party A,

$$NR\Delta_{i+1}\,, \tag{2.1b}$$

at T_{i-1}, for $i = 1,\ldots,m$, where $m < n$.

To summarize, for $i = 1,\ldots,m$, the other party must pay the amount (2.1a), at T_i, while Party B must pay, at T_{i-1}, the amount (2.1b). Additionally, for $i = m+1,\ldots,n$, Party A must pay the amount (2.1a), at T_i, while Party B does not make any payment. Moreover, Party B may elect to cancel the swap, at T_i, for $i = 0,\ldots,n-1$.

The valuation model is a "disconnected" tree discretization of a two-factor, risk-neutral Black-Karazinski (BK) short-interest rate process; in particular, the SDEs governing the short-interest rate process admit respective deterministic mean reversion and volatility parameters. The disconnected tree discretization above is non-recombinant by design, but employs an interpolation scheme to approximate short-interest rate values at tree nodes along a time slice.

Calibration of the model parameters is accomplished by matching, in a least squares sense, the model price against the market price for each respective European style payer swaption or caplet in a cache of calibration securities.

For the particular live deal above, the Libor average level, L_{avg}^{i} $(i = 0,\ldots,n-1)$, is approximated by the $\Delta-$period Libor rate that fixes at the accrual period midpoint, $\frac{T_i + T_{i+1}}{2}$. We examine the impact of this approximation, on the price calculation, in our testing.

We consider two benchmark models, that is, a single-factor short-interest rate model of the Hull-White (HW) form and a single-factor short-interest rate model of the BK form. We assume that the HW and BK short-interest rate processes satisfy respective risk-neutral SDEs of the form,

$$dr_t = (\theta_t - a_t r_t)dt + \sigma_t dW_t \,, \qquad \text{(4.1.1a)}$$

And

$$d\log r_t = (\theta_t - a_t \log r_t)dt + \sigma_t dW_t \,, \qquad \text{(4.1.1b)}$$

Where

- r_t denotes the short-interest rate,
- a_t is a piecewise constant mean reversion rate,
- σ_t is a piecewise constant volatility function,
- θ_t is chosen to fit the initial term structure of discount factors, and
- W_t is a standard Brownian motion.

We construct a trinomial tree to approximate the short-interest rate process, $\{r_t | 0 < t \le T\}$, based on the algorithm described in [Canale, 1996]. Let

$$\Omega = \{t_i\}_{i=0}^{p} \,,$$

where $0 = t_0 < \ldots < t_p = T$, be a partition of the interval, $[0,T]$, from valuation to the last payment time. We assume that the respective volatility and mean reversion functions, σ_t and a_t, are constant over each interval, $[t_{i-1}, t_i)$, for $i = 1, \ldots, p$. Moreover, we ensure that the set of time slices, Ω, includes the following events,

- the reset point, T_i, for $i = 1, \ldots, n$, and
- the respective sets of volatility time and mean reversion time break points.

One party makes regular payments that depend on the average level of a $\Delta -$period Libor rate over a set of Asian observation points. We note that, for a trinomial tree discretization of a BK short-interest rate process, the computation of a Libor rate may be numerically unstable, if the tree is bushy. However, a HW trinomial tree construction does not suffer from numerical instability in computing a Libor rate. We therefore considered both HW and BK respective benchmark short-interest rate tree constructions.

Observe that the price of a cancelable swap price is given by that of the underlying swap plus that of a Bermudan swaption that reverses the payment flows. We employed the HW tree benchmark to examine the potential error in pricing the swap component, due to the approximation of the Libor rate average value by that of a Libor rate level at a single point.

Next, to avoid pricing mismatches due to distributional differences between the HW and BK short-interest rate respective model assumptions, we employed the BK benchmark, but replaced the Libor rate average value by that of a Libor rate level at a single point.

In particular, recall that the AX2 model pricing of the cancelable swap approximates the Libor rate average level, L^i_{avg} $(i = 0, \ldots, n-1)$, by the $\Delta -$period Libor rate value at the *midpoint*, $\frac{T_i + T_{i+1}}{2}$, of the accrual period, $[T_i, T_{i+1})$; for our BK based benchmark, we instead approximate the Libor rate average level by the $\Delta -$period Libor rate value at the *start*, T_i, of the accrual period.

The benchmark placement of the approximating Libor reset point, at the start of the accrual period, avoids having to compute a Libor rate directly under BK short-interest rate dynamics.

Let $swap_i$, for $i = 0,\ldots,n-1$, denote the value from Party B's perspective, at T_i, of the remaining swap payments over the accrual periods, $\left[T_j, T_{j+1}\right)$, where $j = i,\ldots,n-1$. In particular,

$$swap_i = E\left(\sum_{k=i}^{n-1} L_{avg}^k (T_k;\Delta)\Delta_{k+1} \exp\left(-\int_{T_i}^{T_{k+1}} r(s)ds \right) - \sum_{k=i}^{m} R\Delta_{k+1} \exp\left(-\int_{T_i}^{T_k} r(s)ds \right) \middle| F_{T_i} \right) \qquad (4.1.2)$$

where $E(\cdot)$ denotes expectation under the risk-neutral probability measure. Furthermore, let

$$Euro_i = \max\left(-swap_i, 0\right)$$

denote the value, at T_i, of a European swaption to receive fixed-rate payments in exchange for floating-rate payments. Also, let

$$berm_0 = \max_{\tau \in \{0,\ldots,n-1\}} E\left(Euro_\tau \times \exp\left(-\int_0^{T_\tau} r(s)ds \right) \right), \qquad (4.1.3)$$

where τ is a stopping time, be the value at T_0 of a Bermudan swaption to receive fixed-rate payments in exchange for floating-rate payments. The value at T_0 of the cancelable swap, $cancel_0$, is then given by

$$cancel_0 = swap_0 + Berm_0 . \qquad (4.1.4)$$

Since the average Libor rate level, L_{avg}^i $(i = 0,\ldots,n-1)$, is an interest rate path dependent quantity, we employ crude Monte Carlo simulation to identify short-interest rate paths along our trinomial tree; an estimate of the swap price, $swap_0$, is then given by the arithmetic average of the swap value with respect to each Monte Carlo path. The Monte Carlo estimate of $swap_0$ will converge to the corresponding trinomial tree based value as the number of Monte Carlo paths tends to infinity.

We consider a portfolio of l European style caplets. In particular, the i^{th} ($i = 1,\ldots,l$) caplet is specified as follows,

- Libor fixing time, τ_i, with corresponding accrual period, δ_i ,
- payoff at τ_i equal to $\max(1-(1+X\delta_i)P(\tau_i,\tau_i+\delta_i),0)$ where X is a strike level.

For each option in the portfolio above, we obtain from WM a corresponding Black's implied volatility. We then price the payer swaption above based on Black's analytical formula. Let P_i denote the price for the i^{th} ($i = 1,\ldots,l$) caplet calculated using Black's model as described above.

We assume that the volatility function in (4.1.1a and b), σ_t, is constant over the respective periods, $[0,\tau_1)$, $[\tau_1,\tau_2),\ldots,[\tau_{l-1},T)$. In particular, let $\sigma_1,\ldots,\sigma_l$ be the constant volatility value that corresponds to the respective intervals, $[0,\tau_1)$, $[\tau_1,\tau_2),\ldots,[\tau_{l-1},T)$. Furthermore let $bench(\vec{\sigma};i)$, for $i = 1,\ldots,l$, denote the benchmark option price for the i^{th} caplet where $\vec{\sigma} = [\sigma_1,\ldots,\sigma_l]^T$. We seek to solve

$$\vec{F}(\vec{\sigma}) = \begin{bmatrix} bench(\vec{\sigma};1) - P_1 \\ \vdots \\ bench(\vec{\sigma};l) - P_l \end{bmatrix} = \vec{0},$$

in a least squares sense, for the unknown, $\vec{\sigma}$; that is, we seek to minimize

$$\vec{F}(\vec{\sigma})^T \vec{F}(\vec{\sigma}) = 0$$

with respect to $\vec{\sigma}$. A necessary condition for a minimum is that

$$\left[\frac{\partial}{\partial \vec{\sigma}} \vec{F}(\vec{\sigma})^T\right] \vec{F}(\vec{\sigma}) = \vec{0}^T . \tag{4.2.1}$$

We solve (4.2.1) for the unknown, $\vec{\sigma}$, using Newton's method based on our short-interest rate trinomial tree.

References:

https://finpricing.com/lib/IrCurveIntroduction.html