GIC Coupon Rate Calculation

We calculate the Treasury transfer coupon rate, in the case of zero coupon payment frequency, or the equivalent annualized simple rate in the case of non-zero coupon payment frequency, from which the transfer coupon rate.

The model assumes that the GIC holder receives deterministic payments on specified payment days, and the embedded put option is exercised by the GIC holder when the redemption value exceeds the holding value.

Consider a GIC specified by

- maturity, T,
- set of future payment times, $\{t_i\}_{i=1}^{N}$, where $t_1 < ... < t_N = T$.

Let

- c_{Cst} be the annualized customer coupon rate,
- c_{Tr} be the transfer coupon rate,
- f_c denote the coupon rate compounding frequency (i.e., annual, semi-annual, quarterly or monthly),
- f_p be the coupon payment frequency (i.e., annual, semi-annual, quarterly or monthly),

We define an "equivalent simple annualized rate", which we denote EAR:

$$EAR_{Cst} = f_p \left[\left(1 + \frac{c_{Cst}}{f_c} \right)^{\frac{f_c}{f_p}} - 1 \right], \tag{1a}$$

$$EAR_{Tr} = f_p \left[\left(1 + \frac{c_{Tr}}{f_c} \right)^{\frac{f_c}{f_p}} - 1 \right]. \tag{1b}$$

The payment P_i at time t_i is then

$$P_i = principal \times EAR_{Tr} \times (t_i - t_{i-1}), \tag{2}$$

where time interval, $t_i - t_{i-1}$, is calculated based on the ACT/365 day counting convention. We have reviewed the generation of the payment dates, t_i, and the respective payment amounts, P_i.

In the case of zero payment frequency, the 'bullet' payment at maturity is calculated as

$$bullet\ payment = principal \times (1 + c_{Tr})^T$$

No equivalent simple annualized rate calculation is required in this case.

We solve the equation,

$$pv_{Tr} + p_{Tr} - \text{accrued interest} = \text{principal} , \qquad (3)$$

for the unknown Transfer Coupon Rate, c_{Tr}, where pv_{Tr} and p_{Tr} are the closed GIC present value and the redemption (put) option price respectively. Here,

$$\text{accrued interest} = \text{time since the last coupon} \times EAR_{Tr} . \qquad (4)$$

We note that if the payment frequency, f_p, is not zero, then the spreadsheet outputs EAR_{Tr}, defined by eq. (1b), instead of the transfer coupon rate, c_{Tr}; it is the responsibility of the spreadsheet user to then calculate

$$c_{Tr} = f_c \left[\left(1 + \frac{EAR_{Tr}}{f_p} \right)^{\frac{f_p}{f_c}} - 1 \right]. \qquad (5)$$

In the case of zero payment frequency, we directly output the transfer coupon rate (ref https://finpricing.com/lib/IrCurveIntroduction.html)

We assume a stochastic interest rate environment, with the short interest rate dynamics governed by a HW model of the form,

$$dr = (\theta(t) - ar)dt + \sigma \, dW \,, \tag{6}$$

with constant mean reversion, a, and volatility, σ, parameters. The interest rate model is implemented as a trinomial tree.

We use the HW tree described above to evaluate the terms of eq. (3). It employs Brent's root finding method for solving eq. (3), which is guaranteed to converge if a root exists within the given interval.

One can expect monotonic behavior of the left hand side of equation (3) as a function of the transfer coupon rate, which ensures the uniqueness of the eq. (3) root. This is supported by the graph in Figure 1 that shows the difference between the left and right sides of eq. (3) for the Premium GIC specified in Section 6 below.
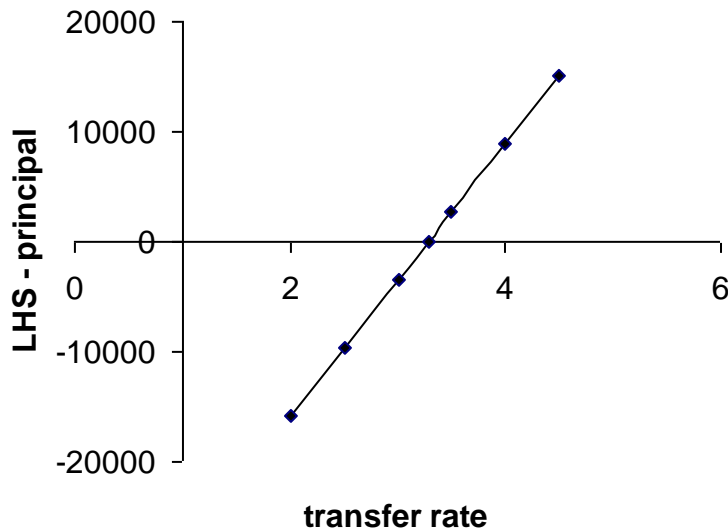
Figure 1. Difference between the left and right hand sides of equation (3) for the Premium GIC specified in Section 6 below. Hull-White volatility is 1.383%.