# Swap Average Term Computation

The average term is calculated for a swap that underlies a European style payer swaption, which is in the calibration portfolio for a Bermudan swaption with amortizing notional (i.e., the outstanding notional is reduced from time-to-time). Given the payer swaption maturity and the average swap term pair, we then look up, from a table indexed by payer swaption maturity and underlying swap term, the corresponding Black's implied volatility.

The Black's volatility is then applied to compute the market price for the payer swaption above; this market price is required to calibrate the short-rate volatility for pricing the Bermudan swaption above.

We consider a single currency Bermudan style swaption, which has underlying swap specified as follows,

- reset point, $T_i$ (expressed in years), for $i = 0,\ldots,M$ , where $0 < T_0 < \ldots < T_M$ , with corresponding notional amount, $N_i$, where $N_0 \geq \ldots \geq N_M \geq 0$ and $N_j > N_{j+1}$, for some $j \in \{0,\ldots, M-1\}$,

- notional payment, $N_i - N_{i+1}$, at time, $T_{i+1}$, for $i = 0,\ldots,M-1$,

- floating-leg payment, $N_i\, L(T_i;\Delta_i)\, \Delta_i$, at settlement time, $T_{i+1}$ , for $i = 0,\ldots,M-1$, where
    - $\Delta_i = T_{i+1} - T_i$ ,

- $L(T_i;\Delta_i)=\frac{1}{\Delta_i}\left(\frac{1}{P(T_i,T_{i+1})}-1\right)$ is the simple interest rate applicable at $T_i$ for the accrual period, $[T_i,T_{i+1})$,

- fixed-leg payment, $N_i\,R_i\,\Delta_i$, at settlement time, $T_{i+1}$, for $i=0,\ldots,M-1$, where $R_i$ is a simple, annualized rate (ref. https://finpricing.com/lib/IrBasisCurve.html).

Consider a European style payer swaption, which is in the calibration portfolio for the Bermudan swaption in Section 2. In particular assume that the payer swaption has maturity, $\tau_0$, where $T_0 \le \tau_0 < T_M$, and that its underlying swap has last reset point, $\tau_L$, where $\tau_L = T_M$.

Let $j_{\tau_0} \in \{0,\ldots,M-1\}$ be such that $T_{j_{\tau_0}} \le \tau_0 < T_{j_{\tau_0}+1}$. Then

$$\lambda=\frac{\sum_{i=j_{\tau_0}}^{M-1}(N_i-N_{i+1})(T_{i+1}-\tau_0)}{\sum_{i=j_{\tau_0}}^{M-1}(N_i-N_{i+1})},$$
$$=\frac{\sum_{i=j_{\tau_0}}^{M-1}(N_i-N_{i+1})(T_{i+1}-\tau_0)}{N_{j_{\tau_0}}-N_M},$$

is defined as the average term, expressed in years, of the swap that underlies the payer swaption above. Next, the average swap term is expressed in days, assuming that there are 360 calendar days in a year,

$$\lambda_{\text{day}}=\lambda\times 360.$$

Given the swaption maturity and average swap term pair, $\tau_0$ and $\lambda_{\text{day}}$, for the payer swaption above, we then look up, from a table indexed by payer swaption maturity and underlying swap term, the corresponding Black's implied volatility. The Black's volatility is then applied to compute the market price for the payer swaption above; this market price is required to calibrate the short-rate volatility for pricing the Bermudan swaption.

We recommend, for operational robustness, that we include a check that the resulting average swap term is positive; if the average swap term is non-positive, an exception should be raised and handled.

We consider a European style payer swaption specified by

- maturity, June 30, 2005, and
- underlying swap's last reset date, June 30, 2014.

In the table below we show the calculation of the cumulative swap term, $A_i$, for $i = 0,\ldots,L-1$, which is specified as follows. Specifically,

$$A_0 = \left(N_{j_{\tau_0}} - N_{j_{\tau_0}+1}\right)\left(T_{j_{\tau_0}+1} - \tau_0\right)$$

and, for $i = 1,\ldots,L-1$,

$$A_i = A_{i-1} + \left(N_{j_{\tau_0}+i} - N_{j_{\tau_0}+i+1}\right)\left(T_{j_{\tau_0}+i+1} - \tau_0\right).$$

The cumulative term values were obtained via appropriate print statements.