Credit Risk Calculator

The purpose of the credit risk calculator is to ensure that the expected loss that can occur from the guarantor's (CMHC's) perspective is covered by the guarantee fee. From CMHC's perspective, the risk of loss will occur if an AAA/AA swap counterparty fails instantaneously without any rating migration to a lower state (i.e., AA to A). Under a normal rating migration, the swap counterparty to the Trust would have to collateralize its exposure.

Thus the expected loss from the guarantor's perspective is maximum loss×probability of default×(1-recovery rate). The maximum loss is the difference between CMB value and MBS value. We use ratings transition to estimate the probability of default and assume that the recovery rate is zero.

The two legs of the swap are as follows: a bond and a VRM (once reinvestment is considered, the amortizing nature of the VRM disappears). Thus the swap is like a conventional "receive fixed pay floating" swap.

As the duration (sensitivity) is essentially one month, the price of VRM-MBS will always be close to par. It follows that the worst-case scenario will happen in a downward interest rates environment where the CMB price goes up and exceeds the MBS value. However, the CMB price will get back to par as the maturity approaches. Consequently, the worst-case scenario is likely to occur sometime in the middle of the term.

As described in the Credit Risk Calculator documentation, we can assume that the forward rates follow a lognormal process as follows:

$$F_{t,T} = F_{0,T} \times \exp\left\{\left(\mu_T - \tfrac{1}{2}\sigma^2\right)t + \sigma\sqrt{t}\varepsilon\right\}$$

Where

$F_{t,T}$ is one-month forward rate at time t for time T, $t \leq T$

$F_{0,T}$ is current one-month forward rate for time T

μ_T is the drift

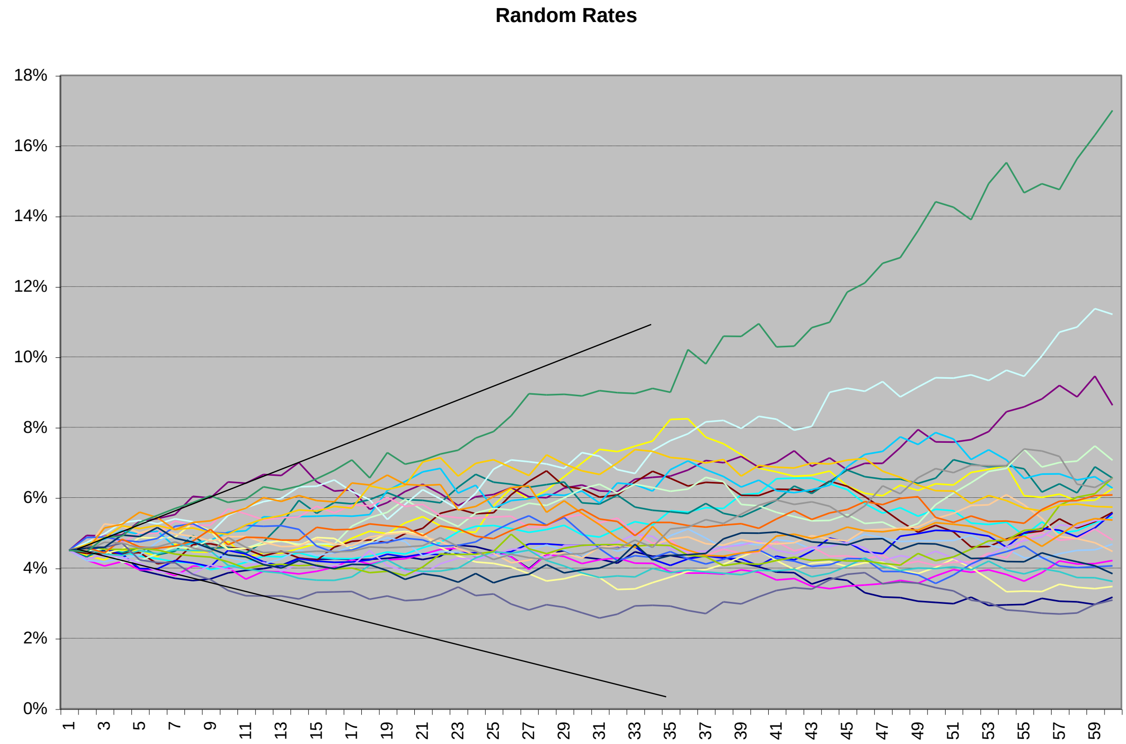
σ is the volatility

ε is a standard normal random variable

The key determinant in Monte Carlo simulation is the underlying stochastic process. Monte Carlo simulation can be used to generate a "random walk" based on the underlying stochastic process. When Monte Carlo approach is used, we first generate a series of random numbers based on the distribution of random term in the PDE. By combining this random effect and the main trend (drift) of the stochastic variable, a path is simulated. By repeating this process, we can produce more paths for underlying variable. However, a great amount of trials should be undertaken so as to generate an unbiased result.

The figure below is a Monte Carlo simulation of short-term rates (not the forward rates in our case --we will explain the relationships later). As illustrated in the diagram, if we use a 95% confidence interval to build upper and lower boundaries according to the underlying stochastic process, we have 95% chance for the rates simulated to fall within these limits.

As you can see in the diagram, Monte Carlo simulation is basically reproducing the underlying distribution already defined by the underlying stochastic process. In the credit risk analysis, we don’t care what is happening in the middle of the interval when using Monte Carlo simulation. What matters to us is the worst-case scenario, which is the lower bound. Consequently, we can calculate directly the lower bound and find out the worst-case scenario.



Back to our forward rates process.

$$F_{t,T} = F_{0,T} \times \exp\left\{\left(\mu_T - \tfrac{1}{2}\sigma^2\right)t + \sigma\sqrt{t}\varepsilon\right\}$$

If we use Monte Carlo approach, we generate a series of random numbers for the normally distributed random term ε in the process above.

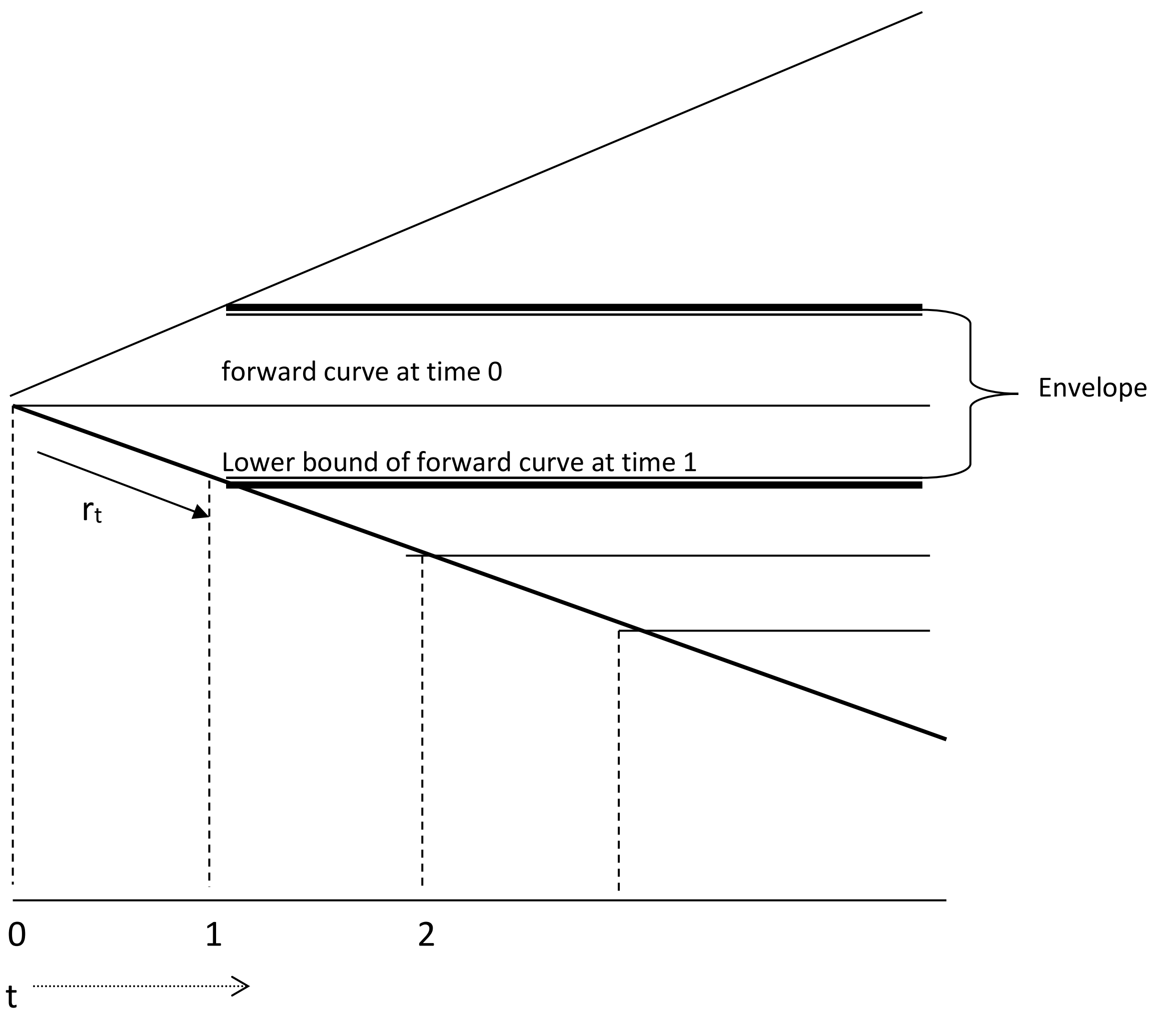
The drift μ_T for interest rates is usually small enough to be ignored. Similar to the previous example for short-term rates, we can establish an upper bound and a lower bound for the forward rates based on a given confidence interval. But what really matters for us here is the lower bound, i.e., how far the rates can go down. Therefore, as we look forward into the future, a lower bound of $F_{t,T}$ with a 95%(or 99%) confidence interval can be established by:

$$F_{t,T}{}^{Down} = F_{0,T} \times \exp\left\{-\tfrac{1}{2}\sigma^2 t - 1.645(or 2.33)\sigma\sqrt{t}\right\}$$

This lower bound is proportionate to the time horizon, the confidence interval and the volatility. Consequently, as we move forward into the future (i.e., as t takes successively 1,2…etc.), a lower bound can be calculated according to the formula for each individual forward rate in the forward rates curve (see https://finpricing.com/lib/IrCurve.html) at each time step. As we can see from the equation above, given t, the maximum size of downward movement is the same for all the forward rates, i.e., all the forward rates will make the same maximum downward shift at time t. Actually, as the entire forward rates curve goes down, we create an envelope of the downward paths for the short-term rate r_t.

Regarding the non-parallel shift, as we use a one-factor model and assume a constant volatility for all forward rates, we cannot simulate a non-parallel shift of the term structure. What will make the worst-case scenario worse is the non-parallel downward shift at the longer-term end of the term structure, i.e., long-term rates will go down even further when the term structure declines. However, we can simulate this event by looking at a larger volatility which will shift the lower bound even lower.

The main idea is illustrated in the figure below.



At each time step, the forward curve can move up, down and twist and can take any shapes within the upper and lower boundaries. However, we only consider the lower bound situation in which the worst case can happen.

This analytical approach is similar to using Monte Carlo simulation or using a tree. We should obtain similar results by using whichever of these three methods provided the underlying process of interest rates is the same. Hence, this approach seems to be a good approximation to estimate the credit risk exposure for VRM-MBS.

As demonstrated in the stochastic process above, we assume that each individual forward rate on the forward curve follows a log-normal distribution. We do not assume that they are perfectly correlated. However, we do assume that all the forward rates have the same volatility. Consequently, at any given time, the lower bound for these forward rates is the same, which does not mean the forward rates will move together perfectly.

Otherwise, we assume the perfect correlation between the swap curve and the valuation curve, which is basically the swap curve plus a liquidity spread.