

Pricing Variable Rate MBS

The variable rate mortgage (VRM) has provisions that allow the mortgagor to convert his mortgage, without penalty, to a fixed rate mortgage. Consequently, VRM MBS are subject to an additional source of early principal repayment.

The VRM MBS calculator treats conversions using a constant annualized conversion rate that is applied as a monthly mortality rate. This treatment of conversions ignores the impact of ageing, seasonality and interest rate volatility. However, since the VRM coupon resets each month, we expect ageing/seasonality/ interest rate volatility to impact the price marginally; therefore, the extra spread to be paid to investors because of the early principal payment, especially due to interest rate volatility, is marginal as well.

The VRM MBS calculator can be used to estimate the order of magnitude that interest rate volatility would cost an investor on an NPV basis by comparing the average of MBS prices calculated under different scenarios to the price obtained using the average of inputs over the various scenarios

The calculator used for MBS valuation of FRM assesses the expected cash flows from the mortgages while considering the potential for not earning those cash flows due to early return of principal from prepayments and mortgage refinancing, and determines the appropriate fixed coupon for the MBS accordingly. However, VRM have different characteristics that determine the cash flow behaviour in particular a changing interest rate and the possibility for conversion to a new FRM.

An NHA MBS passes through to its holders, the investors, a portion of the cashflows generated by a pool of amortizing insured residential mortgages which are sold on a fully serviced basis. More specifically, an NHA MBS passes through to its holders all principal payments and a portion of the interest payments generated by the underlying mortgage pool. The principal payments include the scheduled monthly payments, partial prepayments, and liquidations. Interest is paid monthly on the outstanding principal at the specified MBS coupon rate.

It is proposed that the VRMBS be backed by standard VRM which requires a borrower to make fixed payments regardless of changes to the interest rate on the mortgage. However, the portion of the payment that will go to interest or principal will depend on the interest rate level and particular features of the mortgage. These features typically include the rate movement schedule, the index the rate is based on, the discount offered against the index, any interest rate ceiling that may be offered, and the level of bank prime effective for that month. Depending on the mortgage specifics, penalty interest may also be charged to the borrowers for early prepayment/liquidation of their mortgage, or according to the flexibility surrounding conversion to a FRM mortgage. Certain of the current NHA MBS pool types pass through to its holders all or part of the penalty interest paid by mortgagors when liquidating their fixed rate mortgages.

It is critical to establish a means of specifying interest rate movement because the timing and shifts in the rate will determine the future rates of the mortgages. The interest payable on the mortgage principal will serve to determine the coupon for the MBS. Since the mortgage rates float, a valuation model must have a means of specifying the expected future mortgage rates and then determining an appropriate MBS coupon rate.

It has been decided to set the MBS coupon off the 1-month CDOR setting (Bankers Acceptance (BA)), since this index is used by all swap desks to set the floating leg of monthly pay swaps. Forward 1-month BA rates are predicted using the swap curve because the swap curve (see

<https://finpricing.com/lib/IrCurveIntroduction.html>) is readily observable on any day. The issue then is how should the discount factors be determined and how should prime be determined (as this affects future cash flows).

Discount rates are based on a cost-of-funds (COF) curve, which is input by the user of the calculator. It represents an MBS curve or any curve the user desires to value the VRMBS.

Prime is an administered rate. The way the MBS has been structured is that the prime – BA basis risk is retained by the issuer/seller. The question of how the mortgage rate (i.e. prime evolves) is still important as it is a determinant of the evolution of mortgage and hence MBS cash flows.

Mortgages are set off of prime. Prime however, is not a market rate. However, since the Bank of Canada changed how Bank Rate is set (it is set off the target overnight rate as opposed to being set off of 3-month treasury bills), the market has evolved such that prime moves when the central bank changes the target overnight rate. Since there is a close correlation between prime and shorter-term money market rates the prime rate can be model as a constant spread to BAs.

Unlike fixed-rate mortgages there is a risk of negative amortization with a VRM. As rates rise more of the fixed mortgage payment goes to paying the interest on the mortgage. Should the current mortgage rate increase to the point where your payment no longer covers the interest portion of the mortgage, the mortgagor is required to increase the regular principal and interest payment so that the mortgage will be repaid in full over the registered amortization period, less the amount of time that has elapsed since the initiation of the mortgages. The mortgagor can also request that the P&I be adjusted to reflect the original amortisation period, cover interest only or convert the VRM to a FRM product of 3 year tenor or longer . In order for the mortgage to remain in the pool in the first scenario, the originator would provide a full pass through of the interest to the investor.

The creation of a new NHA MBS product based upon the characteristics of a VRM pool requires the development of a new model for the valuation and pricing the characteristics of the pool. Since the VRMBS will have a coupon rate that is set monthly, the calculator computes the value of the MBS according to projected cash flows and predicted interest rate movement. It is structured to accounts for characteristics common to eligible underlying mortgages in particular rate (discount, teaser rates), prepayment behaviour, and conversion assumptions. The model will be used for structuring a new issue and for monthly valuations of the VRMBS.

The VRMBS product has a fixed monthly payment with a variable rate that resets upon the bank prime. The 3-year fixed mortgage rate is used to determine the borrower's fixed monthly payment. As mentioned above, the monthly payment is fixed regardless of interest rate changes, consequently the principal and interest portions of the payment will change according to the movement of the mortgage rate that is based off of bank prime movements. If the interest rate goes down, the mortgage will pay down faster and converse is true as well, however in the event of negative amortization where the interest owing is not met by the monthly payment and the original amortization schedule is not being satisfied, the borrower will be required to adjust the fixed payment or pay a lump sum of accrued interest to ensure that the mortgage will be repaid in full over the registered amortization period. During the teaser period, the mortgage rate is 101 bps below the bank prime, and there is a 25 bps normal discount for the rest of the mortgage term.

MBS coupon will be established based on BAs plus a spread. Since the mortgage rate is based off the bank prime minus a discount, a big discount during the teaser period and a normal discount afterwards, and the mortgage rate will reset whenever the bank prime changes. The bank prime is usually based on BAs plus a spread. This spread can change overtime according to economic conditions, etc.

Based on this rate relationship there are two very important inputs for VRM MBS valuation: one is the swap curve and the other is the valuation curve. The swap curve is used to forecast the one-month forward BA rates, and based on these one-month forward rates, we can estimate the future cash flows for the VRM and the MBS accordingly. Once these estimated cash flows are generated, a valuation curve is used to generate the appropriate discount factors for discounting the future cash flows to a present value for the MBS. While the valuation curve can be a bank's cost funds curve, for MBS in the CMB program, the CMB curve is used as the valuation curve, because the issuer is funding through CMB issuance.

The input rates R_i define the swap curve. The first rate is the over night deposit rate. The next 5 rate are the 1-month, 2-month, 3-month, 6-month, and 12-month CDOR rates. These are annual rates with interest is paid on maturity. The remaining 4 input rate are the 2-year, 3-year, 4-year and 5-year all-in swap rates. These rates are semi-annual rates. All these rates can be obtained from a market data source such as Bloomberg.

Payment dates are generated using modified following convention. If a payment date falls on a weekend payment is move to the following Monday. Holidays are ignored. The payment time is calculated as actual over 365/fixed.

t_i = payment date

d_i = discount factors. For the over night and CDOR rates these are calculated as follows.

$$d_i = \frac{1}{(1 + t_i R_i)}$$

This gives discount factors up to one year.

Discount factors beyond one year are calculated at 6-month intervals starting at 1.5 years and ending at 5 years. The swap rates corresponding to the half-year points are calculated by linear interpolation between the whole year points. The one-year semi-annual swap point R_{1y} is calculated as follows.

$$R_6 = R_{1y} = \left[\frac{2(1 - d_6)}{(d_5 + d_6)} \right]$$

This is used to calculate the 1.5-year swap rate. The swap input curve is redefined as follows to include the half-year points.

$$R_8 = R_7; \quad R_{10} = R_8; \quad R_{12} = R_9; \quad R_{14} = R_{10}; \quad R_9 = (R_8 + R_{10})/2;$$

$$R_{11} = (R_{10} + R_{12})/2; \quad R_{13} = (R_{12} + R_{14})/2; \quad R_7 = R_{1.5y}$$

Discount factors from 1.5 years to 5 years at half year intervals is calculated as follows

$$d_j = \frac{1 - t_5 d_5 - d_6(t_6 - t_5) - \sum_{i=7}^{i=j-1} (t_i - t_{i-1}) d_i}{1 + (t_j - t_{j-1}) R_j}$$

This formula is used to calculate the discount factors for both the swap curve and valuation curve. Afterwards, we interpolate the swap curve discount factors according to reset dates and use these interpolated discount factors to generate forward BA rates. As for the valuation curve discount factors, they are simply used to discount cash flows.

The exact same procedure is used to generate the valuation curve payment times and discount factors.

Future mortgage rates are generated from the forward prime rate less any discount specified in the mortgage documentation. The forward prime rate is equal to the forward 1-month CDOR rate calculated from the swap curve plus an assumed constant spread between prime and 1-month CDOR. The mortgage payment dates T_i are generated off the maturity of the MBS and are adjusted on a modified following, holidays are again neglected.

The forward CDOR rate $CDOR_i$ used in the calculation of interest owed paid at time T_{i+1} is

$$CDOR_i = (d_i / d_{i+1} - 1) / (T_{i+1} - T_i)$$

The discount factors correspond to payment times T_i and T_{i+1} . The mortgage rate r_i at time i is equal to

$$r_i = CDOR_i + \text{prime_CDOR_spread} - \text{discount}$$

The forward MBS rate m_i is equal to the forward CDOR rate plus the MBS spread.

Contractual mortgage cash flows are modified to account for partial prepayment, liquidation, and conversions. In each calendar year, the mortgagor may prepay up to 15% of the original principal amount of the mortgage without notice, bonus or penalty. This is modeled as partial prepayment. Liquidation models mortgagors that leave the bank, this attracts a penalty equal to three month's interest costs charged at the current CIBC Prime Rate. Conversions model mortgagors

converting their variable rate mortgage into a fixed rate mortgage. There is no penalty charge on conversion.

These rates are input as annual rates and then converted to monthly rates as follows

partial prepayment,

$$p = 1 - (1 - p)^{1/12}$$

liquidation,

$$l = 1 - (1 - l)^{1/12}$$

conversion,

$$c = 1 - (1 - c)^{1/12}$$

The mortgage originator/issuer retains all bonuses and penalties.

Partial prepayments reduce outstanding principal but do not affect the contractual monthly payment. Liquidations and conversions reduce outstanding principal and reduce the monthly payments. It is assumed that regular principal payments are made first, then liquidation, conversions and partial prepayments.

The interest payment on the mortgage I_i^m on the opening balance B_i due at the end of the month is equal to

$$I_i^m = B_i r_i$$

If the contractual monthly payment at the start of the month is A_i , the regular principal payment P_i^r is

$$P_i^r = A_i - I_i^m$$

If the swap curve is sufficiently “steep”, negative amortization may occur:

$$P_i^r = A_i - I_i^m < 0.$$

If this situation does occur, a new regular payment, A_i^{New} , is calculated for the current period based on the rate for a 3-year fixed-rate mortgage at time t_i . The rate, M_i , for a 3-year fixed-rate mortgage at time t_i is modeled as a constant fixed spread to prime

$CDOR_i + prime_CDOR_spread$. It is

$$M_i = CDOR_i + prime_CDOR_spread + mortgage_prime_spread$$

If negative amortization occurs all of the following payments are based on the new regular payment. The new regular payment is calculated as follows:

$$A_i^{New} = \frac{B_i \cdot M_i \cdot (1 + M_i)^{WAM-i+1}}{\left((1 + M_i)^{WAM-i+1} - 1 \right)}$$

is the remaining amortization. *RAM* is calculated as :

$$RAM = \text{Current Remaining Amortization} - \text{period of negative amortization} + 1$$

Current Remaining Amortization is an input to the calculator. The +1 is because no principal has been paid.

The principal liquidated P_i^l is equal to

$$P_i^l = (B_i - P_i^r)l$$

The penalty associated with liquidation p_i^l is three months interest at prime

$$p_i^l = 3(r_i + \text{discount})P_i^l$$

The principal converted to fixed P_i^c is

$$P_i^c = (B_i - P_i^r - P_i^l)c$$

Partial prepayment P_i^p is

$$P_i^p = (B_i - P_i^r - P_i^l - P_i^c)p$$

The outstanding principal at the end of the month is

$$B_{i+1} = B_i - P_i^r - P_i^l - P_i^c - P_i^p$$

provided $B_i - P_i^r > 0$.

It may happen that $B_i - P_i^r \leq 0$. If this occurs then $P_i^l = P_i^c = P_i^p = 0$ and $P_i^r = B_i$, and all future payments are set equal to zero.

The new monthly payment is

$$A_{i+1} = A_i(1-l)(1-c)$$

The interest paid to the MBS holder I_i^b is

$$I_i^b = B_i m_i$$

The full payment C_i^b to the MBS holder at the end of the month is

$$C_i^b = P_i^r + P_i^l + P_i^c + P_i^p + I_i^b$$

The cash payment to the MBS issuer/servicer C_i^m is

$$C_i^m = I_i^m - I_i^b + P_i^l$$

There is a 14 day payment delay on the MBS. The mortgage and MBS cash flows are discounted on the valuation curve. Let the discount factor for MBS payment i be $D(T_i^b)$. The full price of the MBS P_{MBS} with n payments is equal to

$$P_{MBS} = \sum_{i=1}^n C_i^b D(T_i^b)$$

The accrued interest payment on the MBS is equal to the accrual fraction on the mortgage multiplied by the first MBS interest payment. The accrual fraction for the mortgage is equal to

$$\text{accrual fraction} = \frac{\text{days from last payment}}{\text{next payment date} - \text{last payment date}}$$

The accrued interest on the MBS AI is

$$AI = \text{accrual fraction} \times I_1^b$$

The clean price of the MBS is

$$P_{MBS}^c = P_{MBS} - AI$$