Prepayment Neural Net Method

A model of mortgage prepayment rates based on the neural net approach is proposed. The model for insured, closed, five-year term mortgages has been developed. The neural net prepayment model behaves consistently across the training and testing sets and outperforms a simpler predictor, the linear regression model.

Mortgage prepayment rate is affected by a large number of economic, social and demographic factors and is to a significant degree a random variable. We have identified six principal determinants of the prepayment rate and built a neural network that uses these determinants as input parameters. Given these inputs the model is supposed to predict the corresponding prepayment rate.

The prepayment rate is considered a random variable whose distribution function depends on the input parameters. The neural net model predicts the mean of this distribution given the inputs. Since many of the input parameters are themselves random variables, a more precise statement is that the model estimates the expectation of the prepayment rate conditional on the fixed set of parameters.

The model is a compound function of the form:

$$r(x_1,\cdots,x_6)\equiv r(\mathbf{x})= f(a+\sum_{i=1}^{3} w_i g(b_i+\sum_{j=1}^{6} v_{ij}x_j)) \qquad (1)$$

where

$$f(t)=\frac{1}{1+\exp(-t)},$$

$$g(t)=\frac{\exp(t)-\exp(-t)}{\exp(t)+\exp(-t)},$$

and a, b_i, w_i, v_{ij} are the model's parameters. The 25 parameters are estimated by fitting the function to a training set of data using the Levenberg-Marquardt method.

The six variables the model depends upon are

- k, age of the pool at the beginning of the current month,
- volatility of mortgage rate, which is the sum of squared changes of the mortgage rate over the three previous months:

$$vol_k=\sum_{i=k-3}^{k-1}(5yMrtgRate_i-5yMrtgRate_{i-1})^2$$

- slope, which is defined as the excess of the 5yr mortgage rate over the 3-month GOC T-Bill rate in the previous month,
- $pvpb_k$, the ratio of the present value of the mortgage minus penalty to the remaining principal balance at the beginning of the current month,
- $pvpb_{k-1}$, similar ratio for the previous month,
- the average liquidation rate over the previous six months.

We used the mean square absolute error as a measure of goodness of the fit.:

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}(r(\mathbf{x}_i)-R_i)^2}$$

where

- $r(\mathbf{x}_i)$ is given by equ. (2) for a given vector of variables,
- R_i is the actual rate that occurred for those variables.

The mean square absolute error is also the measure that is used by the Machine Learning Group for the Levenberg-Marquard model fitting.

Three kinds of test have been carried out here. One, a qualitative test, compares the goodness of the fit of the prepayment model to the training data set with that to the testing data set. The second test compares the performance of the neural net model on the testing data set with that of a linear regression model (a semi-quantitative test). The third, also a qualitative test, generates scenarios of continuously rising *pvpb* (falling interest rates) of falling *pvpb* (rising interest rates).

Figure 1 demonstrates the qualitatively correct behavior of the predicted prepayment rates: they rise with rising *pvpb* (or falling interest rate) and vice versa.

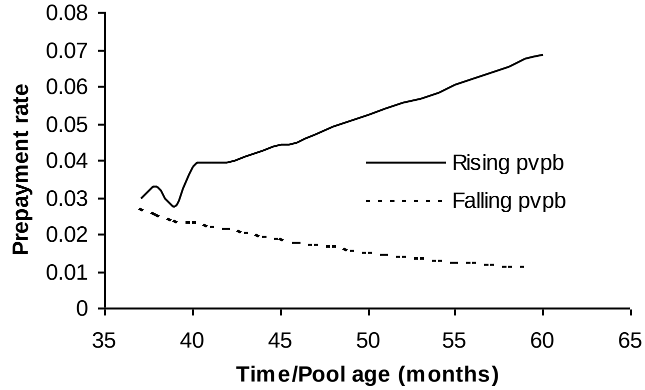
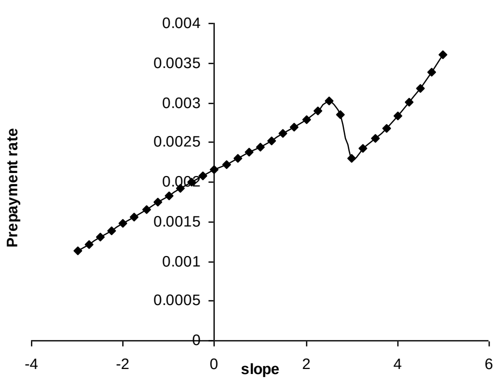
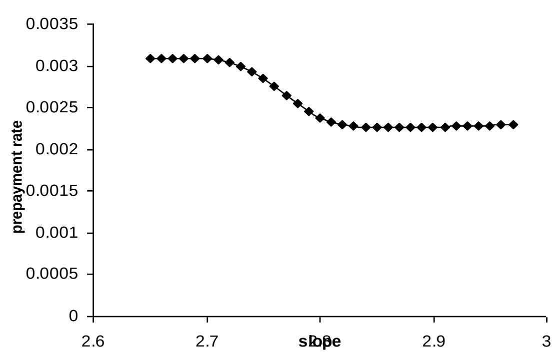


Figure 1. Prepayment rate dynamics of an aging pool of mortgages with continuously falling (solid line) or rising (dashed line) interest rates.

Prepayment function (1) is supposed to be used as a part of other models, such as a model for valuing mortgages under stochastic interest rates (see https://finpricing.com/lib/IrInflationCurve.html). These bigger models may require a certain degree of smoothness of the prepayment function. Technically, function (1) is analytic, i.e. infinitely smooth. It is possible, however, that in certain domains of its argument space the function exhibits relatively steep gradients, which under discrete sampling may appear as discontinuities. That this may indeed be the case is illustrated by Figs. 1a and 1b, which are based on an unlikely scenario of varying the slope input to the prepayment function while the other inputs fixed: Fig. 1b zooms the region of the large gradient and shows that the function is still smooth.





a) apparent discontinuity b) steep slope

Fig.1. An apparent discontinuity of the prepayment function which is in fact a steep local gradient