MBS Model with Liquidation Rate

We calculate the price of an MBS based on future cashflows that are assumed to be deterministic. One of the factors affecting future cashflows is a liquidation rate. In its current implementation the user has two options for specifying a liquidation rate, that is, it can be assumed to be constant or vary deterministically according to a Standard Vector prepayment model. For the Standard Vector model, the liquidation rate is calculated as

$$LQR = c \cdot V,$$
$$c = 0.85 \cdot \exp(40 \cdot (WAC - REFI))$$

where

- V is the Standard Vector,
- WAC is weighted averaged mortgage rate,
- $REFI$ is current refinancing rate defined as an average of one, three and five year mortgage rates.

The Standard Vector V is defined as increasing linearly for the first 42 months from 1.75% to 12% in 0.25% increments and then decreasing linearly from 12% to 6% in 0.33% decrements for the next 18 months.

When the standard vector is used, the MBS PE attempts to compute an equivalent constant liquidation rate, i.e., a constant liquidation rate which produces the same price as the variable liquidation rate based on the above formula. In certain rare cases, however, an equivalent constant liquidation rate does not exist. The existing version of PE in those cases displayed equivalent constant liquidation rate and prepayment rate as zero.

The proposed enhancement computes the constant liquidation rate for which the remaining principal balance at the month before the first maturing principal tranche is the same as remaining principal balance (RPB) using variable liquidation rate (Standard Vector). The MBS PE then also displays a warning message that a different constant equivalent liquidation rate was computed.

The constant liquidation rate is computed by using Newton's method to solve an equation for RPB as a function of the liquidation rate, keeping the other inputs constant. Since RPB decreases as the liquidation rate increases, RPB with zero liquidation rate is higher than RPB using standard vector, and RPB with 100% liquidation rate is lower than RPB using standard vector, the solution for the equation always exists, and the equivalent constant LQR rate can always be found.

Consider an MBS pool specified by

- number of tranches, M, where $M > 0$,
- i^{th} tranche's remaining principal balance as of the valuation date, N_i, for $i = 1,\ldots,M$,
- i^{th} tranche's remaining term as of the valuation date, n_i, expressed in months,

- pool's notional at inception, N,
- scheduled principal pre-payment rate, P, expressed as an annualized percentage,
- coupon paid to MBS holders, C, expressed as a semi-annually compounded percentage,
- weighted average mortgage rate, R, expressed as a semi-annually compounded percentage,
- i^{th}-year reference mortgage rate, R_i, for $i = 1,3,5$,
- pool remaining amortization, m, expressed in months,
- modified interest adjustment date, D_o, (this date is a weighted average of the respective originating date for each tranche in the pool),
- current valuation date, D_c.

Next consider the i_j^{th} tranche where $i_j \in \{1,\ldots,M\}$. Let

$$\alpha = \frac{\sum_{i=1}^{M} N_i}{N}$$

be the proportion of the pool's original principal amount that remains unpaid as of the valuation date. Then

$$\alpha_{i_j} = \frac{N_{i_j}}{\sum_{i=1}^{M} N_i}\alpha, = \frac{N_{i_j}}{N},$$

is the proportion of α that is attributable to the i_j^{th} tranche's remaining principal balance at the valuation time.

Suppose that the tranche generate cash flows at time t_k (expressed in years), for $k = 1,\ldots,n_{i_j}$, where $t_k = t_1 + \frac{k-1}{12}$, if $k > 1$, and t_1 (where $0 < t_1 \le \frac{1}{12}$) is a stub interval of time. At time t_k ($k \in \{1,\ldots,n_{i_j}\}$), the tranche generates a regular annuity payment (which includes interest and principal amounts), a scheduled (penalty interest-free) principal pre-payment and liquidated principal (which is subject to penalty interest).

The annuity payment at time t_1 is given by

$$A_1 = \frac{\alpha_{i_j} r}{1-(1+r)^{-m}}$$

Where

$$r = \left(1 + \frac{R}{200}\right)^{\frac{1}{6}} - 1$$

is a monthly compounded rate, expressed as a decimal, that is equivalent to R.

For $k = 1, \ldots, n_{i_j}$, let

- A_k denote the annuity payment due at time t_k,
- B_k be the tranche's outstanding principal balance after all principal payments at time t_k,
- η_k denote the principal portion of the annuity payment, A_k, at time t_k.

Then

$$\eta_k = \min\left(A_k - B_{k-1} r, B_{k-1}\right)$$

where $B_0 = \alpha_{i_j}$.

Let ζ_k denote the remaining principal balance, at time t_k, after the annuity payment, A_k. Then

$$\zeta_k = B_{k-1} - \eta_k .$$

Let l_k, for $k \geq 1$, denote the monthly principal liquidation rate at time t_k. We assume that principal liquidation rates are either constant or time-varying. If we assume that liquidation rates are constant, then

$$l_k = 1 - \left(1 - \frac{L}{100}\right)^{\frac{1}{12}},$$

for $k \geq 1$, where L is an annually compounded liquidation rate, which is expressed as an annualized percentage. Time-varying liquidation rates are based on the Standard Vector pre-payment model for liquidations; here

$$l_k = 1 - \left(1 - \omega f(k + \upsilon)\right)^{\frac{1}{12}},$$

for $k \geq 1$, where $f : I^{+} \to \Re$ is defined by

$$f(K) = \begin{cases} 1.75 + 0.25(K-1), & 1 \leq K < 42, \\ 12 - \frac{1}{3}(K-42), & 42 \leq K < 60, \\ 6, & 60 \leq K, \end{cases}$$

and

$$\omega = .85e^{\frac{2}{5}\left(C - \frac{R_1 + R_3 + R_5}{3}\right)}.$$

We set υ equal to the number of months since the modified interest adjustment date. Specifically, let

- y_c and y_o, where $y_{c,}y_o \in I^+$, be the year corresponding to the respective dates D_c and D_o,
- ν_c and ν_o, where $\nu_{c,}\nu_o \in \{1,\ldots,12\}$, correspond to the respective month that the dates D_c and D_o fall on.

Then $\upsilon = 12(y_c - y_o) + \nu_c - \nu_o$; for example, if D_c denotes the date September 14, 2000, and D_o represents the date December 1, 1996, then $\upsilon = 12(2000 - 1996) + 9 - 12 = 45$.

Let θ_k denote the penalty interest, at time t_k $(1 \leq k < n_{i_j})$, arising from liquidation of principal. Here

$$\theta_k = 3\xi_k r\beta$$

where β, with $0 \leq \beta \leq 1$, is a penalty interest ratio. Furthermore we set $\theta_{n_{i_j}} = 0$.

Let γ_k denote the scheduled principal pre-payment at time t_k where $k < n_{i_j}$. Then

$$\gamma_k = (\zeta_k - \xi_k)\left(1 - \left(1 - \frac{P}{100}\right)^{\frac{1}{12}}\right).$$

Let φ_k denote the total principal payment at time t_k. Then

$$\varphi_k = \begin{cases} \eta_k + \xi_k + \gamma_k, & \text{if } 1 \le k < n_{i_j}, \\ B_{k-1}, & \text{if } k = n_{i_j}. \end{cases}$$

The remaining principal balance at time t_k, where $1 \le k \le n_{i_j}$, is given by

$$B_k = B_{k-1} - \varphi_k .$$

The total cash flow to the MBS holder, λ_k, at time t_k, where $1 \le k \le n_{i_j}$, is given by

$$\lambda_k = B_{k-1}c + \varphi_k + \theta_k .$$

Reference:

https://finpricing.com/lib/IrCurveIntroduction.html