Mortgage Pool Valuation

A model is presented for the calculation of the fair value and the hedge ratios, Delta, Vega and Gamma, with respect to pools of Canadian commercial and residential mortgages. Commercial mortgages are closed and either insured or not insured, while residential mortgages are separated into

- insured open or closed mortgages, and
- non-insured open or closed mortgages.

The implementation is based on a single factor Hull-White interest rate trinomial tree. Below we give a summary overview the mortgage pool functionalities.

We examine one closed and two open mortgage pools under three different market scenarios. We define the *relative difference*, symbolically, as

$$\frac{\text{DVP quantity} - \text{Benchmark quantity}}{\text{Benchmark quantity}}$$

and the *absolute difference* as

$$\left|\text{DVP quantity} - \text{Benchmark quantity}\right|.$$

Our benchmark employs a parallel shift of five basis points in the Base Rates to calculate the hedge ratios, Delta and Gamma, and a one percent shift in Black's implied swap rate volatility to calculate Vega.

Consider a closed mortgage pool specified by a monthly payment, P_i, from Canadian Mortgages Inc. (CMI) to Treasury at the payment time, t_i, for $i=1,...,N.$ We note that the payment P_i ($i=1,\ldots,N$) includes both scheduled and unscheduled principal pre-payments. Then, the fair value of the closed mortgage is given by

$$\sum_{i=1}^{N} P_i d_i \qquad (3.1.1)$$

where d_i is the discount factor based on Treasury's cost of funds rate for the term t_i.

In addition to the notations defined in Section 3.1, let $\hat{P}_i$ denote the monthly payment to CMI from the homeowner, and let $\hat{R}_i$ denote the remaining principal balance after the payment, $\hat{P}_i$, for $i=1,...,N.$ Furthermore, let t^* denote the first time at which the mortgage pool is open to refinancing and τ, where $t^* \le \tau < t_N$, be the stopping time for the homeowner to pay the remaining balance, accrued interest and the penalty interest. From the homeowner's point of view, the open mortgage pool refinancing option price is then given by

$$\sup_{t^* \le \tau < t_N} E\left(\exp\left(-\int_0^{\tau} r(s)ds \right) \left(\sum_{t_i > \tau} \hat{P}_i B(\tau, t_i) - \left(\hat{R}_{j(\tau)} + \hat{A}_{j(\tau)} + \hat{K}_{j(\tau)} \right) \right)^+ \right)$$

where

- $E(\cdot)$ denotes the risk-neutral measure,
- the short-interest rate, r, corresponds to the homeowner's cost of funds,
- $B(t,T) = E\left(\exp\left(-\int_t^T r(s)ds \right) \middle| F_t \right)$ where F_t denotes the information set up to time t,
- $j(\tau) = Max\left(j \in \{1,...,N\} \middle| t^* \le t_j \le \tau \right)$,
- $\hat{A}_{j(\tau)}$ is the accrued interest based of homeowner coupon over the interval $\left\lfloor t_{j(\tau)}, \tau \right\rfloor$,
- $\hat{K}_{j(\tau)}$ is the penalty interest.

Moreover, the cost to Treasury from the homeowner's refinancing option is given by

$$E\left(\exp\left(-\int_0^\tau r'(s)ds \right) \left(\sum_{t_i > \tau} P_i B'(\tau, t_i) - X'_\tau \right) \right) \tag{3.2.1}$$

Where

- r' denotes the Treasury cost of funds short interest rate,
- $B'(t,T) = E\left(\exp\left(-\int_t^T r'(s)ds \right) \middle| F_t \right)$,
- X'_τ is the remaining principal balance and accrued interest at time τ.

By the mortgage price, we mean the quantity, (3.1.1), less the quantity, (3.2.1). By the option price, we mean Formula (3.2.1).

Let T_i and R_i, for $i = 1, \ldots, n$, denote respectively a key term and the corresponding base rate (see https://finpricing.com/lib/IrBasisCurve.html). By the mortgage delta and gamma, we mean, respectively,

$$\sum_{i=1}^{n} \frac{\partial M}{\partial R_i}$$

And

$$\sum_{i,j=1}^{n} \frac{\partial^2 M}{\partial R_i \, \partial R_j},$$

where M denotes the mortgage price. By the option delta and gamma, we mean, respectively,

$$\sum_{i=1}^{n} \frac{\partial O}{\partial R_i}$$

And

$$\sum_{i,j=1}^{n} \frac{\partial^2 O}{\partial R_i \, \partial R_j},$$

where O denotes the embedded option price.

Consider a two-dimensional grid of Black's implied swaption volatilities,

$$(\sigma_{ij}), \text{ for } i = 1, \ldots, n \text{ and } j = 1, \ldots, m,$$

which is indexed by the swaption's forward start time and maturity. By the option Vega, we mean

$$\sum_{j=1}^{m} \sum_{i=1}^{n} \frac{\partial\, O}{\partial\, \sigma_{ij}}$$

where O denotes the option price.