# Mortgage Transfer Coupon Rate Computation

With respect to a closed commercial mortgage certificate, the transfer coupon rate is defined as an annualized, monthly compounded interest rate, such that the fair value of the closed mortgage certificate, from Treasury's point of view, is par. The model calculates the transfer coupon rate for forward starting, closed Canadian commercial mortgage certificates.

We assume that there are no scheduled and unscheduled principal payments during the life of the mortgage certificate.

Let n denotes the mortgage term in months, and t_0 and t_n denote the forward start date and the maturity date of the mortgage, respectively. Furthermore, let $\tau_1, ..., \tau_n$ denote dates that are obtained by rolling back the maturity date of the mortgage monthly until the forward start date of the mortgage. If the maturity date is the end of the month, then each τ_i $(i = 1, ... n-1)$ corresponds to the end of the month date. For example, if the mortgage term is 60 months and the maturity date is April 30, 2009, then τ_{60} is April 30, 2009, τ_{59} is March 31, 2009, τ_{58} is February 28, 2009, and, finally, τ_1 is May 31, 2004.

The payment dates, $t_1, ..., t_n$ are given by the following business day rule. The ith payment date, t_i, is equal to τ_i, for $i = 1, ..., n$, if τ_i is a weekday. On the other hand, if τ_i $(i = 1, ..., n)$ is either a Saturday or a Sunday, then t_i is set equal to the following Monday, provided that it falls in the same month; otherwise, t_i is set equal to the Friday immediately preceding τ_i.

See the schematic below for a pictorial description of the valuation date, forward start date and payment dates. Here, t^* denotes the valuation date; moreover, $t^* < t_0 < \ldots < t_n$.
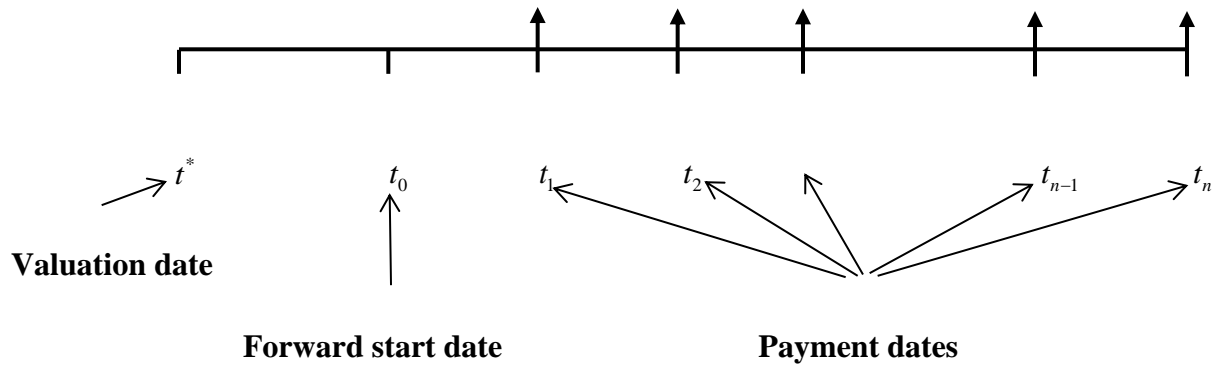
5.2 Discount Factors Generation for Key Terms

We use the previously vetted par-bond bootstrapper, employed by the previously vetted mortgage pricing spreadsheet, *Mtg.xls*, to generate discount factors based on Treasury rates (see https://finpricing.com/lib/IrCurveIntroduction.html) for key terms. This spreadsheet takes a set of terms and corresponding rates as input parameters. Table 5.2.1 below specifies the set of input terms used in our testing.

The output is a two-dimensional array, $\left\{ \left(\Delta_k, D_k \right) \right\}_{k=1}^{64}$, where D_k is the discount factor associated with the key term Δ_k. The output key terms range from one month to 30 years, that is,

$$\Delta_1 = \frac{1}{12} \text{ yr.}, \ \Delta_2 = \frac{2}{12} \text{ yr.}, \ \Delta_3 = \frac{3}{12} \text{ yr.}, \ \Delta_4 = \frac{6}{12} \text{ yr.}, \ \Delta_5 = \frac{9}{12} \text{ yr.}, \ \Delta_6 = 1 \text{ yr.},$$

And

$$\Delta_k = \Delta_{k-1} + 0.5 \text{ yr.}, \text{ for } k = 7,...,64.$$

Table 5.2.1 – Input Terms

	Term (Yr.)	Term (Yr.)	
	Table 6.1.	2	
	Table 6.1.	3	
corresponding rates with	0.250	4	corresponding rates with
terms less than one year	0.500	5	terms longer than one year
are expressed as	0.750	7	are expressed as an
annualized, simple	1.000	10	annualized semi-annually
interest rates		12	compounded rates
		15	
		20	
		30	

5.3 Calculating Discount Factors for Payment Dates

Let

$$\delta_i = \frac{\text{number of actual days between } t_i \text{ and } t^*}{365.25}, \quad \text{for } i = 0,...,n.$$

Moreover, let j be an integer such that $\Delta_j \le \delta_i < \Delta_{j+1}$, where Δ_j and Δ_{j+1} are key terms described in Section 5.2. We obtain the discount factor d_i for the term δ_i by log-linear interpolation,

$$d_i = exp\left[-\left(\frac{-ln(D_j)}{\Delta_j} + \frac{(\delta_i - \Delta_j)}{(\Delta_{j+1} - \Delta_j)} \times \left(\frac{-ln(D_{j+1})}{\Delta_{j+1}} + \frac{ln(D_j)}{\Delta_j} \right) \right) \times \delta_i \right],$$

where D_j and D_{j+1} are the discount factors associated with the terms Δ_j and Δ_{j+1}.

5.4 Monthly Principal Balance Generation

Let *MP* denotes the monthly mortgage payment. Then

$$MP = \frac{B_0 \times C/12}{\left[1 - \frac{1}{\left(1 + C/12\right)^N} \right]}$$

Where

- B_0 is the principal at the forward start time, t_0,

- C is the annualized monthly compounded coupon, and

- N is the number of amortization months of the mortgage.

We calculate the ith $(i=1,...,n)$ month end balance, B_i, as follows,

$$B_i = B_{i-1} - \left(MP - B_{i-1} \times C/12 \right).$$

5.5 Transfer Coupon Rate Calculation

We solve

$$B_0 \times d_0 = \sum_{i=1}^{n} \left(B_{i-1} - B_i \right) \times d_i + B_n \times d_n + \sum_{i=1}^{n} B_{i-1} \times \left(r/12 \right) \times d_i$$

for the unknown, r, that is,

$$r = \frac{B_0 \times d_0 - \sum_{i=1}^{n} \left(B_{i-1} - B_i \right) \times d_i - B_n \times d_n}{\sum_{i=1}^{n} B_{i-1} \times d_i/12}. \tag{1}$$

Next, we show that $r \geq 0$. Observe that

$$B_0 = \sum_{i=1}^{n} \left(B_{i-1} - B_i \right) + B_n \tag{2}$$

and assume that

$$0 < d_i < d_0 \leq 1, \text{ for } i = 1,...,n. \tag{3}$$

From (2) and (3),

$$B_0 \times d_0 - \sum_{i=1}^{n}(B_{i-1} - B_i) \times d_i + B_n \times d_n \geq 0,$$

which leads to the conclusion.

To check that payment dates are generated properly, we set the maturity date of one of them equal to a month end. Moreover, in one of the examples, we set the forward start date equal to the valuation date. In another example, we set the number of mortgage months to 120 instead of the standard number of 60. We define the transfer coupon rate relative difference, symbolically, as

$$\frac{\text{Treasury transfer coupon rate} - \text{Benchmark transfer coupon rate}}{\text{Benchmark transfer coupon rate}}.$$

We observed that the benchmark payment dates agreed with the payment dates generated