Mortgage Cash Flow Modelling

We model the closed monthly cash flows from a pool of mortgage. Here cash flows consist of principal and interest payments. Principal payments arise from the regular amortization of principal, as well as from scheduled and unscheduled principal pre-payments.

The homeowner's initial monthly payment is calculated from a formula that depends on the average remaining amortization months. When the number of months is a whole number, this formula is equivalent to that for a standard annuity. The formula does not otherwise have an apparent financial interpretation. For more accuracy we recommend, when the number of months is not a whole number, that the initial monthly payment be calculated according to a practical financial treatment.

For a closed mortgage, the penalty from liquidation is legally the full interest rate differential. In practice the penalty interest from liquidating a closed mortgage may be taken as bigger than zero but less than the full interest rate differential. Due to lack of data, we model the penalty interest as that from an open mortgage with penalty. This treatment provides for a conservative estimate of the mortgage's price. For greater accuracy, however, we recommend that the penalty interest for a closed mortgage be estimated more robustly (e.g., from empirical data).

A liquidation event is assumed to be independent of the interest rates (see https://finpricing.com/lib/IrCurve.html) prevailing at the time of its occurrence.

The closed cash flows from a pool of mortgages are calculated using

- the remaining principal balance, P,
- the weighted average coupon, c,
- the average remaining amortization months, n,
- a scheduled principal pre-payment rate, s,
- an unscheduled principal pre-payment rate, u,
- a term in months, m.

Here the term, m, is the maximum number of future monthly cash flows to be computed; a fewer number of cash flows may be calculated, however, if the remaining principal balance goes to zero at a month prior to the m^{th}. The remaining principal balance is as of the payment date immediately prior to the valuation date.

We assume that the cash flows from a mortgage pool occur on the first day of each future month. Here the i^{th} first day of the month from the valuation date is mapped to the value

$$\frac{i}{12}$$

less an offset. Let d be the difference, in actual days, from the valuation date to the payment date immediately preceding it. Then the offset is taken as

$$\frac{d}{365.25}.$$

We understand that a robust day counting scheme, which also takes into account holiday schedules

In the above, the weighted average coupon is expressed as an annualized, semi-annually compounded percentage, and the scheduled and unscheduled principal pre-payment rates are both given as annualized, annually compounded percentages. Based on the day counting convention above, an equivalent, monthly compounded, average coupon is given by

$$C = \left(1 + \frac{c}{200}\right)^{\frac{1}{6}} - 1 .$$

An equivalent, monthly compounded scheduled pre-payment rate is given by

$$S = 1 - \left(1 - \frac{s}{100}\right)^{\frac{1}{12}} .$$

An equivalent, monthly compounded unscheduled pre-payment rate, U, is similarly calculated. These cash flows are determined sequentially as described below.

Initial Monthly Payment

We set the monthly payment on the first day of the month after the valuation date equal to

$$\frac{CP}{1 - \frac{1}{(1+C)^n}} .$$

Observe that, if the average remaining amortization months, n, is a whole number, then the formula above is equivalent to a standard annuity payment,

$$\frac{P}{\sum(1+C)^{-i}}.$$

If n is not whole, then the formula for the initial monthly payment does not have an apparent financial interpretation. The initial monthly payment, however, has the correct qualitative behaviour when n is non-whole, since it is monotonically decreasing as a function of n (for $n > 0$).

Interest and Principal From Regular Amortization

Let M denote the monthly payment of regular amortized principal and interest due on the first day of a future month. The interest owing on this day equals

$$BC,$$

and the regular amortized principal owing equals

$$M - BC,$$

where B denotes the principal balance remaining on the first day of the previous month.

Scheduled Principal Pre-Payment

If the remaining principal balance after the monthly payment above,

$$R = B - (M - BC),$$

is positive, then a scheduled principal pre-payment is taken as a percentage of R. In particular, the scheduled principal pre-payment on this day is taken as

$$SR.$$

Unscheduled Principal Pre-Payment

Unscheduled principal pre-payment is taken as a proportion of the principal balance that remains after the scheduled principal payment. Specifically the unscheduled principal payment is taken on this day as

$$R(1-S)U.$$

Penalty Interest

Unscheduled principal pre-payments model liquidation events, and are subject to penalty interest. Since the mortgage is closed, the homeowner is legally bound to cover the full interest rate differential (IRD) upon liquidation. **The treatment of penalty interest varies from branch to branch, and that the homeowner does not typically pay the full IRD.** In practice the penalty interest from liquidating a closed mortgage may be taken as bigger than zero but less than the full IRD.

We model the penalty interest as a conservative proportion,

$$\min(3\%, 3R),$$

of the unscheduled principal payment above. This fee has the form of that from an *open mortgage with penalty*.

Total Payment

In summary the total monthly payment on the first day of the month, except for possible penalty interest, equals

$$M + R\left(S + U[1 - S] \right),$$

and the remaining principal balance is given by

$$R(1 - S)(1 - U).$$

The monthly payment due on the first day of the next month is taken as the current, but proportionately reduced by the percentage of unscheduled principal pre-payment, that is,

$$(1 - U)M \ .$$

Let

- a denote the MBS coupon expressed as an annualized, semi-annually compounded percentage,
- ω denote the ratio of the penalty interest paid to the MBS issuer.

Cash flows for a mortgage backed security (MBS) are based on those generated for the homeowner.

Total Payment

The MBS total monthly payment is given by the sum of the regular principal payment,

$$M - BC,$$

scheduled principal payment,

$$SR,$$

and unscheduled principal payment,

$$R(1 - S)U,$$

calculated as in Section 2.3.1 for the homeowner, plus regular interest based on the MBS coupon,

$$BA,$$

where A is the equivalent, monthly compounded MBS rate.

Penalty Interest

The penalty interest from liquidation is taken as the fraction,

$$1 - \omega,$$

of that calculated for the homeowner.

Let N denote the normal servicing fee, expressed as a monthly compounded rate. Then the monthly total payment for normal servicing is taken as the remaining principal balance from the previous month times the servicing fee,

$$BN,$$

where B is calculated from Section 2.3.1 for the homeowner.

The monthly total payment for excess servicing is taken as

$$B(R - A - N)$$

where

- B is the remaining principal balance, calculated for the homeowner, as of the previous month,
- R is the homeowner's coupon,
- A is the MBS coupon, and
- N is the normal servicing fee.