Pricing Partial Barrier Option

A model is presented for pricing certain types of European, continuously monitored partial barrier options. The method is based on certain analytical formulas, for pricing such options.

We consider certain types of European, continuously monitored partial barrier

- down and out (D_O),
- down and in (D_I),
- up and out (U_O), and
- up and in (U_I)

call and put options. By a partial barrier option, we mean an option where the barrier monitoring period is limited to a subinterval of the option's lifetime. Specifically, we consider options where the barrier monitoring period either

- begins at option onset and ends at a time before the option's expiry (called Type A), or
- starts at a time before option expiry, but after the option onset, and ends at option expiry (called Type B2).

A European D_O, Type A partial barrier option specification, for example, includes the option tenor, T_{opt}, a strike level, K, a barrier level, H, and a window endpoint time, T_{win}, where

$0 < T_{win} \leq T_{opt}$. The payoff at maturity for the option above is that of a standard call option, if the underlying security price lies above the barrier level for all time in the window $[0, T_{win}]$; otherwise, the option knocks out, and the payoff is zero. Formally the payoff at maturity is defined as

$$\begin{cases} \max(S_{T_{opt}} - K, 0), & \text{if } S_t > H, \text{ for all } t \in [0, T_{win}], \\ 0, & \text{otherwise,} \end{cases}$$

where S_t denotes the price of the underlying security at time *t*.

Using the notation above, the payoff for a European D_I, Type B2 partial barrier call option is that of a standard call option, if the underlying security price lies below the barrier at some time in the window $[T_{win}, T_{opt}]$; otherwise, the payoff is zero. The payoff at maturity is formally defined as

$$\begin{cases} \max(S_{T_{opt}} - K, 0), & \text{if } S_t \leq H, \text{ for some } t \in [T_{win}, T_{opt}], \\ 0, & \text{otherwise.} \end{cases}$$

The method for pricing the types of partial barrier options assumes that the underlying security follows geometric Brownian motion with constant drift and volatility. Furthermore the method is based on certain analytical pricing. We note that these formulas include certain bivariate cumulative normal distribution terms, which the method approximates using an analytical technique based on Drezner’s approach.

However, for particular option parameter values (e.g., including, but not limited to, extremely low volatility or where there is a large spread between the barrier level and initial underlying security value), certain partial barrier option pricing formulas may greatly amplify the truncation error in Drezner's approximation. If this is found to be a problem in practice, then a more accurate approximation to the bivariate cumulative normal distribution should be employed.

With respect to Type B2 options, the method constrains the barrier level and initial value for the underlying security, H and S_0, as follows

- $S_0 > H$ for D_O and D_I options, and
- $S_0 < H$ for U_O and U_I options.

We considered options specified as follows

- option tenor equal to three or nine months,
- barrier endpoint equal to one half of option tenor,
- window specified as Type A or Type B2,
- spot, risk free rate and dividend yield respectively fixed at 100, 10% and 5%,
- strike chosen to produce at-the-money, in-the-money, or out-of-the money options (specifically, strike was set equal to 90, 100 or 110),
- barrier level chosen either below the lowest strike or above the highest strike level (specifically, the barrier level was set to either 85 or 115),
- volatility equal to 10%, 25% or 50%.

Notice that the value of a Type A option, where the barrier endpoint is set to the option expiry, is equal to that of a corresponding standard single barrier option (i.e., where the barrier is monitored throughout the option's lifetime). Furthermore, analytical formulas are available for pricing standard single barrier options. We priced a certain single barrier down and out call option using both the implementation and a theoretical pricing formula. For this test case, the theoretical and option price agreed to 4 decimal places.

Reference:

https://finpricing.com/lib/IrBasisCurve.html