Delta Gamma Vega Value at Risk Model

The Delta Gamma Vega (DGV) methodology is developed to estimate Value-at-Risk (VaR) for portfolios of equities and equity options in order to comply, in regard to market risk measurement. The model can accurately estimate over-night VaR for portfolios with non-zero convexity or linear risk.

In principle, the accuracy of the DGV methodology can be easily improved by using more sensitivities, but in practice the difficulty is measuring the sensitivities other than delta, gamma, Vega, rho, and theta.

VaR over D*t* is given as follows:

$$VaR[V(t)] = E[\Delta V(t)] - 3\sqrt{E[\Delta V^2(t)] - E^2[\Delta V(t)]} \qquad (1)$$

where D*V*(*t*) is a random variable describing the change in portfolio value over D*t* . The DGV methodology takes delta, gamma, Vega, rho, and theta sensitivity measures of the equity-options book as inputs to an analytical approximation to D*V*(*t*) for an equity/equity-option portfolio.

The DGV methodology assumes that the log-relative changes in equity-prices, implied volatilities, and interest-rates (see https://finpricing.com/lib/FxForwardCurve.html) are normally distributed. The method also uses linear regression to describe stock returns in terms of market index returns (CAPM) to reduce the dimension of the variance-covariance matrix and to capture stock specific risk.

Our method consists of using Monte Carlo and re-implementing DGV to estimate $E[\Delta V(t)]$, $E[\Delta V^2(t)]$, $\text{var}[\Delta V(t)]$, which is the variance, and $VaR[V(t)]$ for the following portfolios when Δt =1/250 years.

Portfolio I: 1 stock and 0 options
Portfolio II: 4 stocks and 0 options
Portfolio III: 1 stock and 2 options with no hedges
Portfolio IV: 1 stock and 1 option where delta is hedged
Portfolio V: 1 stock and 2 options where delta, gamma, and Vega are hedged
Portfolio VI: 1 stock and 5 options where delta, gamma, Vega, rho, and theta are hedged.

When a sensitivity is “hedged” it means that the sensitivity is zero. For example, in Portfolio IV where delta is hedged, this means that the delta of this portfolio is identically zero. The options used in the various portfolios and distributional assumptions are given in appendices 2 and 3, respectively.

The Monte Carlo estimate of $VaR[V(t)]$ is given as follows :

$$VaR_{MC}\left[V(t)\right] \equiv E_{MC}\left[\Delta V(t)\right] - 3\sqrt{\text{var}_{MC}\left[\Delta V(t)\right]}$$

where $E_{MC}[\Delta V(t)]$ and $\text{var}_{MC}[\Delta V(t)]$ are the Monte Carlo estimates of $E[\Delta V(t)]$ and $\text{var}[\Delta V(t)]$, respectively.

The re-implementation part of the testing is used to establish VaR benchmarks that the DGV methodology must agree with. The Monte Carlo part of the testing is used to measure the impact of sensitivities not captured by the DGV methodology. For example, the DGVRT approximation omits terms

The DGV methodology does not capture the sensitivity in the change in portfolio value whereas the Monte Carlo method accounts for this sensitivity and all others. The Monte Carlo VaR estimates should show large deviations from the DGV VaR estimates for portfolios V and VI because an most of the DGVR sensitivities are perfectly hedged in these portfolios.

The implementation of the DGV methodology matched our benchmarks. The Monte Carlo estimates agree with the DGV estimates for $E[\mathrm{D}V\ 2\ (t)]$, $\mathrm{var}[\mathrm{D}V(t)]$, and $VaR[V(t)]$ to within a few percent for portfolios I to IV. The exceptional case appears to be the Monte Carlo estimate for $E[\mathrm{D}V(t)]$ in portfolio II which is 3.46 %. In absolute terms, however, this is well within the standard probabilistic error estimate ± var [( )] / MC DV t N where N is the number of trials.

There are significant differences between the Monte Carlo and DGV estimates of $E[\mathrm{D}V(t)]$, $E[\mathrm{D}V\ 2\ (t)]$, $\mathrm{var}[\mathrm{D}V(t)]$, and $VaR[V(t)]$ for portfolios V and VI because in portfolio V, the delta, gamma, and Vega sensitivities are hedged and in portfolio VI all of the sensitivities used in the DGV methodology are hedged.

When portfolios are not gamma hedged, the DGV methodology can be quite accurate in estimating over-night VaR although the accuracy diminishes as the convexity of the portfolio approaches zeros.

As more of the DGV sensitivities are hedged, the methodology does a poor job of capturing over-night VaR.

It should be noted that in theory it is always possible to construct a portfolio where any finite number of sensitivities are hedged. In practice, however, it is very unlikely that all of the DGV sensitivities are hedged. A more typical situation is where only delta is hedged and our testing done with portfolio IV indicates that for over-night VaR, the DGV methodology can be quite accurate in estimating VaR.

Furthermore, the DGV methodology can be easily extended to capture as many sensitivities as is required but sensitivities other than delta, gamma, Vega, rho, and theta are not typically measured in a standard equity-options book.

In portfolios I , III, IV, V, and VI it is assumed that the daily log-relative change in the stock price is normally distributed with a mean 0.001048 and variance of 4.441x10-5 . The daily log-relative changes in the interest rates and implied volatilities are also assumed to be normally distributed with means 0.00012 and 0.00021 and variances 0.00016 and 0.00021 respectively. In portfolio II , the daily log-relative changes in the stock prices are assumed to be normally distributed with zero means and standard deviations and correlation matrix.