Asian Swap Model

A model is developed for valuing a swap1 between party A and party B. Here party A receives a fixed amount and makes a single variable payment at swap maturity. The payment amount can be modeled as the value of a European discrete Asian call option on a basket of indices. Here the basket price consists of an arithmetic average of various stock and bond indices. The call option payoff at maturity is equal to maximum of zero and an arithmetic average of basket values at certain points in time prior to option expiry less a fixed strike.

We assume that each index price follows, under the corresponding risk neutral probability measure, geometric Brownian motion with drift. Since the call option pays in Canadian dollars, however, the index price process is represented with respect to the Canadian risk neutral probability measure by means of a quanto adjustment.

Observe that the basket price process does not follow geometric Brownian motion with drift; furthermore, the arithmetic average of basket values is not lognormally distributed. We approximate the basket price process, based on an analytical moment matching technique consistent with [Levy, 1992], using a single geometric Brownian motion with drift. Relevant defining drift and volatility values for this single geometric Brownian motion are computed.

The arithmetic average of the resulting approximate basket price process is further approximated, based on a different analytical moment matching technique, using a shifted lognormal random variable. The call option price is then computed as a discounted expected value of the maximum of zero and the shifted lognormal random variable value less the fixed strike. Here, relevant defining parameters for the shifted, lognormal random variable are computed.

In addition, European discrete Asian options were specified as above, but with a single observation point. In these cases the option price was bench-marked using the Monte Carlo method described above, as well as, a low-discrepancy sequence (cyclotomic point) integrator. Similar results as above were observed.

Let *Sti* , □a*i* and *Li* , for *i* = 1,..., *M* (where *M* = 5), respectively denote

- the price of the *i th* index at time *t* ,
- the weight corresponding to the *i th* index, and
- the initial level corresponding to the *i th* index.

Then the variable amount payable by party A at swap maturity, *T* , is defined as.

$$\text{Notional} \times \max\left[\sum_{i=1}^{M} \hat{\alpha}_i \left(\frac{\left[\frac{1}{N} \sum_{j=1}^{N} S_{t_j}^{i} \right] - L_i}{L_i} \right),\ 0 \right]. \qquad (2.1)$$

Observe, however, that (2.1) is equal to

$$\text{Notional} \times \max\left[\left(\frac{1}{N} \sum_{j=1}^{N} Z_{t_j} \right) - 1,\ 0 \right]$$

The amount payable by party A, then, is given as the notional amount multiplied by the value of a European discrete Asian call option

- on an underlying basket of indices with price, *Zt* , at time *t* ,
- with discrete observation points, {*t i N*} *i* = 1,..., (with *N* = 12), where 0 1 < *t* < < *t* < *T N* ... ,
- with strike equal to 1,
- with maturity equal to *T* , and
- payoff at maturity equal to $\max\left[\left(\frac{1}{N} \sum_{j=1}^{N} Z_{t_j} \right) - 1,\ 0 \right].$

We assume that the process {*S t T* } *ti* Î[0,] (*i* = 1,..., *M*) satisfies, under the corresponding risk-neutral probability measure, a stochastic differential equation (SDE) of the form

$$dS_t^i = S_t^i\left(\left[r_i - q_i\right]dt + \sigma_i d\hat{W}_t^i\right), \qquad t \in [0, T],$$

where

· *ri* , *qi* and s*i* respectively denote constant risk-free rate (see https://finpricing.com/lib/FxForwardCurve.html), dividend yield and volatility values, and

· {*W*, *t* [,*T*]} *ti* Î 0 is standard Brownian motion.

Let *Cti* , for *i* = 1,..., *M* , denote the value of one unit of foreign currency in terms of Canadian currency units. Assume that the stochastic process {*C t T* } *ti* Î[0,] (*i* = 1,..., *M*) satisfies, under the Canadian risk neutral probability measure, an SDE of the form

$$dC_t^i = C_t^i\left(\left[r_c - r_i\right]dt + \sigma_i^C dB_t^i\right), \qquad t \in [0, T],$$

Then, under the Canadian risk-neutral probability measure, for *i* = 1,..., *M* ,

$$dS_t^i = S_t^i\left(\mu_i dt + \sigma_i dW_t^i\right), \qquad t \in [0, T],$$

Here r*iC* represents the constant, instantaneous correlation coefficient between the Brownian motions {*B t T* } *ti* Î[0,] and {*W t T* } *ti* Î[0,] . We also assume that the Brownian motions {*W t T* } *ti* Î[0,] and {*W t T* } *tj* Î[0,] , for *i*, *j* = 1,...,*M* , have constant, instantaneous correlation coefficient equal to r*ij* .

Let a*i* (with a*i* ³ 0), for *i* = 1,.., *M* , denote a weight corresponding to the *i th* index; furthermore assume that at least one of the weights is positive. Let

$$Z_t = \sum_{i=1}^{M} \alpha_i S_t^i$$

denote the value of a basket of indices at time t . Observe, then, that the process {Z t [T]} t Î 0, does not follow geometric Brownian motion with drift. FP, however, approximates the basket price process using a single geometric Brownian motion with drift. That is, we consider a random variable, Xt , such that the stochastic process {X t T } t | Î[0,] satisfies, under the Canadian risk neutral probability measure, an SDE of the form

$$dX_t = X_t\left[\mu_X dt + \sigma_X dW_t^X\right], \qquad t \in [0, T],$$

where {W t T } tX | Î[0,] is standard Brownian motion. The aim is to determine the parameters m X and s X , in the SDE (3.2), to match the mean and variance of XT with that of ZT .

Since the approximate basket price process, {X t T } t Î[0,] , follows geometric Brownian motion with drift, the arithmetic average,

$$\hat{Y} = \sum_{k=1}^{N} \beta_k Z_{t_k} ,$$

where bk > 0 (k = 1,..., N), is not lognormally distributed. FP, however, approximates the arithmetic average above using a shifted, lognormally distributed random variable. That is, we consider a random variable of the form

$$Y = a + \exp(b + c\varepsilon)$$

where e is a standard, normally distributed random variable. Here the parameters a, b and c are uniquely determined by matching the first three moments of Y with the corresponding moments of CY (i.e., we require E([Y]) E(Y)D i = i , for i = 1,...,3).