Pricing Swap with Better-of Cliquet Option

A model is developed for pricing a swap with better of cliquet option. The floating amount payer makes semi-annual payments based on USD-LIBOR-BBA minus a spread. The fixed rate payer makes a single payment at swap maturity based on the arithmetic average of the S&P 500 Index price over certain pre-specified windows of ten consecutive trading days.

The swap's fixed leg price is given by the value of a certain path-dependent European option. This option value is computed using Monte Carlo, by simulating the price of the S&P 500 Index at the point in each trading day window corresponding to the latest date. The price at each such point is then taken as the value of the S&P 500 Index average over the corresponding window of trading days.

The rate of return for each averaging day window is based on the relative change in the arithmetic average of the S&P 500 Index price over this window as compared against the average over the immediately preceding window, but bounded above and below by 13.5% and 0 respectively. The payoff at swap maturity is based on the notional USD amount multiplied by the better of 25.5% and the sum of the rate of return over each averaging day window.

Formally let AV denote the arithmetic average of the S&P 500 Index price over a ten day averaging window. Then the rate of return for the ith (i = 1,...,8) window is given by

$$RET_i = \max\left(0,\ \min\left[13.5\%,\ \frac{AV_i - AV_{i-1}}{AV_{i-1}}\right]\right).$$

The amount payable at swap maturity is given by $N \times PAY$ where

$$PAY = \max\left(25.5\%,\ \sum_{i=1}^{8} RET_i\right)$$

and *N* denotes the notional

FP represents the S&P 500 Index price based on a stochastic process, which follows geometric Brownian motion with piecewise constant drift and constant volatility, and a related stochastic process. Let the time t *i* correspond to the last day in the *i th* averaging window. Then

$$dI_t = I_t\left(\mu_i dt + \sigma dW_t\right), \qquad t \in (\tau_{i-1}, \tau_i],$$

where

$$\mu_i = r_i - q\,,$$

$$r_i = \frac{\log\left(\dfrac{df_{\tau_{i-1}}}{df_{\tau_i}}\right)}{\tau_i - \tau_{i-1}} \quad (\text{with } \tau_0 = 0),$$

q and s respectively denote constant dividend yield and volatility parameters, and *W* denotes standard Brownian motion.

The related process, _ *I*, is given by

$$\log \hat{I}_t = \log I_{\tau_{i-1}} + \left(\mu_i - \frac{\hat{\sigma}^2}{2}\right)\left(t - \tau_{i-1}\right) + \hat{\sigma}\left(W_t - W_{\tau_{i-1}}\right), \qquad t \in (\tau_{i-1}, \tau_i]\,.$$

Observe that

· ! *I* is piecewise continuous, with discontinuities at the points t*i* ,

· ! I depends on the same Brownian motion as does the process I , but on a *different* constant volatility parameter, and

· ! I does not follow geometric Brownian motion with drift (since its sample paths are not continuous).

We approximate the S&P 500 Index price arithmetic average, AV , based on the values of I and ! I. In particular, *RETi* is approximated by

$$\hat{R}_i = \max\left(0,\ \frac{I_{\tau_i} - I_{\tau_{i-1}}}{I_{\tau_{i-1}}}\right) - \max\left(0,\ \frac{\hat{I}_{\tau_i} - I_{\tau_{i-1}}}{I_{\tau_{i-1}}} - .135\right).$$

Upon inspection of the pricing method, we note several technical points:

1. The FP method neglects the averaging of the S&P 500 Index price over the various averaging windows.

2. The FP method approximates the S&P 500 Index price based on a combination of a process, which follows geometric Brownian motion, and an associated piecewise continuous process, which does not follow geometric Brownian motion with drift.

3. The option price is based on the discount factor (see https://finpricing.com/lib/FxForwardCurve.html) for the tenth business day of September, 2006 (whereas the swap maturity is on the 13th business day of September, 2006).

4. When valuing the option on a day belonging to any of the averaging day windows specified in Section 2, the method does not take into account the S&P 500 Index price at previous days in this set. This may affect the accuracy of the arithmetic average value over this window of trading days

we assume that the S&P 500 Index price, *S* , follows geometric Brownian motion with piecewise constant drift and volatility; furthermore the process *S* is driven by single Brownian motion. Here we simulate all points in the various averaging day windows

Entries under the Shift in Spot column represents a shift in the spot S&P 500 Index price relative to the *base spot value* , that is, new spot value = base spot value ´ (1+ shift) . Entries under the Shift in Volatility column are similarly defined, but with respect to a base volatility value. GA and FP option prices were both based on 100,000 Monte Carlo paths.

The method neglects the averaging of the S&P 500 Index price over the various averaging day windows.

The method approximates the S&P 500 Index price based on a combination of a process, which follows geometric Brownian motion, and an associated piecewise continuous process. The associated process is driven by the same Brownian motion, but depends on a *different* constant volatility parameter.

When valuing the option on a day belonging to any of the averaging day windows, the method does not take into account the S&P 500 Index price at previous days in this window. This may affect the accuracy of the arithmetic average value over this window of trading days.