Asian Basket Relative Performance Option Model

A pricing model for basket Asian relative performance options (RPO) is proposed by using Monte Carlo simulation. In a basket Asian RPO, payoff is determined by the difference between the performance of a reference stock and that of a basket of stocks, where performance is defined as the ratio of (weighted) average of two sets of averaging dates.

Let S_0 be a reference stock and $S_1, \cdots, S_N$ be *N* stocks in a given basket, $S_j(t)$ be the price process of the *j*th stock and $0 \le j \le N$. Let $\{t_1 < \cdots < t_n\}$ be an "initial" set of reset dates and $\{T_1 < \cdots < T_m\}$ a "final" set of reset dates, and assume $T_m > t_n$. The price of each stock is recorded on these reset dates to obtain price arithmetic averages of A_j^i and A_j^f corresponding to the initial set of reset dates and the final set of reset dates, respectively. Weighted averages of P^i and P^f are then defined as

$$P^i = \sum_{j=1}^{N} \omega_j A_j^i, \quad \text{and} \quad P^f = \sum_{j=1}^{N} \omega_j A_j^f ,$$

where $\omega_j, j = 1, \cdots, N$ are the weights for the stocks in the basket.

Let $T \ge T_m$ be a payoff settlement date or maturity. The basket Asian RPO is a European style derivative security whose matured payoff at the settlement date *T* is given by

$$S_0(T_m) \times f(\frac{A_0^f}{A_0^i} - \frac{P^f}{P^i})$$

where the functional $f(\cdot)$ specifies how many shares of S_0 should be rewarded as a function of the relative performance, $(\frac{A_0^f}{A_0^i} - \frac{P^f}{P^i})$. It should be noted here that stock dividends are assumed reinvested at the market price when the dividends are paid, so that the model simply assumes that the stocks do not pay dividends (see https://finpricing.com/lib/FxAccumulator.html).

Let *t* be the current value date, then the current value of this derivative security can be written as

$$df(t,T) \times E_t\left[S_0(T_m) \times f(\frac{A_0^f}{A_0^i} - \frac{P^f}{P^i})\right]$$

where $df(t,T)$ is the discount factor at the value date. The above formulae are in a world that is forward risk-neutral with respect to a specific currency C_p. If an underlying asset j is measured in another currency C_U, the governing price dynamics of this underlying asset in the risk-neutral world of C_p should be written as

$$dS_j(t) = (r_U - \rho\sigma_U\sigma_s)S_j(t)dt + \sigma_s S_j(t)dW_t^j, j = 0, \cdots, N$$

where r_U is the short rate of C_U, ρ is the correlation coefficient between the asset price and the cross-currency exchange rate, σ_s is the volatility of the asset price, σ_U is the volatility of the exchange price, and W_t^j is the Wiener process. Asset prices are correlated with

$[dW_t^j, dW_t^k] = \rho_{j,k} dt$ where $\rho_{j,k}$'s are constant correlation coefficients between the logarithmic asset prices. All these parameters are assumed deterministic.

Monte Carlo simulation associated with stratified sampling variance deduction is employed to evaluate the option. In the first sample transaction, there are six stocks, one as a reference, and the other five in a basket with equal weight. These underling stocks and the option are all measured in CAD.