Efficient solar scene wavefront estimation with reduced systematic and RMS errors

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INTRODUCTION

Wave front sensing for solar telescopes is commonly implemented with the Shack-Hartmann sensors.

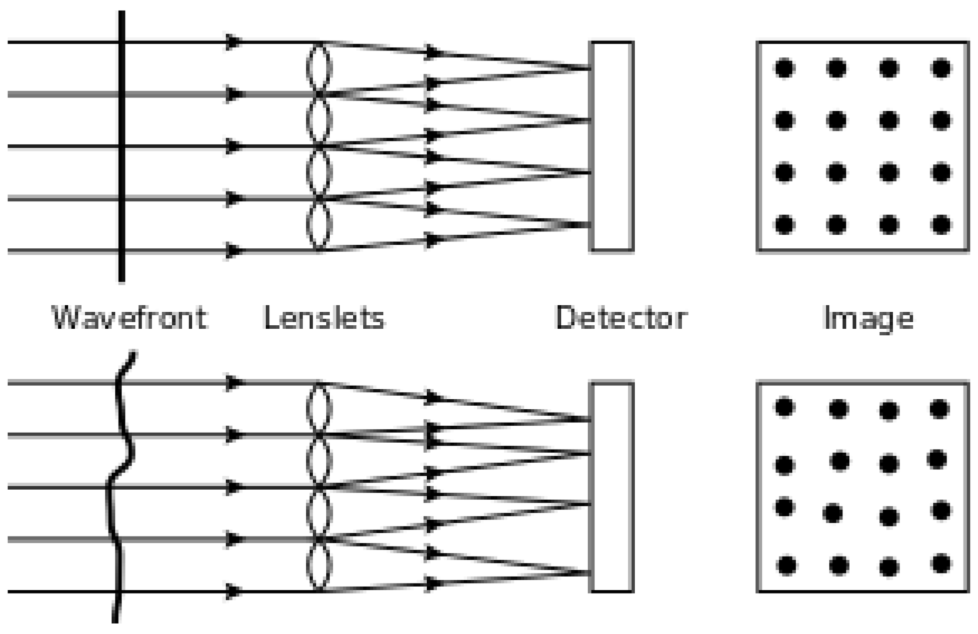


Fig. 1: Shack-Hartmann wavefront sensor.

The Shack-Hartmann lenslet sub-aperture solar image shifts/slopes are usually estimated with correlation algorithms. The sub-pixel precision image shifts are computed by applying a peak-finding algorithm to the correlation peak[1]. Usually, the measured image displacements consist of systematic errors due to pixel locking effects[2] (cf. Fig. 3.a), because correlation matching is limited only to an integer pixel grid. The cross-correlation can be computed in either Fourier or image domain as below

$$\text{CCF}\,(s_x', s_y') = \mathfrak{f}^{-1}\left\{ \mathfrak{f}\left[I_T(x+s_x, y+s_y)\right] \times \left[\mathfrak{f}\left(I_R(x,y)\right)\right]^* \right\}$$

$$\text{CCI}\,(s_x', s_y') = \sum_{x=1}^{N}\sum_{y=1}^{N} I_T(x+s_x, y+s_y) \times I_R(x,y)$$

Where,

CCF- cross correlation in Fourier domain; I_R- reference sub-aperture image; I_T - Target image

CCID - cross correlation in Image domain; s_x - true image shift; s_x' - measured image shift

$\mathfrak{f}$ - Fourier transform, * - complex conjugate

METHODS

The amplitude of the systematic error depends on the combination of the correlation algorithm chosen to compute the correlation peak and the type of peak-finding sub-pixel algorithm.

In this study, the systematic error reduction is carried out in two approaches. First, the performance of different cross-correlation peak finding algorithms is investigated. The algorithms are: parabola fit (PF)[3]; quadratic polynomial fit (QPF)[1]; threshold centre of gravity (TCoG)[4]; Gaussian fit (GF)[5] and Pyramid fit (PYF)[4].

$$\text{PF}: s_x' = x_0 + \frac{P(x_0-1, y_0) - P(x_0+1, y_0)}{2[P(x_0-1, y_0) + P(x_0+1, y_0) - 2P(x_0, y_0)]}$$

$$\text{GF}: s_x' = x_0 + \frac{ln[P(x_0-1, y_0)] - ln[P(x_0+1, y_0)]}{2\{ln[P(x_0-1, y_0)] + ln[P(x_0+1, y_0)] - 2ln[P(x_0, y_0)]\}}$$

$$\text{PYF}: s_x' = x_0 + \frac{P(x_0-1, y_0) - P(x_0+1, y_0)}{2\{min[P(x_0-1, y_0), P(x_0+1, y_0)] - P(x_0, y_0)\}}$$

$$\text{TCoG}: s_x' = x_0 + \frac{P(x_0-1, y_0) - P(x_0+1, y_0)}{6min[P(x_0-1, y_0), P(x_0+1, y_0)] - 2[P(x_0, y_0) + P(x_0-1, y_0) - P(x_0+1, y_0)]}$$

$$\text{QPF}: s_x' = x_0 + \frac{2a_1a_5 - a_2a_4}{a_4^2 - 4a_3a_5}$$

Where, polynomial $f(x) = a_0 + a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2$

Where, (x_0, y_0) is the correlation peak in integer pixels.

Second, a new method is proposed. It works in two steps.

1. First step, the cross-correlation is executed at the original image spatial resolution grid (1 pixel).
2. In second step, the cross-correlation is performed with a sub-pixel level grid by decreasing its image spatial sampling (S) and by confining the field of view to 4 x 4 pixels centred at the first step delivered initial position. The generation of these sub-pixel grid based search windows from the spatially discrete target image is achieved with bi-linear interpolation. This method is called as cross-correlation executed at continuous grid (CCIC).

This technique was previously reported in electronic speckle photography (Sjodahl 1994). This technique is now applied to wave front sensing. The combination of coarse level grid search executed in large field followed by quasi-continuous grid search executed in a small field enables one to achieve high accuracy wave front estimation by reducing the systematic errors with a low computational cost.

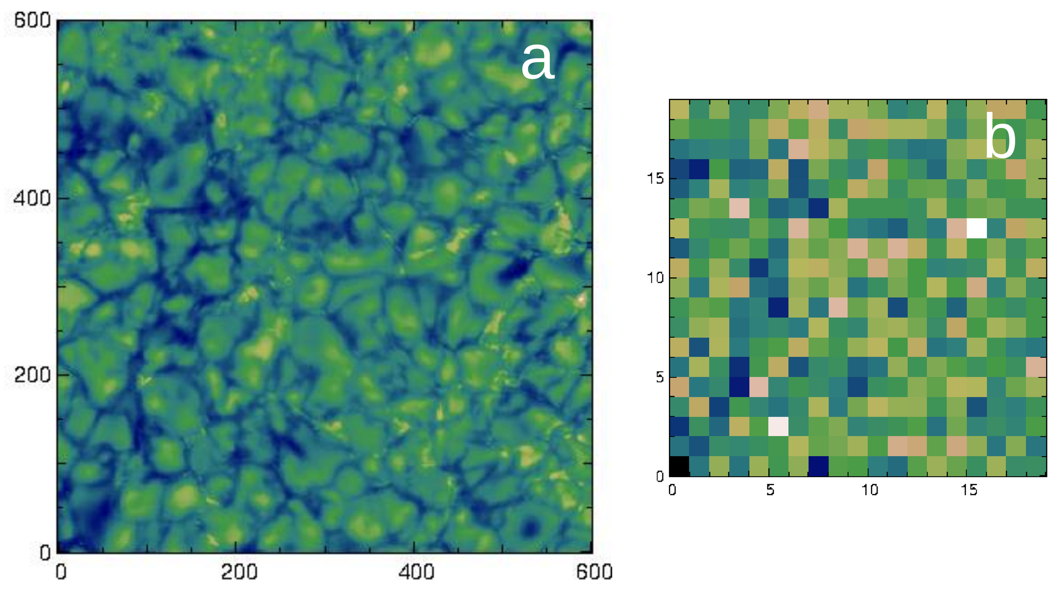
The systematic bias centroid error (measured image shift – true image shift) is

$$\beta_x = s_x' - s_x + \varepsilon_x$$

Where ε_x is the random error.

Total systematic deviation is $\delta = \sqrt{\beta_x^2 + \beta_y^2}$

SYNTHETIC IMAGES



To model a sub-aperture solar image, Swedish Solar Telescope imaged solar granulation image is taken[6]. Sub-aperture images are constructed by re-sampling the Solar image to the 0.41x0.41 arc-sec[2] pixel resolution using bi-cubic spline interpolation.

Fig. 2: a) The solar granulation image; b) A selected solar sub-aperture image.

RESULTS

- ✓ Pyramid fit (PYF) is the most robust to pixel locking effects.
- ✓ Threshold centre of gravity (TCoG) behaves better in low SNR although systematic errors in the measurement are large.
- ✓ No peak finding model is good enough in attenuating both systematic and RMS centroid errors.
- ✓ Proposed method (CCIC) outperforms all the peak finding algorithms performance. It improves the wave front estimation accuracy to a factor of 5 in terms of both systematic and RMS centroid error (75% systematic error reduction, for 0.2 pixel sub-sampling grid), at the expense of twice the computational cost. It is also observed that the method have very low failure rates.

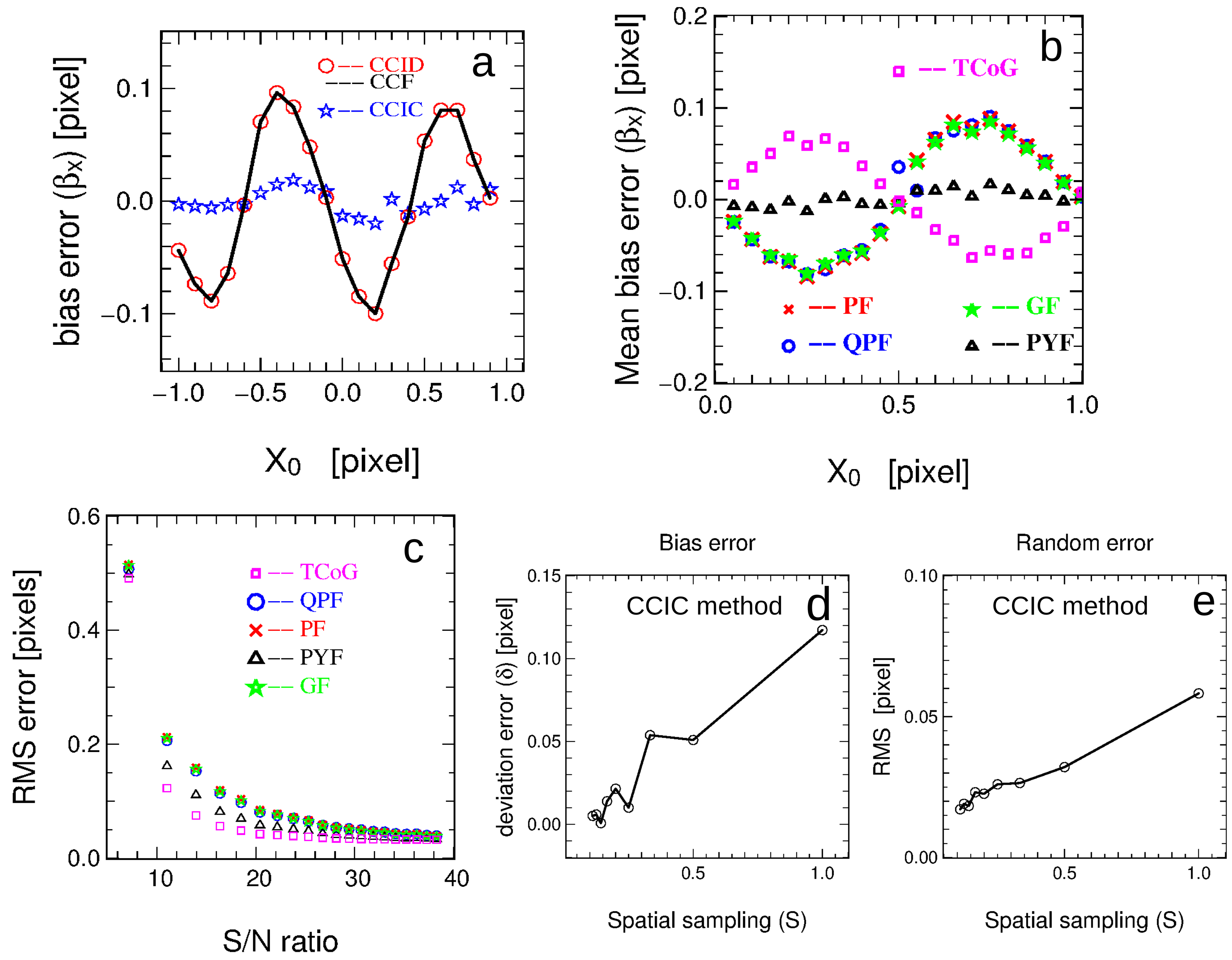


Fig. 3: a) Comparison of the CCI, CCF and CCIC algorithms for the systematic bias centroid errors; Comparison of peak finding algorithms for the bias centroid error (b) and the RMS centroid error (c); The (d) and (e) are the performance of CCIC algorithm with decreasing spatial sampling of image, i.e., correlation carried out at sub-pixel level grid instead of classical implementation of integer level grid.

DISCUSSIONS AND CONCLUSIONS

The CCIC method is strongly recommended for wave front sensing in solar telescopes, particularly in open loop adaptive optics, for measuring large dynamic shifts. Furthermore, by selecting appropriate sub-sampling in trade-off between the aimed sub-pixel image shift accuracy and the computational speed limitation, it can be employed in closed loop adaptive optics effectively.

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