The Role of the Convective Heat Transport in the Fully-convective Stars Dynamo

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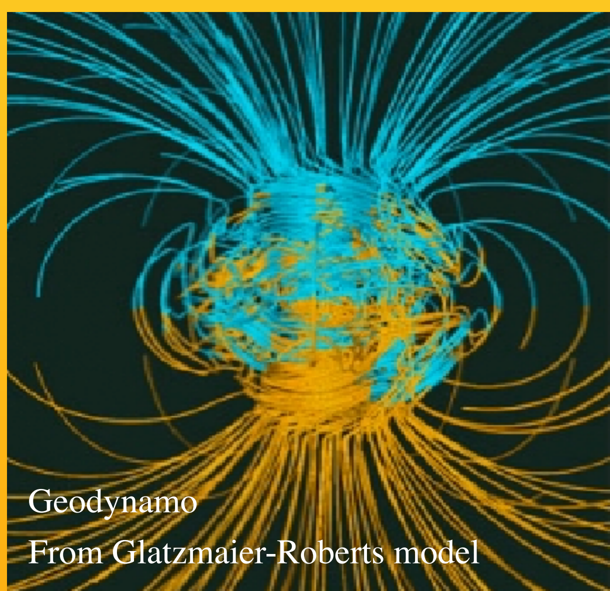
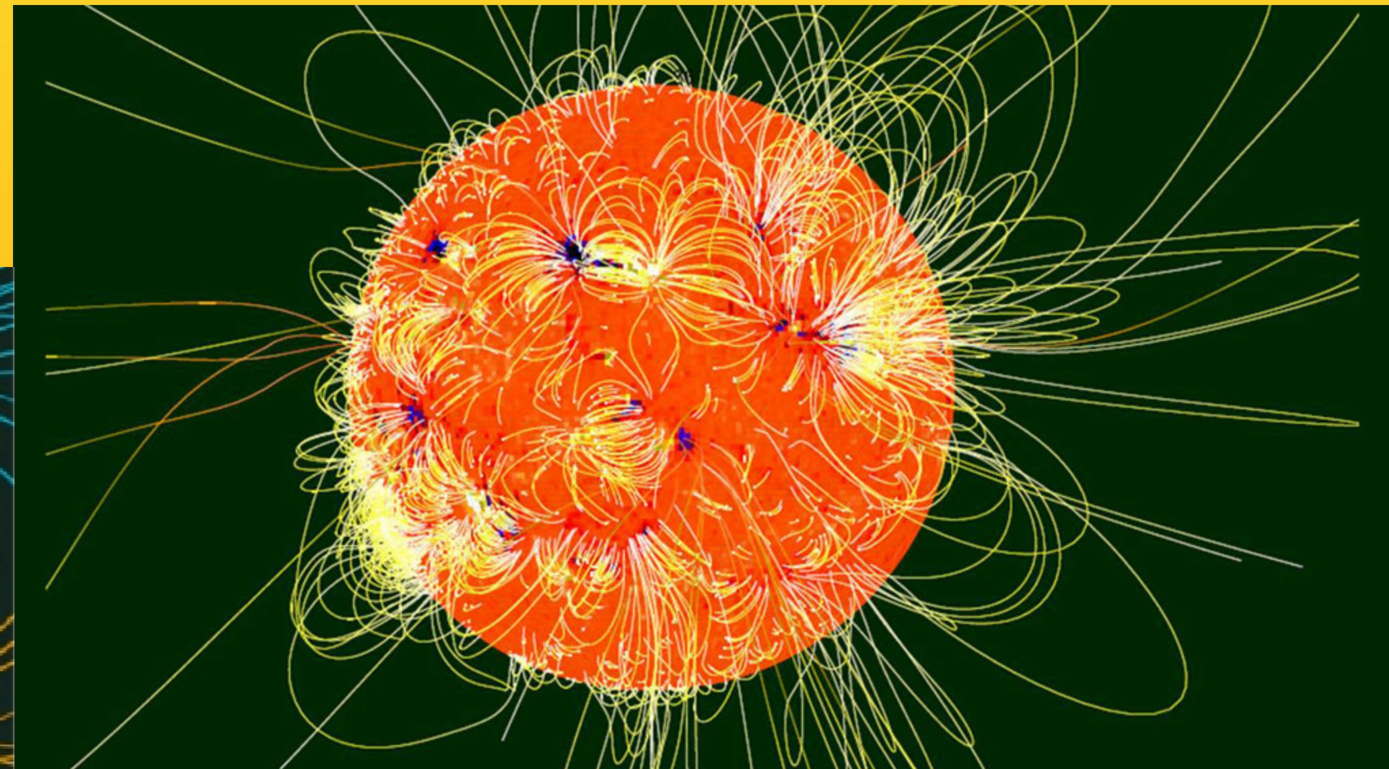
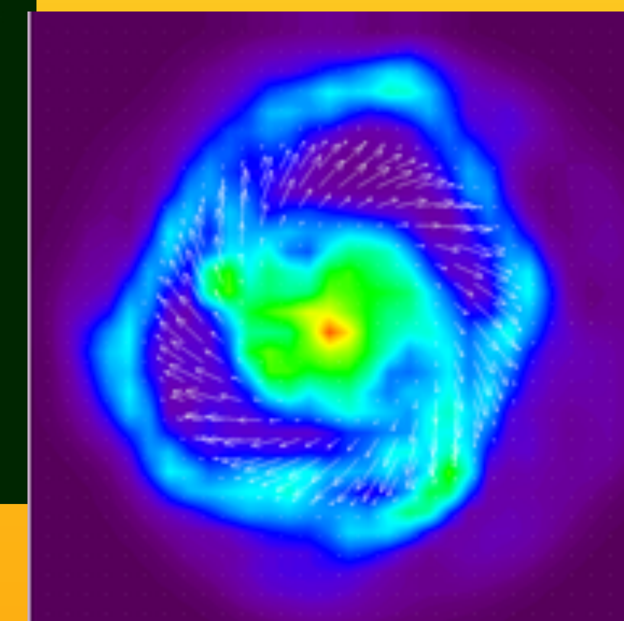
Magnetic Dynamo

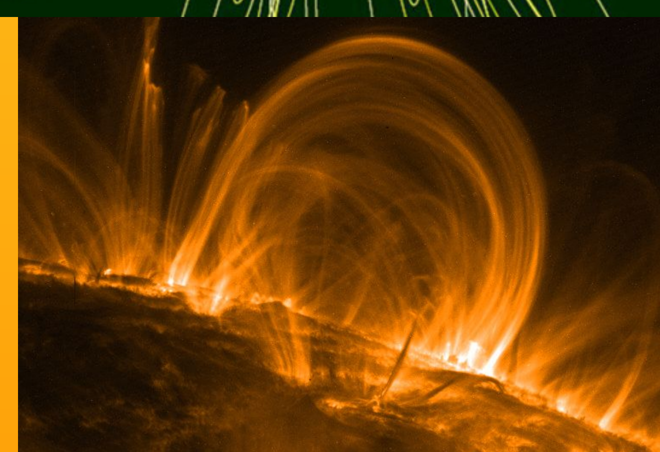
The fundamental phenomenon to explain the origin of the magnetic field in astrophysical systems, like planets, stars, interstellar and intergalactic medium, etc.



Geodynamo
From Glatzmaier-Roberts model

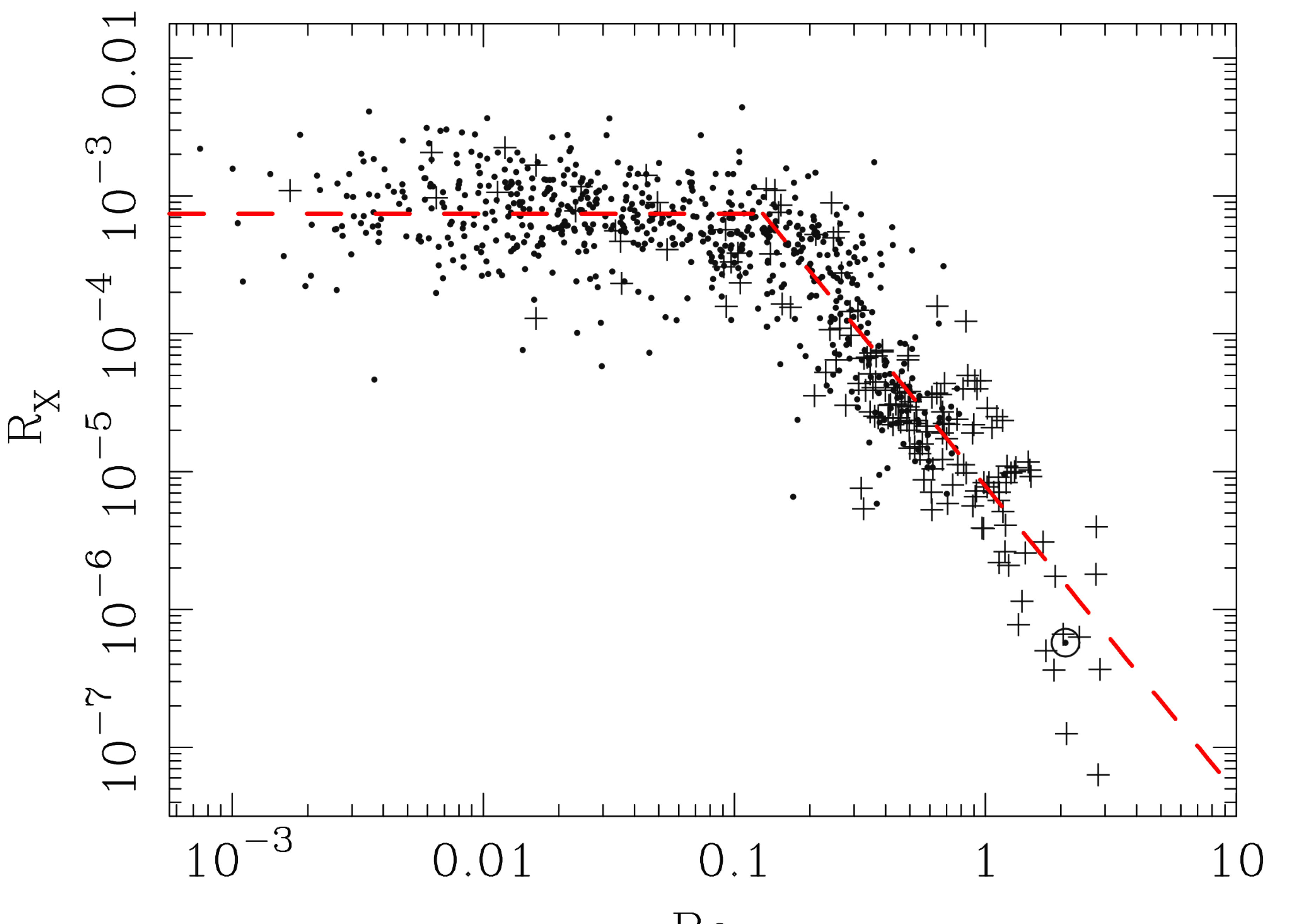






The generation and the dynamics of a magnetic field is described by the induction equation.
A realistic parameter regime is beyond the power of today's supercomputers (larges Re, Rm)

Stellar Dynamo:
The Activity-Rotation Relationship



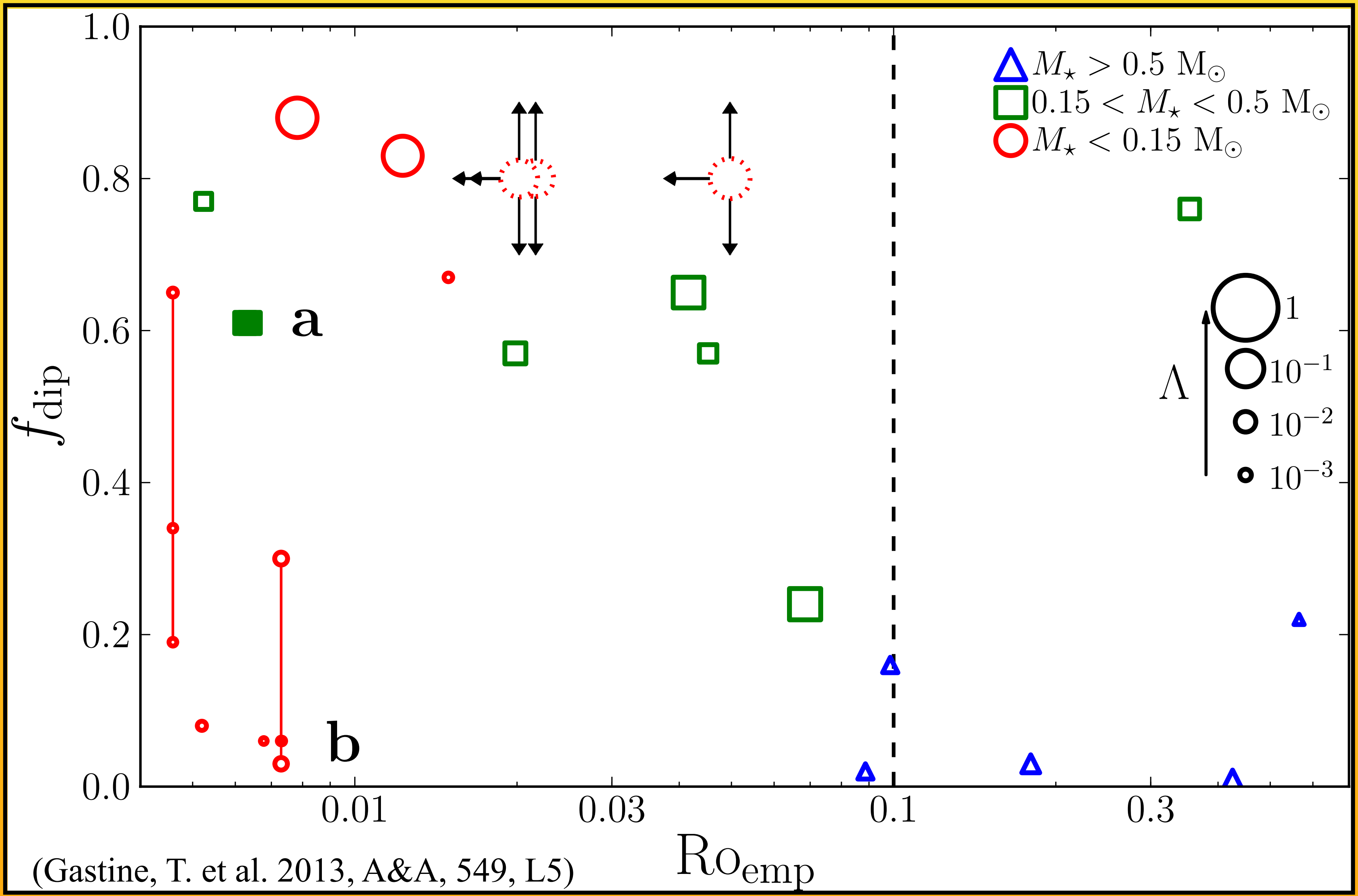
(Wright, N.J., 2011, ApJ, 743, 48)

Rossby number = Ro = Prot /τ , the ratio of the **stellar rotation period**, Prot , and the mass-dependent **convective turnover time**, τ.

α2 - dynamo

α Ω - dynamo

The Observed Magnetic Dichotomy



1. Periods shorter than 4 days
2. Similar masses
3. Stars of quite the same spectral type (from M3/M4 to late-type M dwarfs)

Late M dwarfs (with $M_\star < 0.15$ $M_\odot$) seem to operate in two different dynamo regimes: the first ones show a **stable dipolar magnetic field**. In contrast, others present a **magnetic structure with a significant time variability** (emphasised by the vertical red lines).

f_{dip} = dipolar field strength, i.e., the ratio of the magnetic energy of the dipole to the magnetic energy contained in spherical harmonic degrees up to $\ell_{max} = 11$.

$\Lambda = B_{rms}^2/\rho\mu\lambda\Omega$, where ρ is the density, and μ and λ are the magnetic permeability and diffusivity.

$$Ro_{emp} = P_{rot}/\tau_{conv}$$

Shell Models

$$\left(\frac{\partial}{\partial t} + \nu k^2\right) u_\mu(\mathbf{k}, t) = \sum_p M_{\mu\alpha\beta}\, u_\alpha(\mathbf{k} - \mathbf{p}, t)\, u_\beta(\mathbf{p}, t) \qquad \mu, \alpha, \beta = 1,2,3$$

$$M_{\mu\alpha\beta} = \frac{1}{2i}(D_{\mu\alpha} k_\beta + D_{\mu\beta} k_\alpha)\ , \qquad D_{\mu\alpha} = \left(\delta_{\mu\alpha} - \frac{k_\mu k_\alpha}{k^2}\right)$$

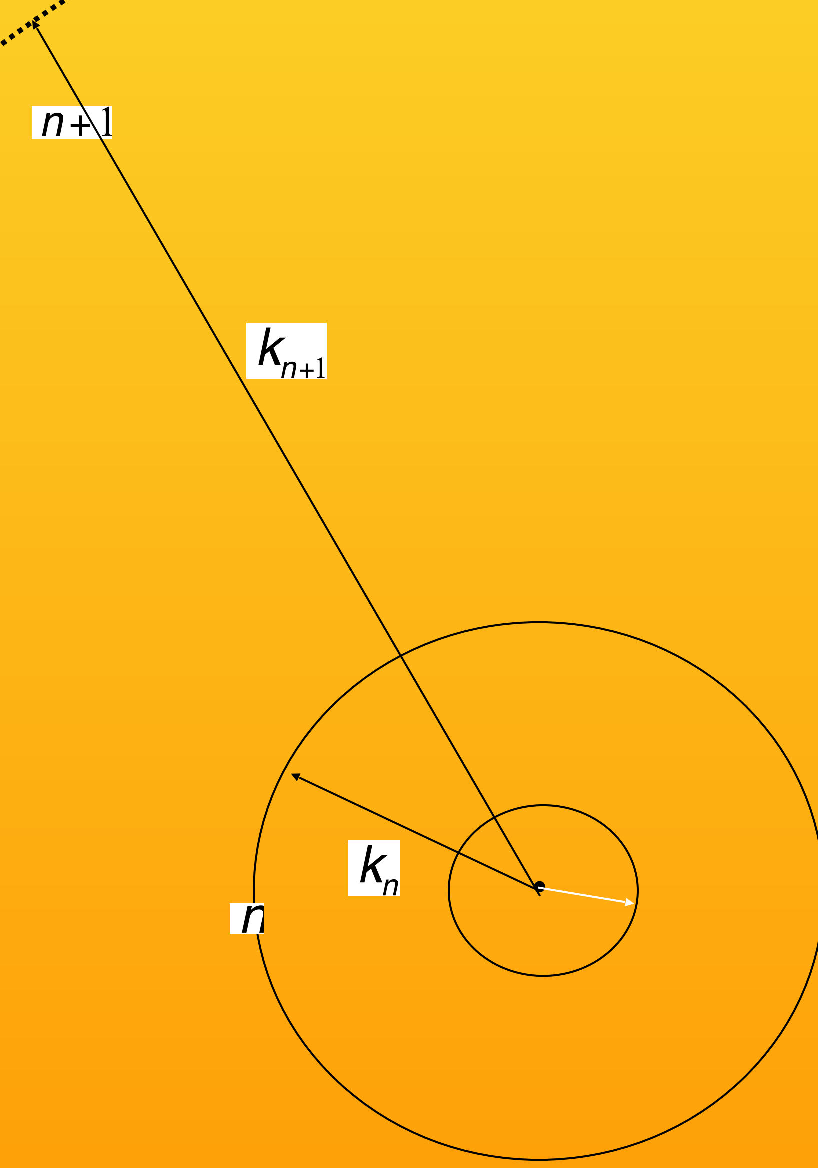
1) Introduce an exponential spacing of the wave vectors space (shells)

2) Assign to each shell dynamical variables

$$l_0 = 2\pi / k_0$$

$$u_n(t)$$

$$k_n = k_0 2^n$$
$$n = 1,2,\ldots, N$$



3) Nonlinear terms are written under the assumption that interactions in k-space are local and imposing that they conserve the quadratic invariants (i.e., kinetic energy in HD case and total energy, cross helicity, and magnetic helicity for an MHD fluid).

A Thermally Driven Shell Model for Magneto-convective Dynamo

$$\left(\frac{d}{dt} + \nu k_n^2\right)u_n = -\tilde{\alpha}\theta_n + ik_n\Big[(u_{n+1}u_{n+2} - b_{n+1}b_{n+2}) - \frac{\epsilon}{2}(u_{n-1}u_{n+1} - b_{n-1}b_{n+1}) - \frac{1-\epsilon}{4}(u_{n-2}u_{n-1} - b_{n-2}b_{n-1})\Big]^* \quad (1)$$

$$\left(\frac{d}{dt} + \eta k_n^2\right)b_n = ik_n\Big[(1-\epsilon-\epsilon_m)(u_{n+1}b_{n+2} - b_{n+1}u_{n+2}) + \frac{\epsilon_m}{2}(u_{n-1}b_{n+1} - b_{n-1}u_{n+1}) + \frac{1-\epsilon_m}{4}(u_{n-2}b_{n-1} - b_{n-2}u_{n-1})\Big]^* \quad (2)$$

$$\left(\frac{d}{dt} + \chi k_n^2\right)\theta_n = ik_n[\alpha_1 u_{n+1}^*\theta_{n+2}^* + \alpha_2 u_{n+2}^*\theta_{n+1}^* + \beta_1 u_{n-1}\theta_{n+1} - \beta_2 u_{n+1}\theta_{n-1} + \gamma_1 u_{n-1}\theta_{n-2} + \gamma_2 u_{n-2}\theta_{n-1}]^* + f_n, \quad (3)$$

G. Nigro, 938, 22, 1 (2022) **ApJ**
G. Nigro et al., under review, **MNRAS**

(GOY) MHD Shell Model

$\epsilon = 1/2 \qquad \epsilon_m = 1/3$

Jensen et al. (PRA, 1992)'s coupling model

$\alpha_1 = \alpha_2 = 1$

$\beta_1 = \beta_2 = 1/2$

$\gamma_1 = \gamma_2 = -1/4$

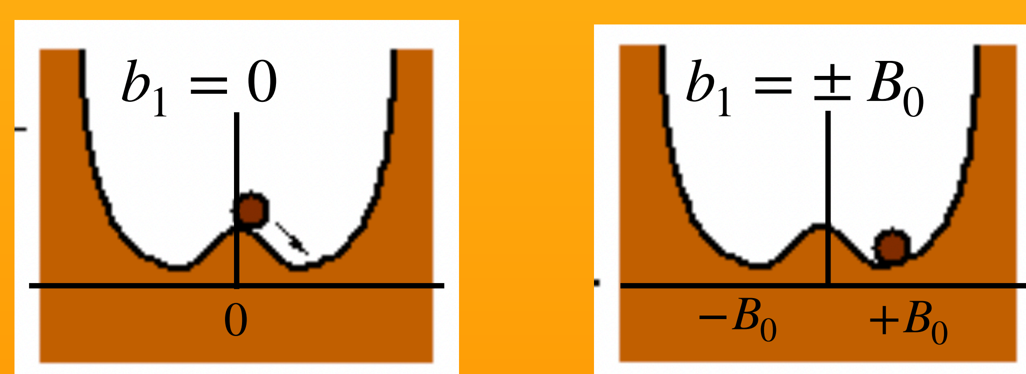
α2 dynamos => We Modified the equation for the largest scale magnetic field

$$\alpha = \mu\left(1 - \frac{b_1^2}{B_0^2}\right)$$

Evolution equation of the large-scale magnetic field:

$$\frac{db_1}{dt} = -\eta k_1^2 b_1 + i\frac{k_1}{6}\left(u_2^* b_3 - b_2^* u_3\right) + \mu b_1\left(1 - \frac{b_1^2}{B_0^2}\right)$$

pitchfork bifurcation

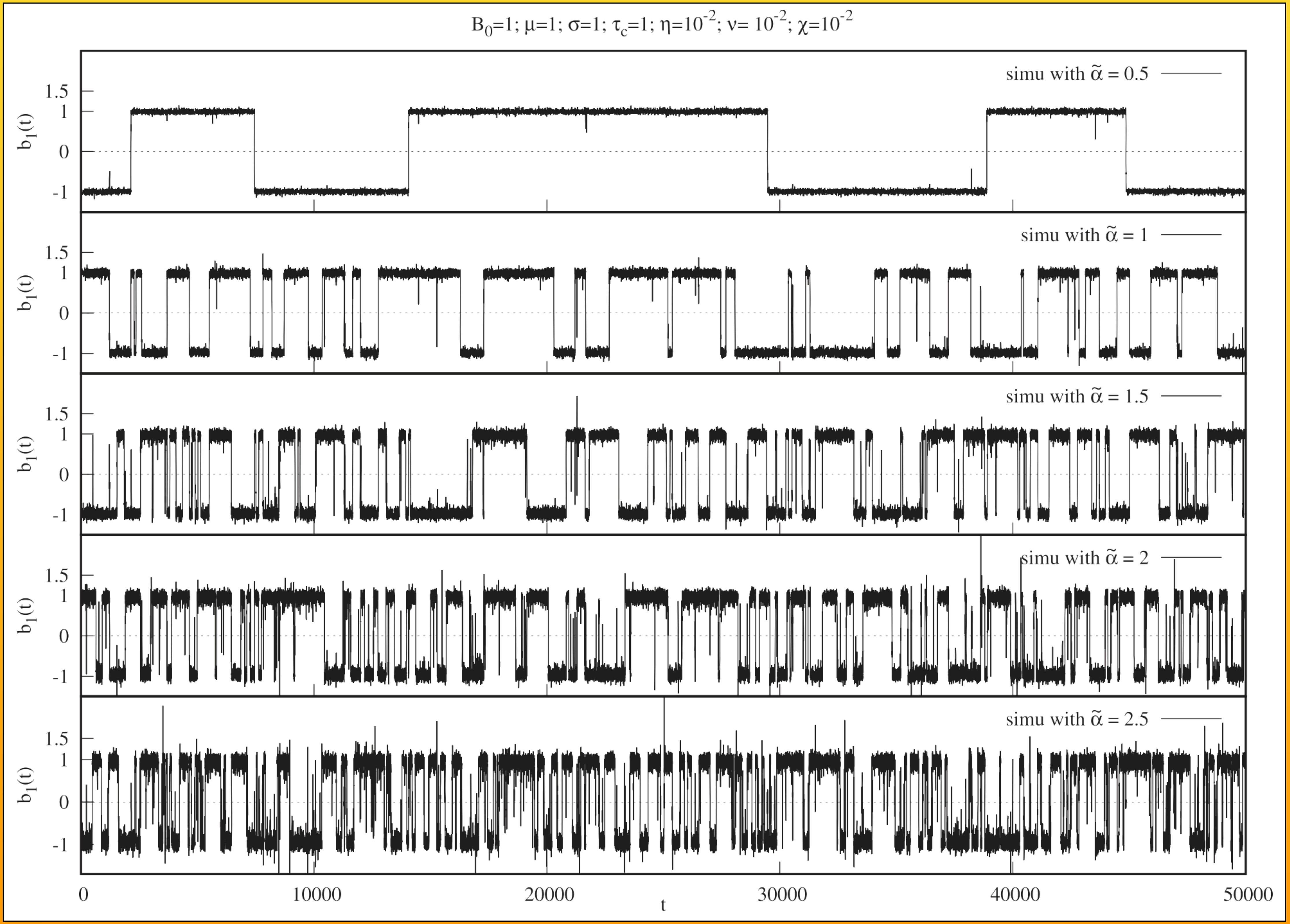


Higher turbulent convection levels make the system more inclined to invert the polarity

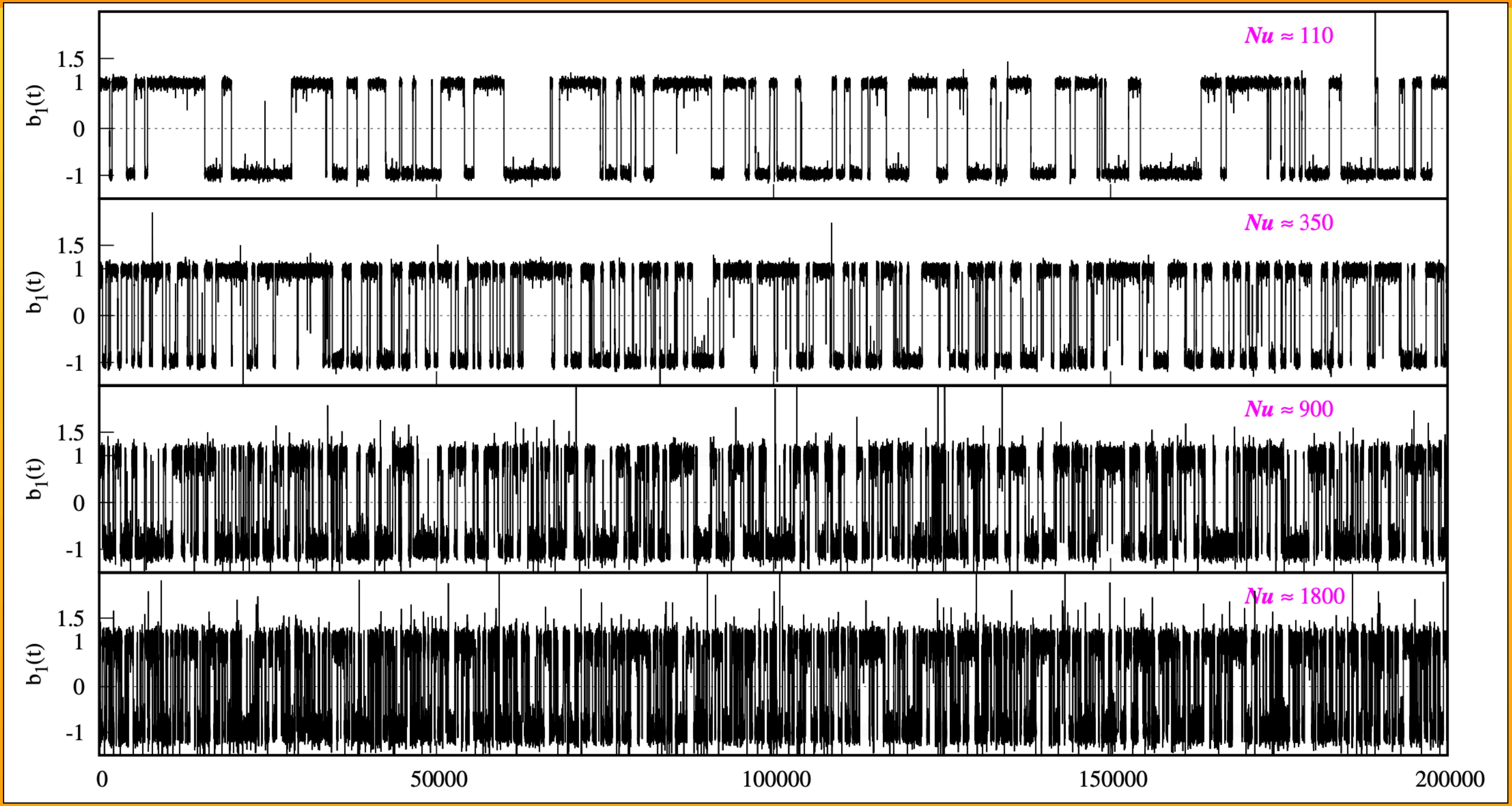
The higher the Rayleigh number

$$Ra = \frac{\tilde{\alpha}\theta_0 L^3}{\nu\chi},$$

The greater the number of reversals.



Simulations with higher Nu tend to develop much more reversals than those with lower Nu



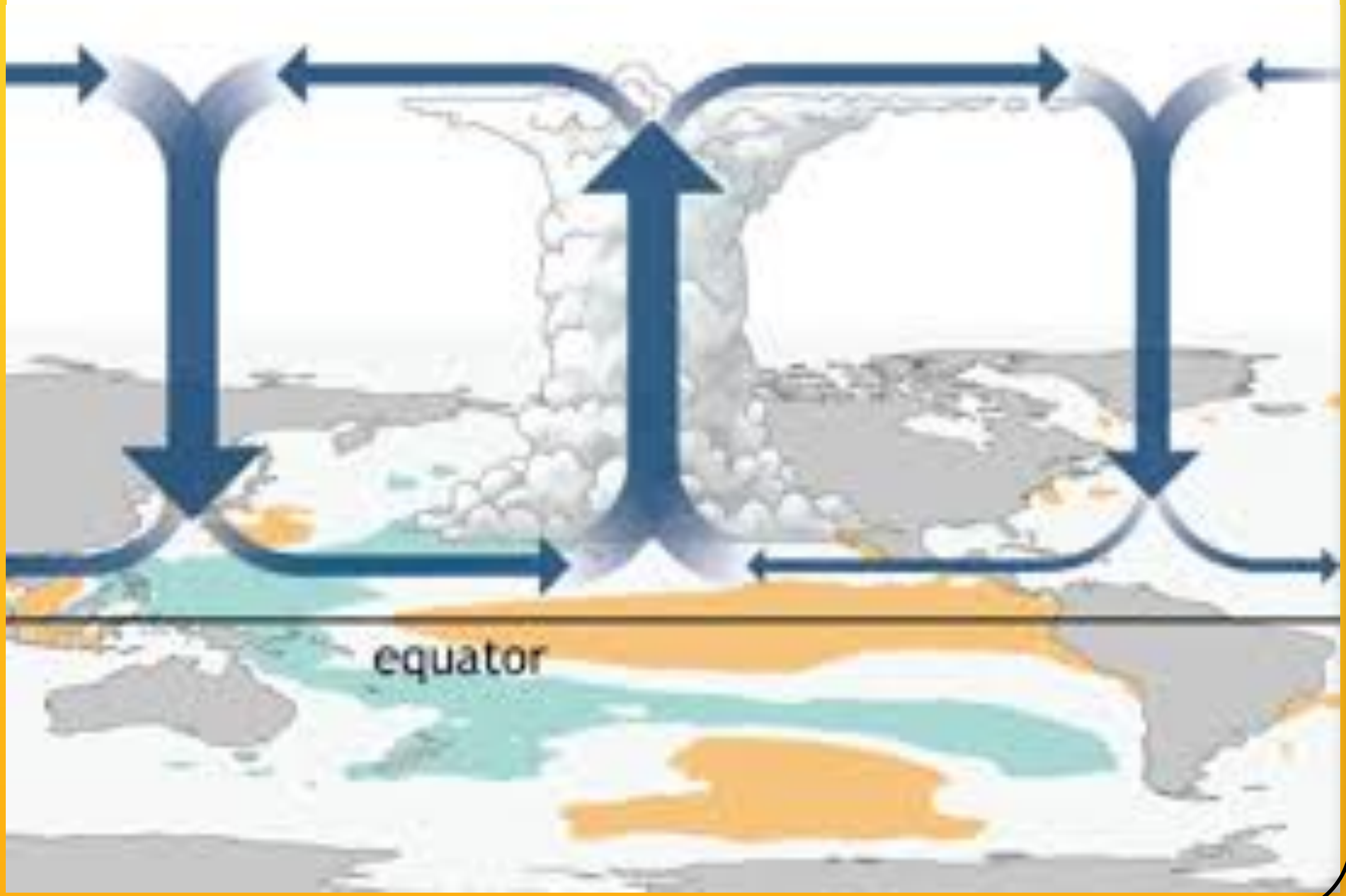
The Nusselt number in the modified shell model: $Nu = \sqrt{\text{Ra Pr}} \langle \sum_{n=1}^{N} [u_n(t)\theta_n(t)^* + u_n(t)^*\theta_n(t)] \rangle_t$

Instantaneous Nusselt Number in RB convection

Nu(t) exhibits instantaneous overshoot above its average value during large-scale circulation reversals (Xi et al. 2016 and Xu et al. 2020)

More coherent flow and plumes that increase heat transfer efficiency

Convective wind of the Earth's atmosphere



Xi et al. 2016 argued that the momentary overshooting behavior in *Nu(t)* could be the **distinguishing feature of the flow reversals among cessations**.

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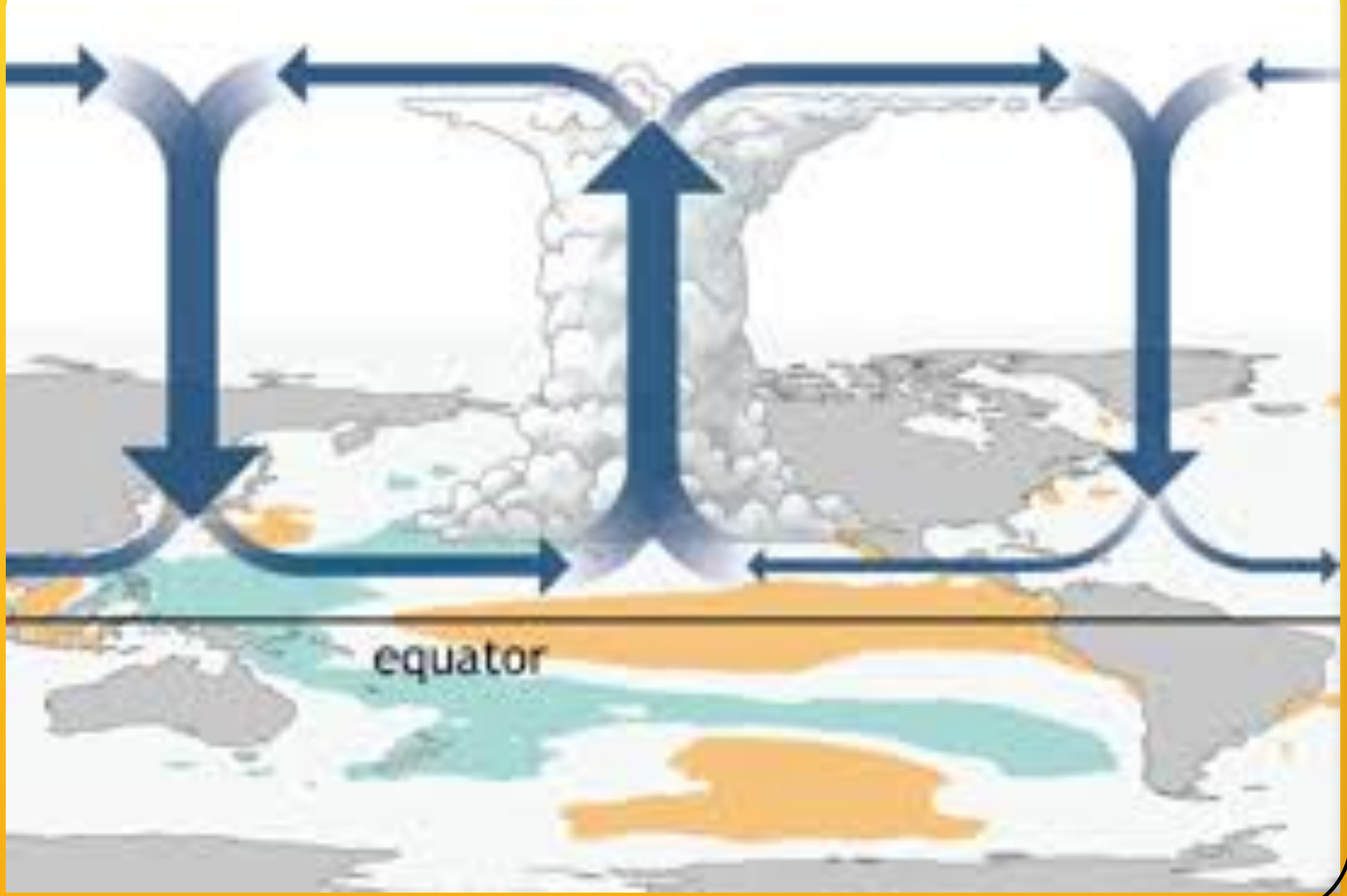
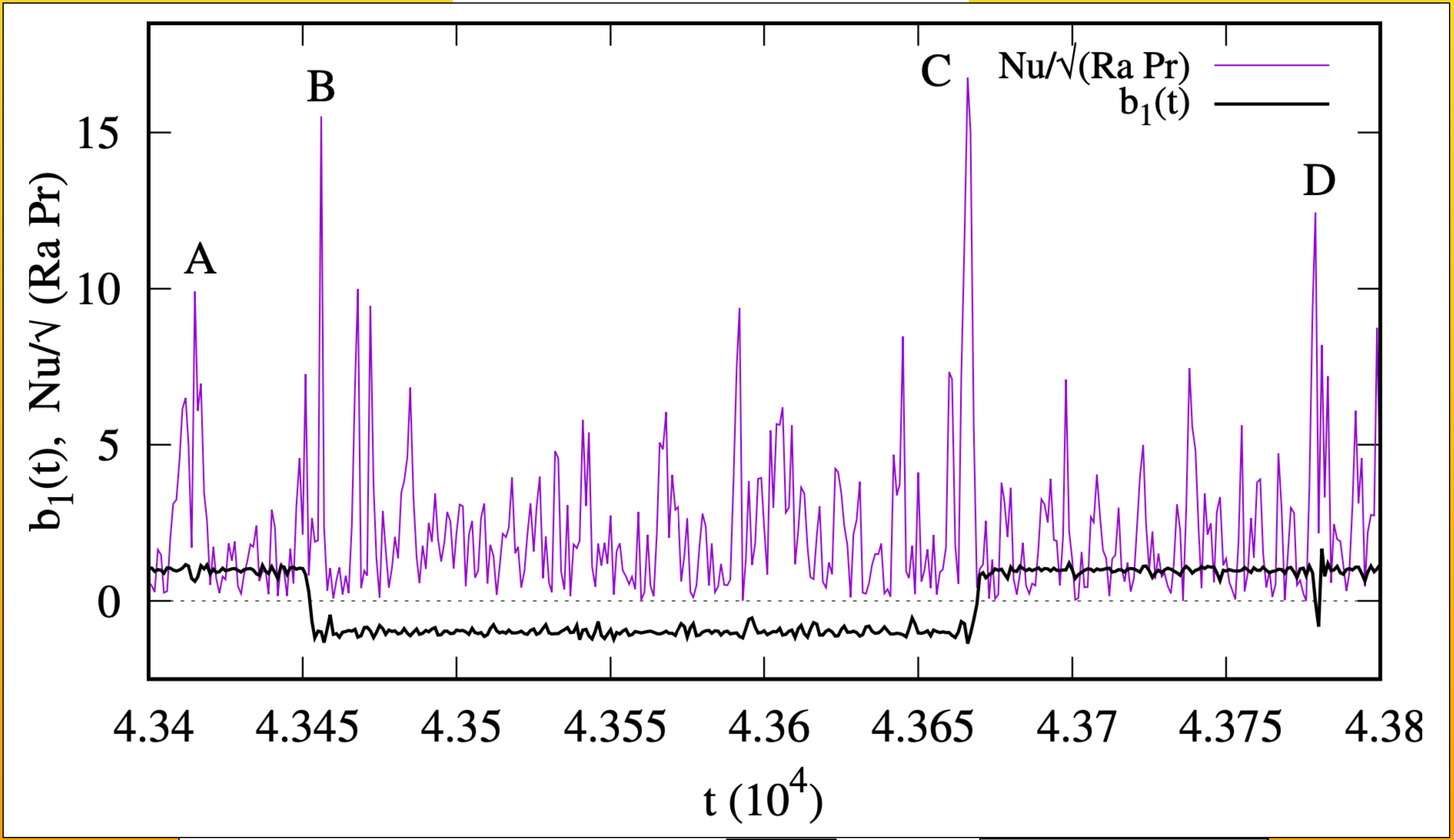


Similarities between large-scale flow reversals in the RB paradigm and magnetic reversals in dynamo (Gallet et al. GAFD 2012, Chandra & Verma PhRvL 2013)

During magnetic polarity reversals, does *Nu(t)* exhibit instantaneous overshoot above its average value?

Highest Nu(t) peaks during magnetic reversals

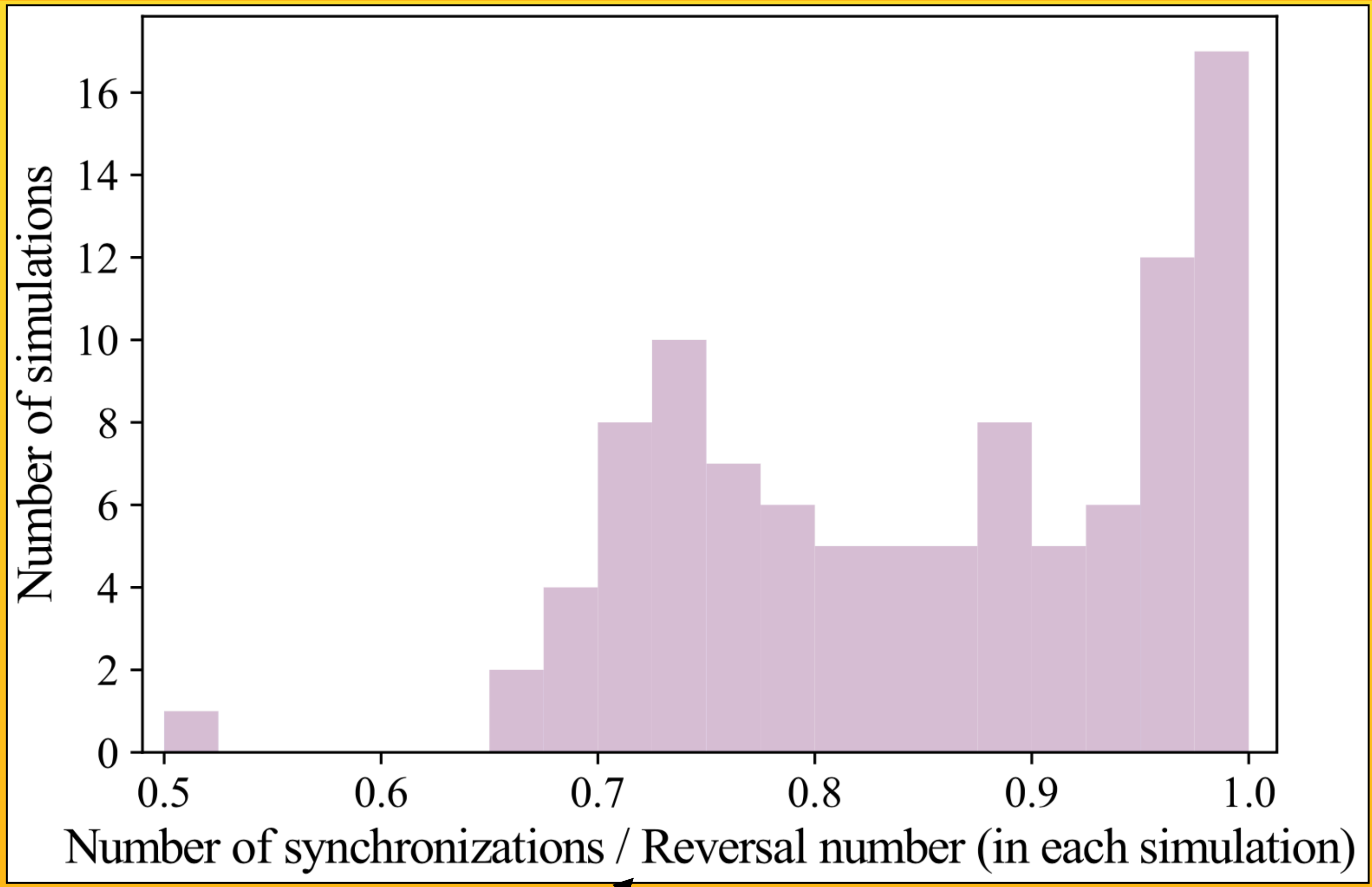
$\chi = \eta = \nu = 10^{-4} \quad \tilde{\alpha} = 0.5$



evaluating a peak as $Nu(t)/\sqrt{\mathrm{Ra\ Pr}} \geqslant C_{thr} = \langle Nu \rangle + 2\,\sigma$

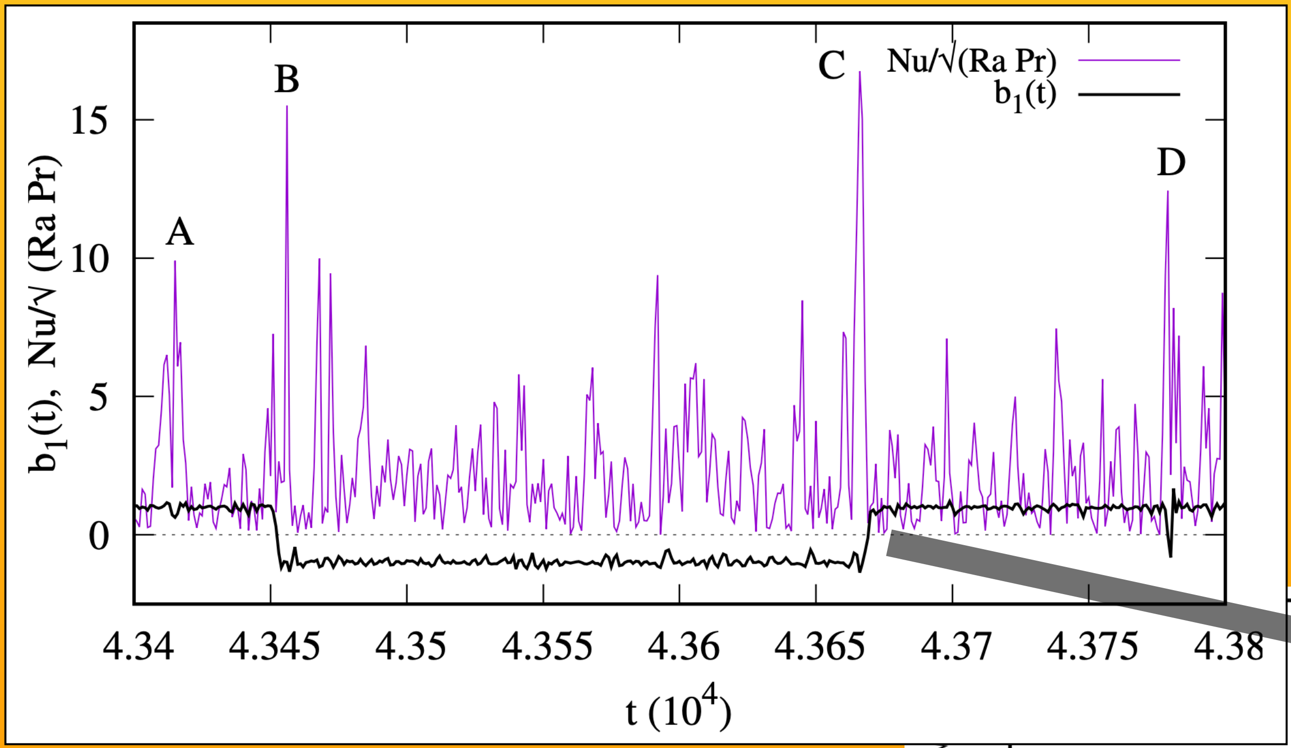
Instantaneous Nusselt Number in $\alpha 2$-dynamo

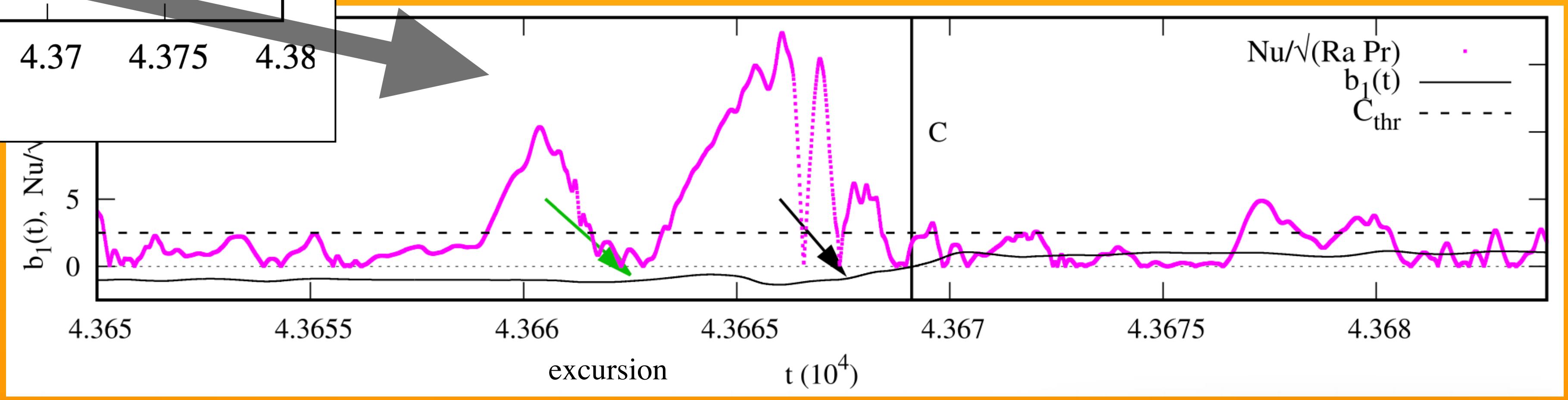
Most simulations have a higher than 70% probability that a reversal occurs during a Nu(t) peak.



Synchronisation probability

Temporal Antecedence





Convergent Cross-Mapping Analysis

The role of the convective heat flux is important for the reversal occurrence

We construct the showdown manifold Mx and My using lagged information of the two-time series, X and Y, respectively.

X => Y

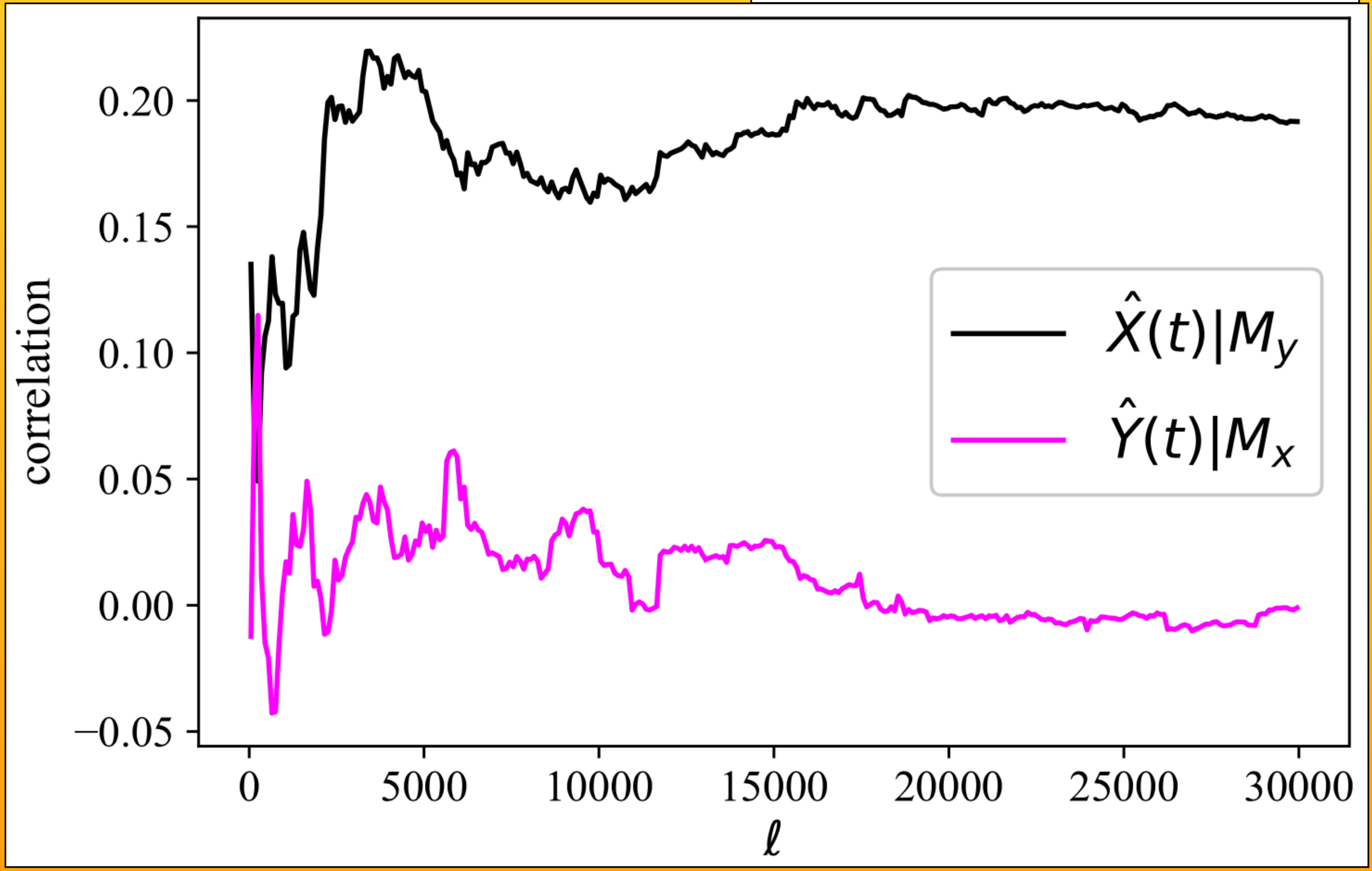
If X causes Y, information from X gets embedded in Y. We can use My to predict X, being *X|My* this prediction.

The accuracy of these predictions is thus adopted, in terms of correlation, as a metric for causality.

If the prediction skill of X increases and saturates **as the entire My is used,** this provides evidence that X is causally influencing Y.

$$X = Nu(t)/\sqrt{Ra\ Pr} \quad \text{and} \quad Y = db_1(t)/dt.$$

$\chi = \eta = \nu = 10^{-4}$ and $\tilde{\alpha} = 0.5$,



CCM analysis shows that **Nu(t) plays a causal role in the magnetic field variation Y**, as indicated by the growth rate in the estimation skill, i.e., correlation, of cross-mapping as **the time series length increases**

Conclusion: The role of the convective heat flux is important for the reversal occurrence

Our results, i.e.,

1. The temporal correlation of the convective heat flux increase and magnetic reversals
2. The antecedence of the convective heat flux spikes relative to magnetic reversals
3. The results of the CCM causality test

lead us to consider the role of the convective heat flux as important for the magnetic reversal occurrence: a long-lasting increase in the convective heat flux can cause magnetic polarity reversal.
The model results show, in general, that **higher levels of convection produce more magnetic reversals.**
Therefore, we propose that late-M dwarfs may show different temporal activity, i.e., more stable or significantly variable activity, depending on the efficiency of the convective heat transport. **In particular, a more efficient convective heat transfer can increase magnetic variability.**

(**G. Nigro**, 938**,** 22, 1 (2022) **ApJ, G.Nigro** et al. under review **MNRAS**)

Thank you for your time!