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The low wave number amplitudes of the solar convective spectrum: why supergranulation

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– Lord, J.W., Cameron, R.H., Rast, M.P., Rempel, M., Roudier, T. 2014, ApJ 793, 24
– Cossette, J.-F. and Rast, M.P. 2016, 829, L17.

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Juri Toomre
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Convective conundrum
Surface driven convection
Entropy rain
Essentially magnetized convection zone
Effectively high Prandtl number flow

Convective conundrum:

Observations:

Sun displays "solar-like" differential rotation: slow pole / fast equator

Giant cells are very difficult to find: Sun shows monotonically decreasing flow amplitudes with decreasing wavenumber below supergranulation

Models:

At solar luminosity and rotation rates, low wavenumber convective amplitudes are too high (Global models: Rossby number problem) $\mathrm{Ro} = \frac{U}{Lf}$ $f = 2\Omega \sin\theta$

Horizontal flows amplitudes monotonically increase to low wave number (Radiative hydrodynamic models: supergranulation problem)

Questions:

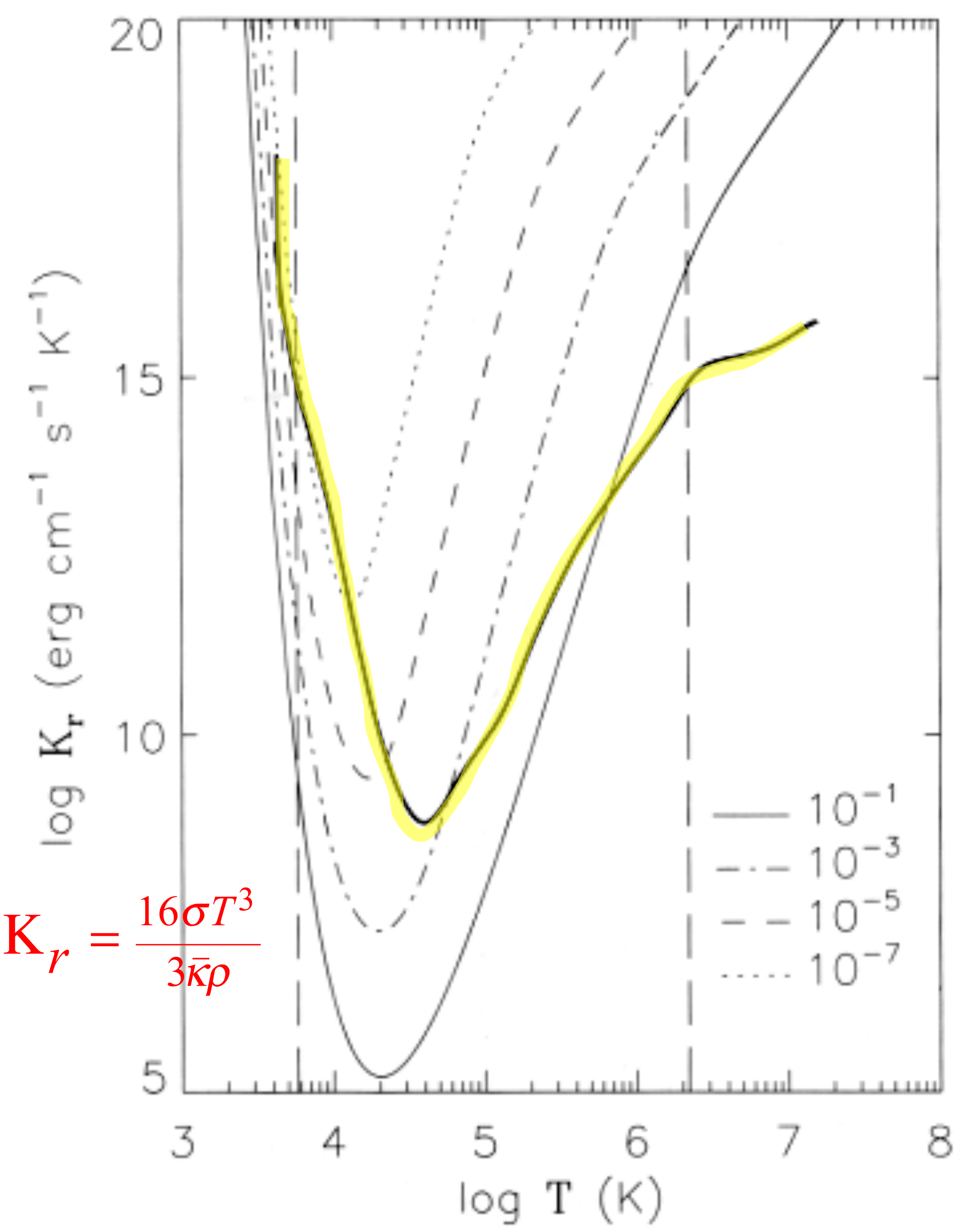
1. What fraction of the solar luminosity should the resolved modes in numerical simulations carry? (N. Featherstone)
2. How does the surface shear layer arise? Is there conversion of large scale convective power to surface shear?
   (does not address rotational constraint at depth)
3. Do simulations correctly capture the highly nonlocal convective transport? (surface driven granular/supergranular downflows descending through an extremely isentropically stratified interior)

Important:

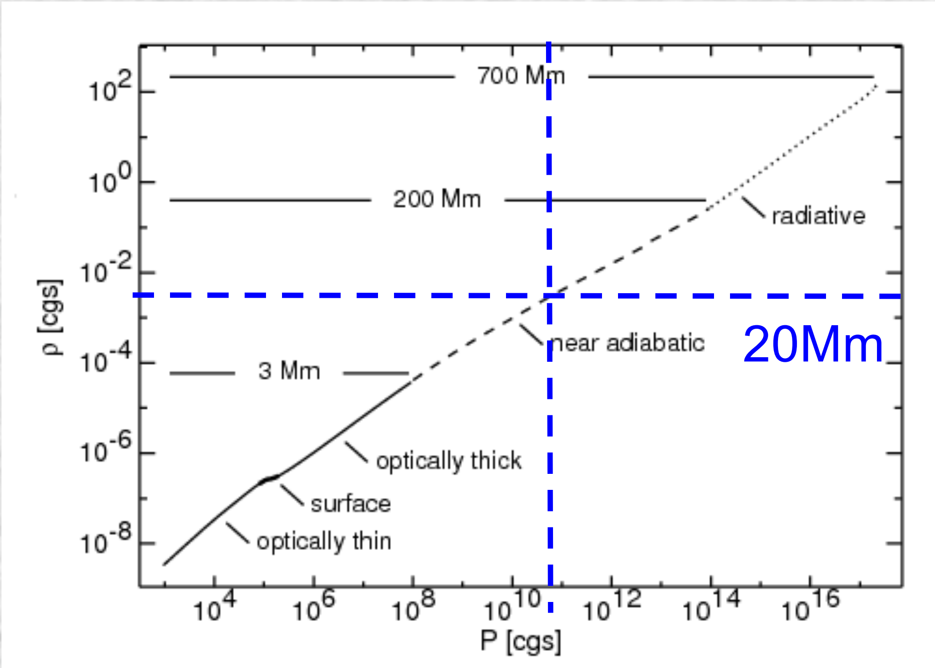
The Sun's thermal boundary layer thickness is not a function of the plasma thermal diffusivity but instead reflects the depth dependent opacity

The photospheric cooling layer is much thicker than the characteristic thermal diffusion length scale

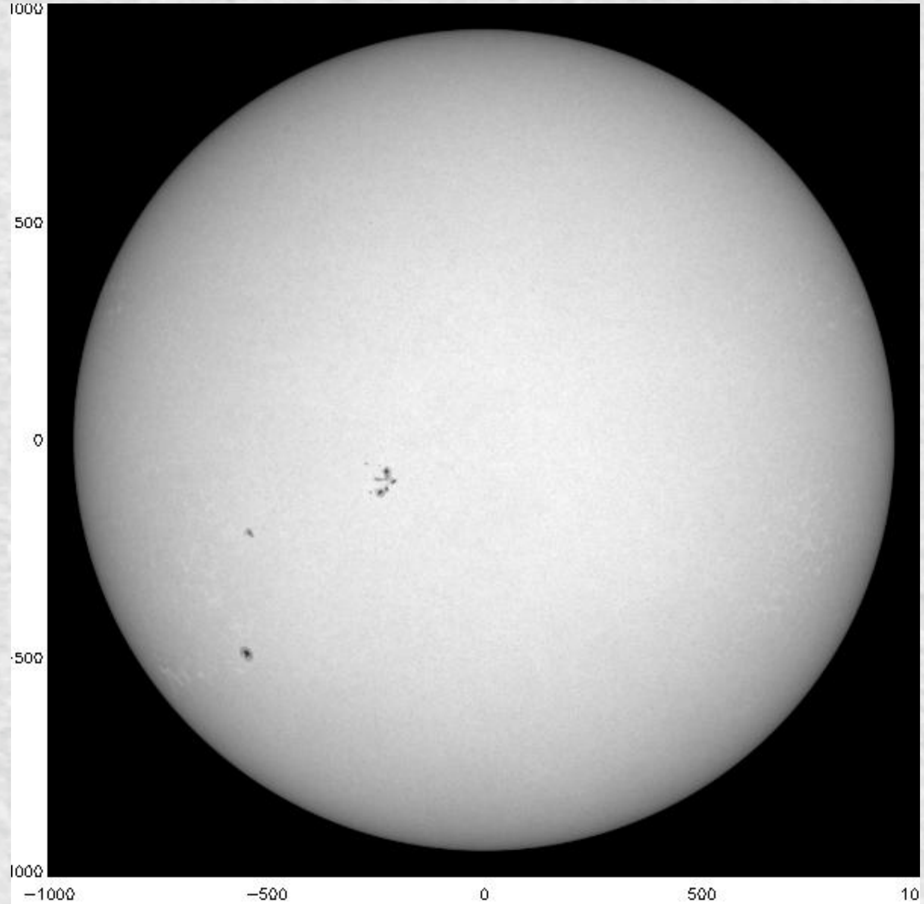
300 – 400 km vs. ~100 m or less



$$K_r = \frac{16\sigma T^3}{3\bar{\kappa}\rho}$$



Nordlund et al. 2009

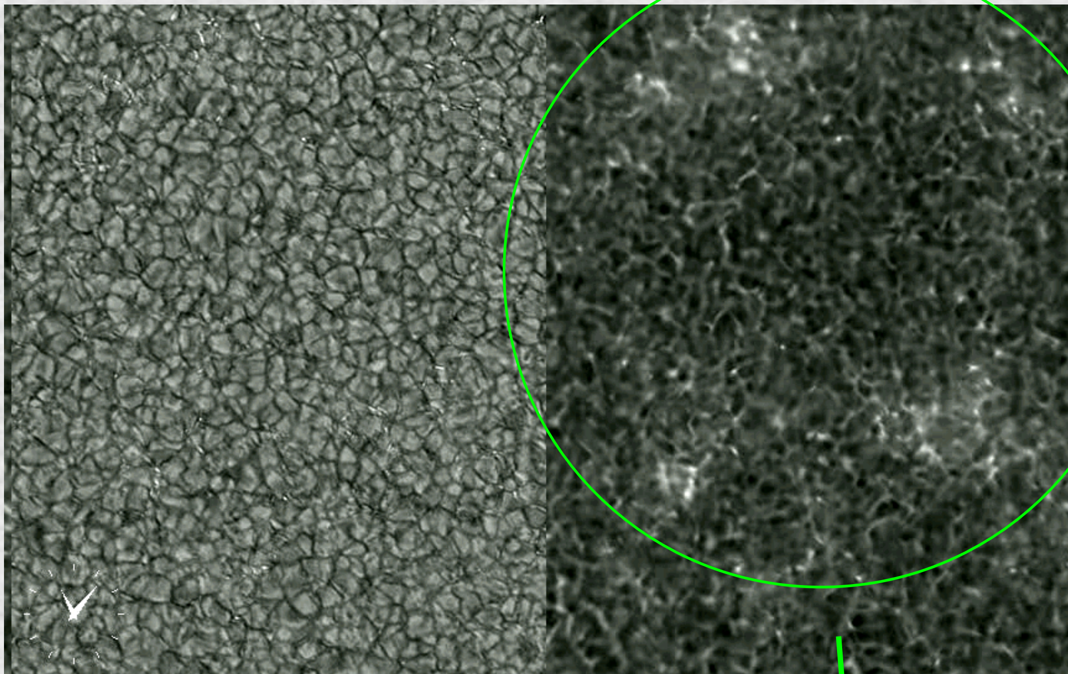


$$R_{CZ} = 0.713 \pm 0.001\, R_{\odot}$$

Density changes by factor of 1e06
Pressure changes by factor of 8e08 across SCZ

Highly stratified: by 20Mm below photosphere (10% depth of convection zone) density and pressure have changed by factors of $1.5e10^{4}$ and $7.7e10^{5}$

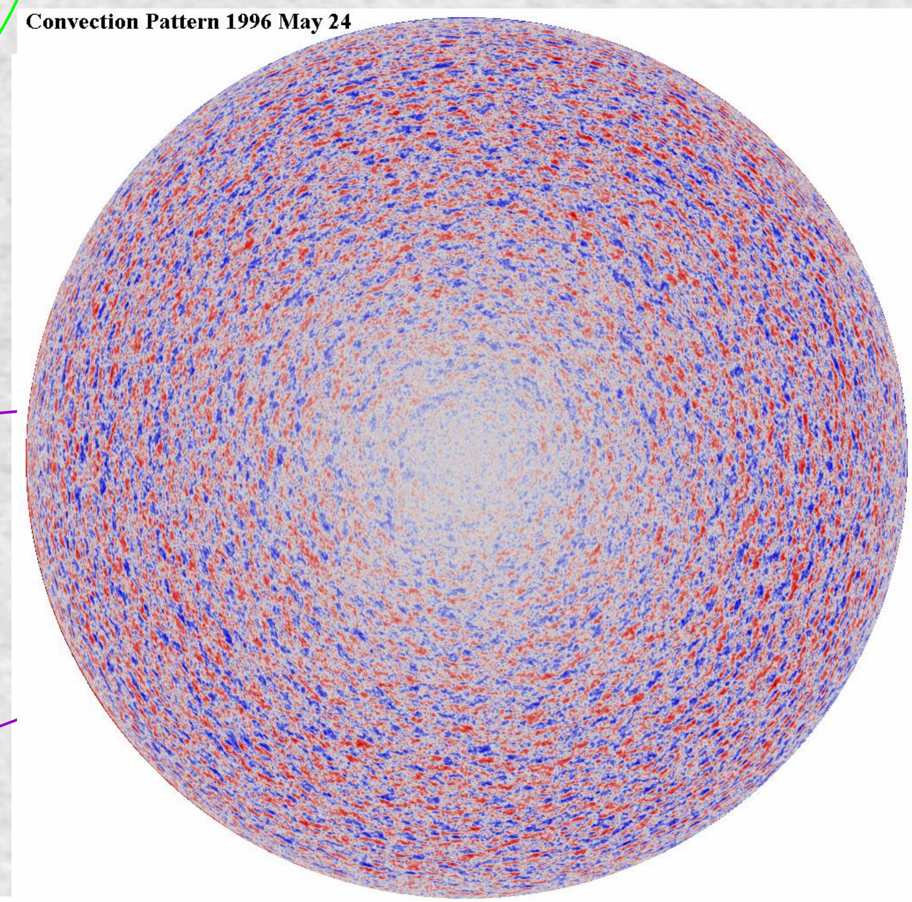
G-band CaIIH



Dutch Open Telescope

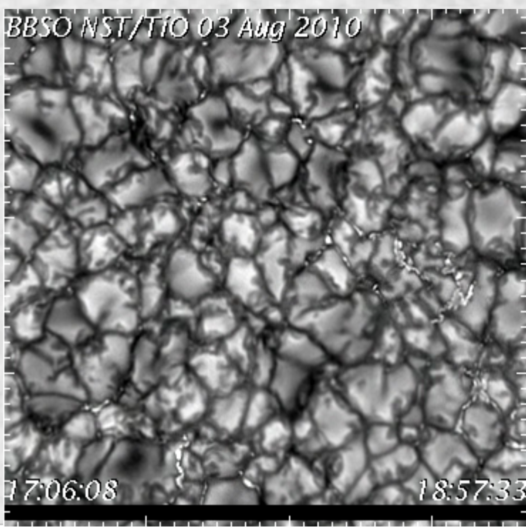
Supergranulation
(Hart 1954, Leighton et al. 1962)
• 32000km scale
• 400m/s horizontal flow
• 20hr lifetime

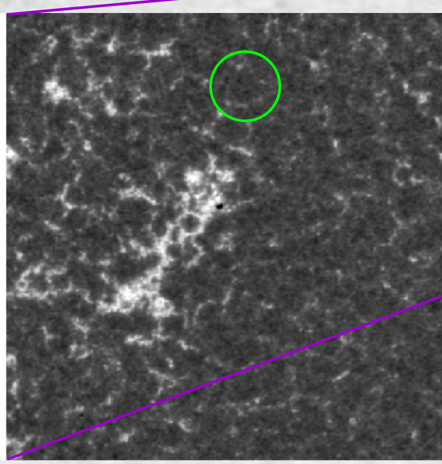
Direct Doppler signal

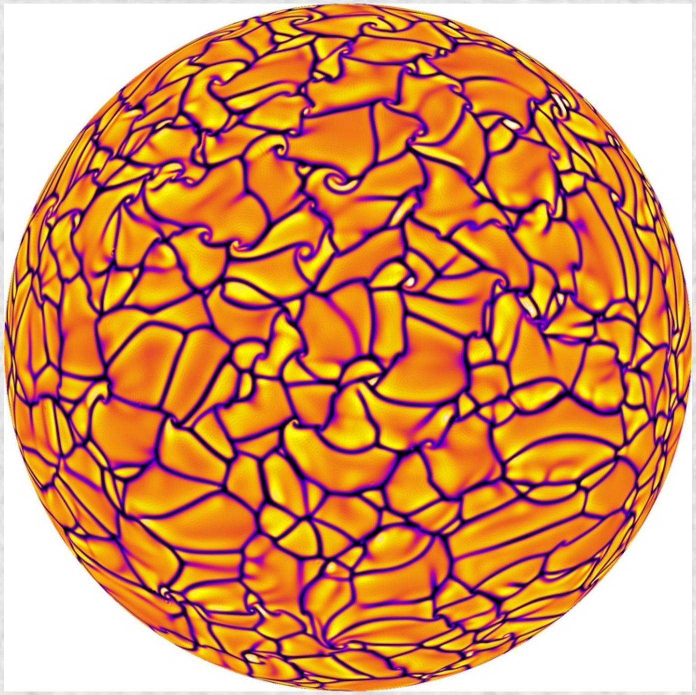


Courtesy D. Hathaway (NASA Marshall)

Granulation (Herschel 1801)
• 1000km scale
• 1000m/s vertical flow
• 0.2hr lifetime





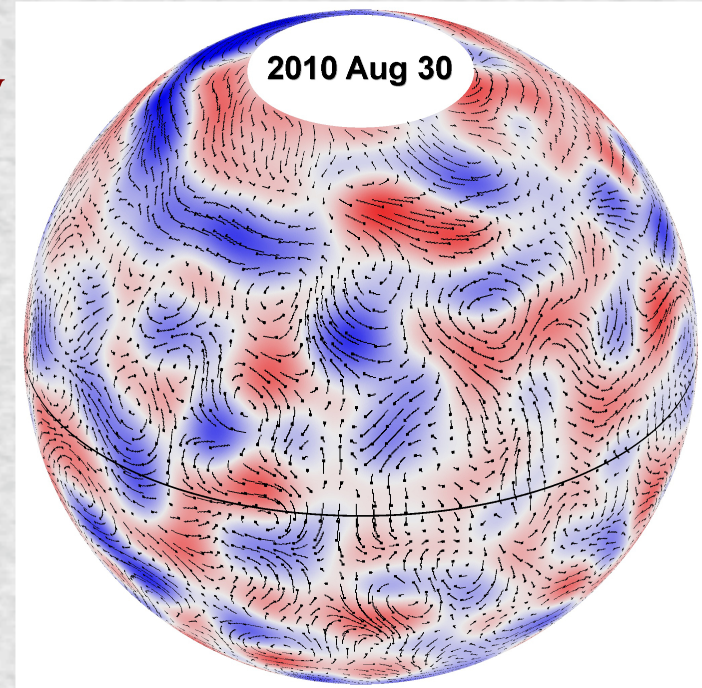


e.g., Miesch et al. 2008, ApJ 673, 557

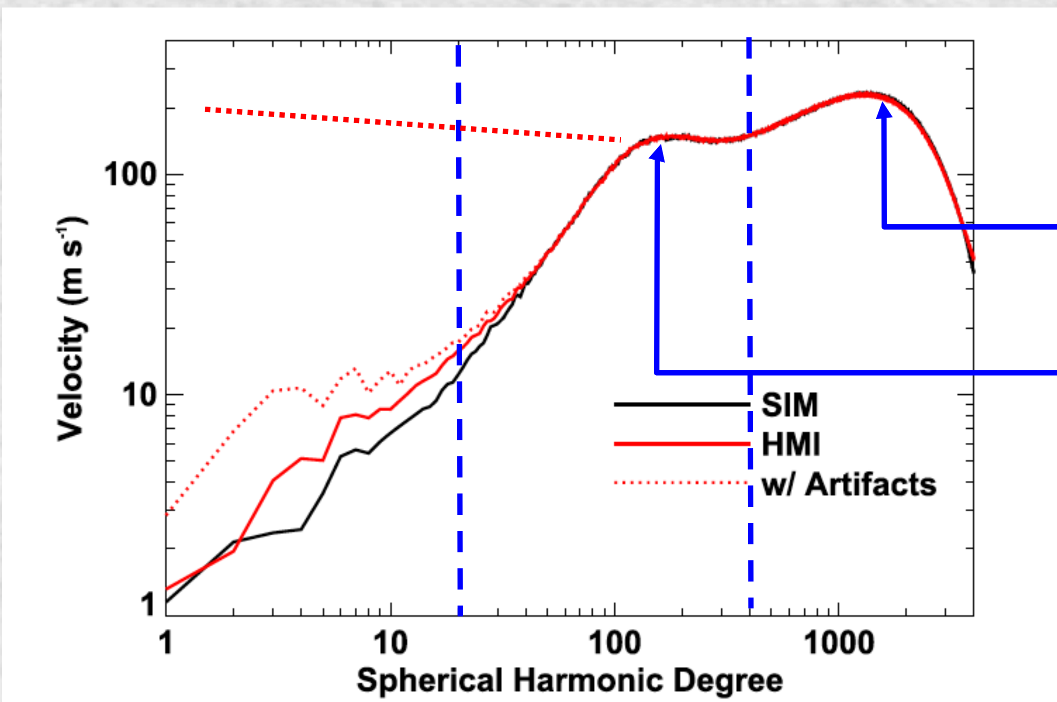
Giant Cells (Hathaway et al. 2013)
- ~200,000km scale
- ~16m/s horizontal flow
- ~1 month lifetime

Why have giant cells been so hard to find?

Why are large scale flows so weak?



Hathaway et al. 2013, Science 342, 1217



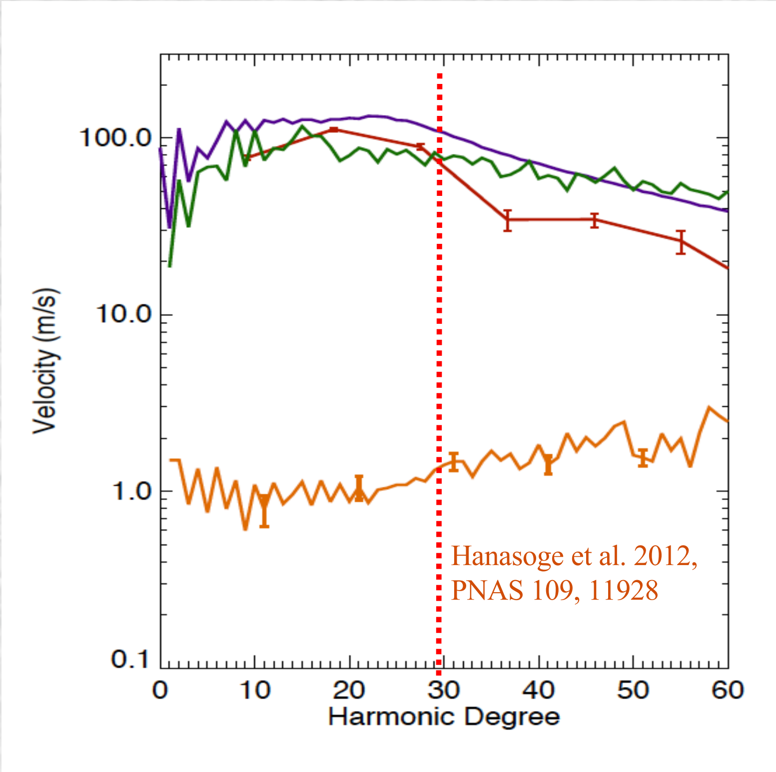
Hathaway et al. 2015, ApJ 811, 105

Granulation: ~1.2Mm, ℓ = 3500

Supergranulation: ~35Mm, ℓ = 120

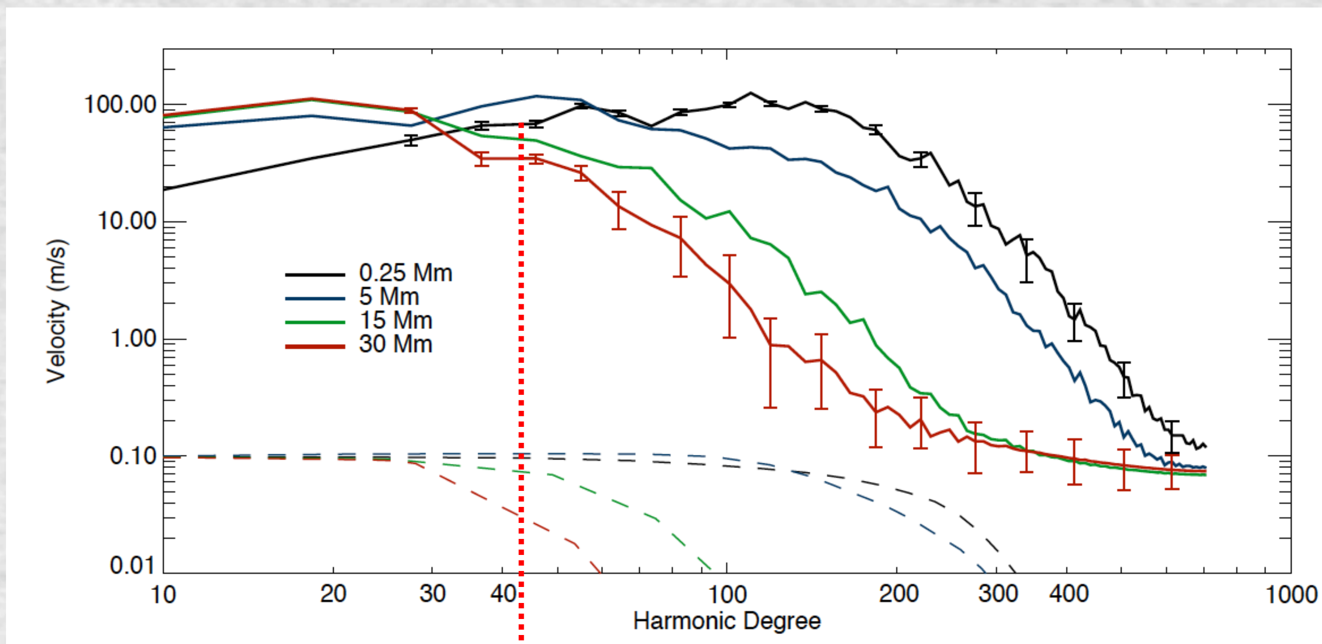
(Mesogranulation: ~10.5Mm, ℓ = 400)

Giant Cells: ~210Mm, ℓ = 20

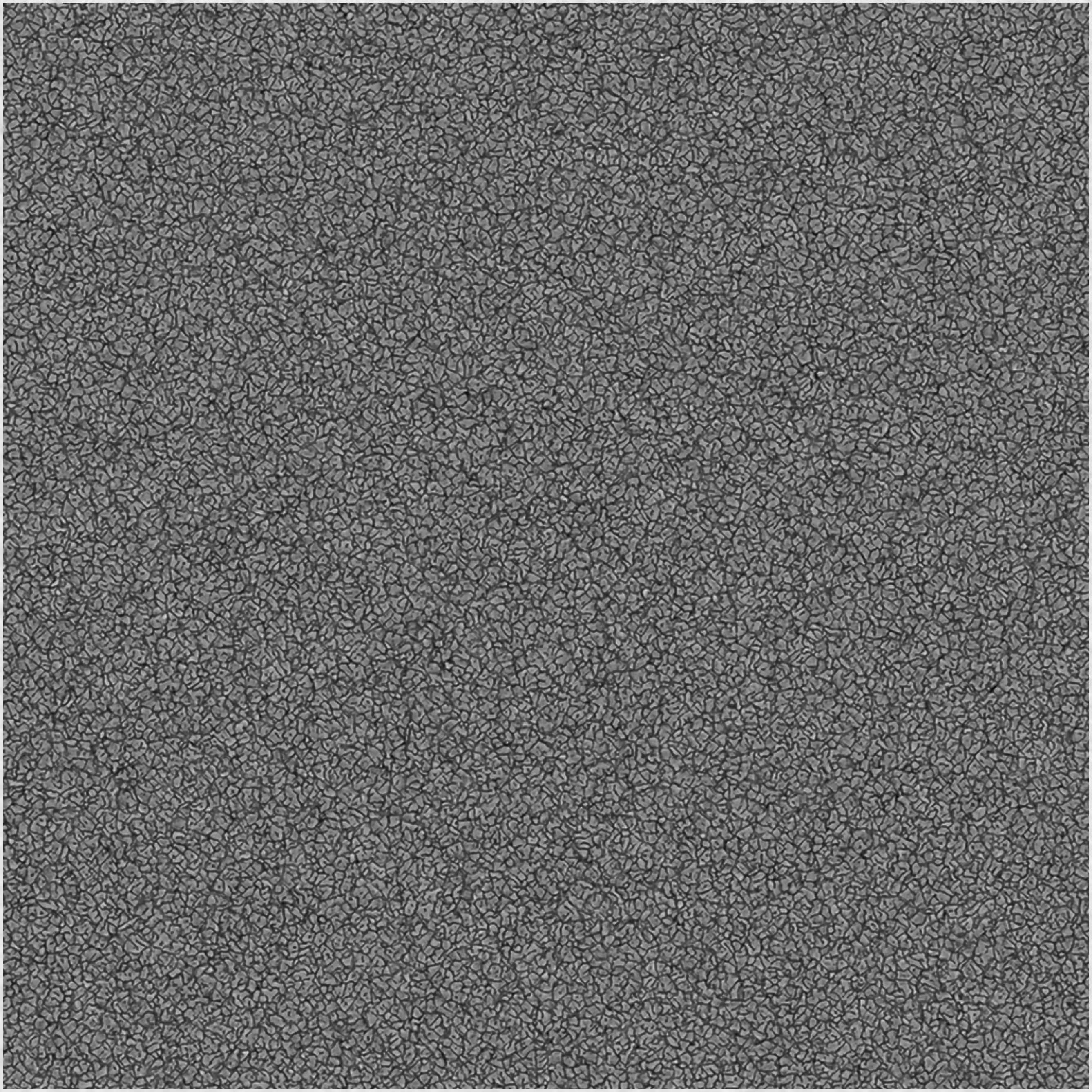


Greer et al. 2015, ApJ 803, L17

- At 30Mm global models show increasing amplitude toward low wavenumber
- Helioseismic measurements of convective amplitudes wildly disagree
- Global models do not get amplitude of the motions correct at solar rotation rate and luminosity
- Some helioseismic results suggest increase of low wavenumber amplitudes with depth



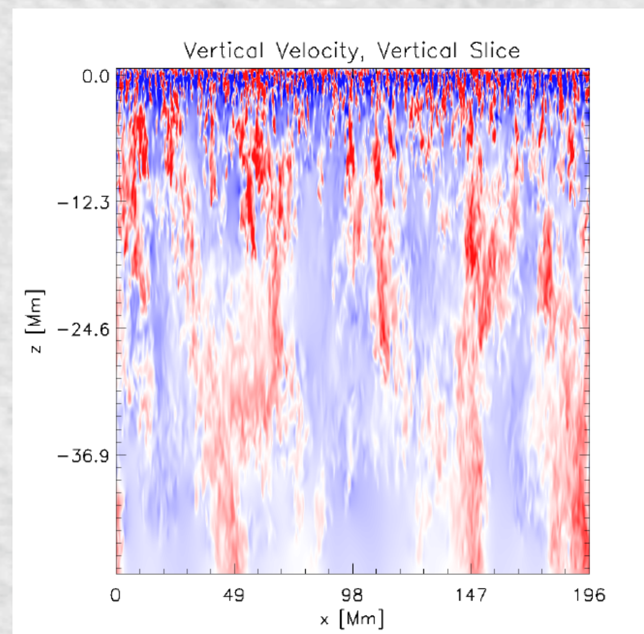
So why the supergranular scale?

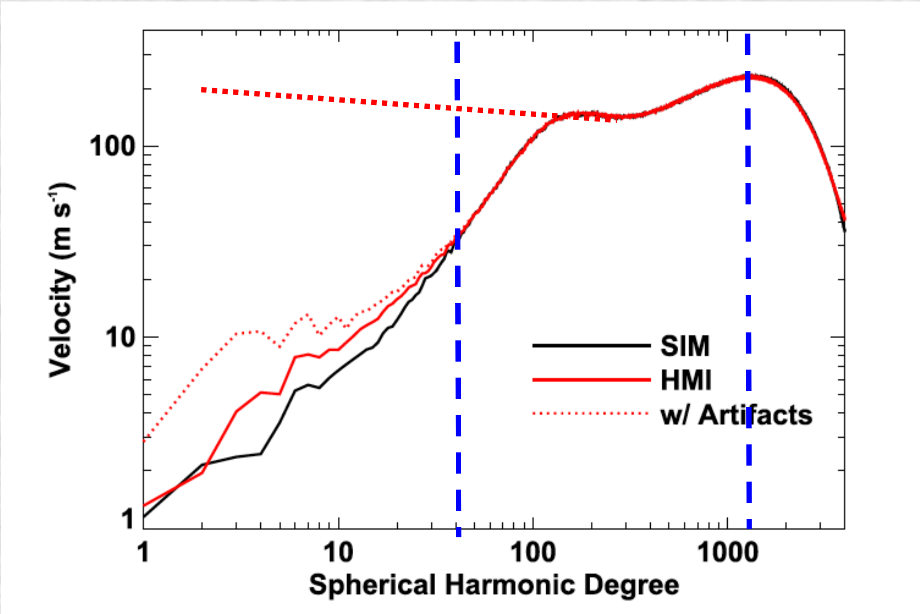


Radiative HD simulations
(MURaM code)

1024x1024x768 grid points
196x196x49Mm

Slope limited diffusion
Grey opacity
Open lower boundary





Granulation: ~1.2Mm, ℓ = 3500

(Mesogranulation: ~10.5Mm, ℓ = 400)

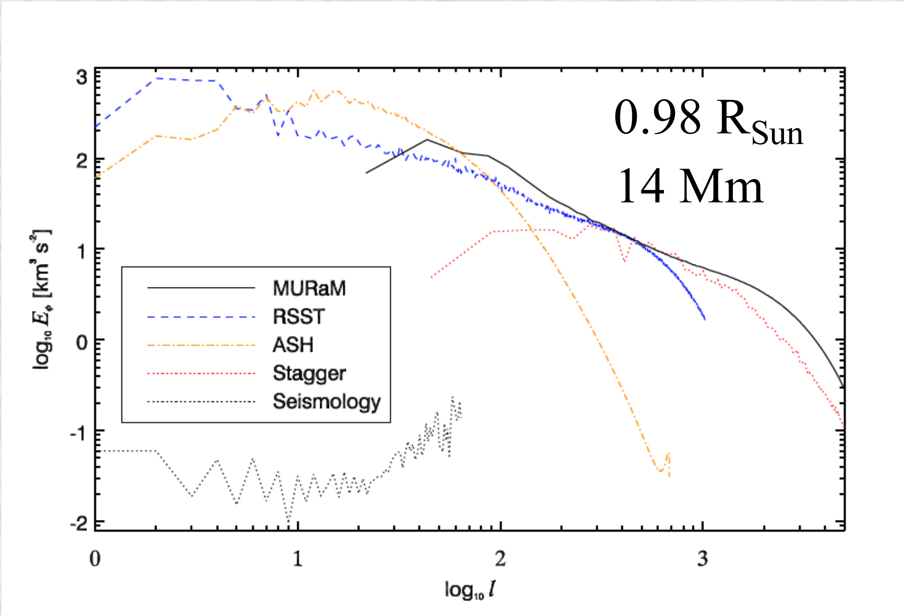
Supergranulation: ~35Mm, ℓ = 120

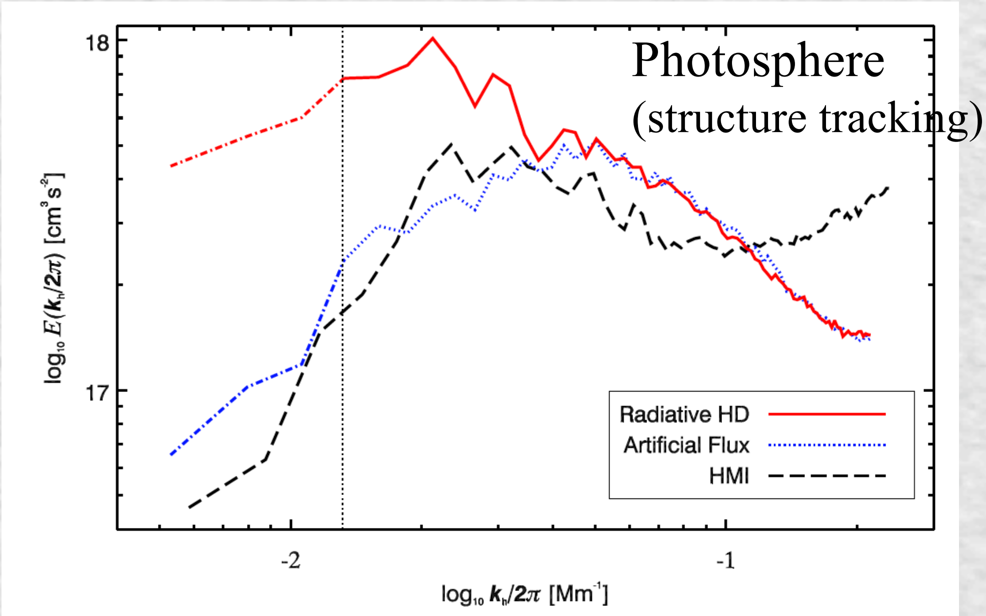
$\log_{10}(1/35) = -1.54$

Giant Cells: ~210Mm, ℓ = 20

$\log_{10}(1/210) = -2.32$

- Local area radiative magnetohydrodynamic simulations do not get spectrum of horizontal motions in photosphere correct

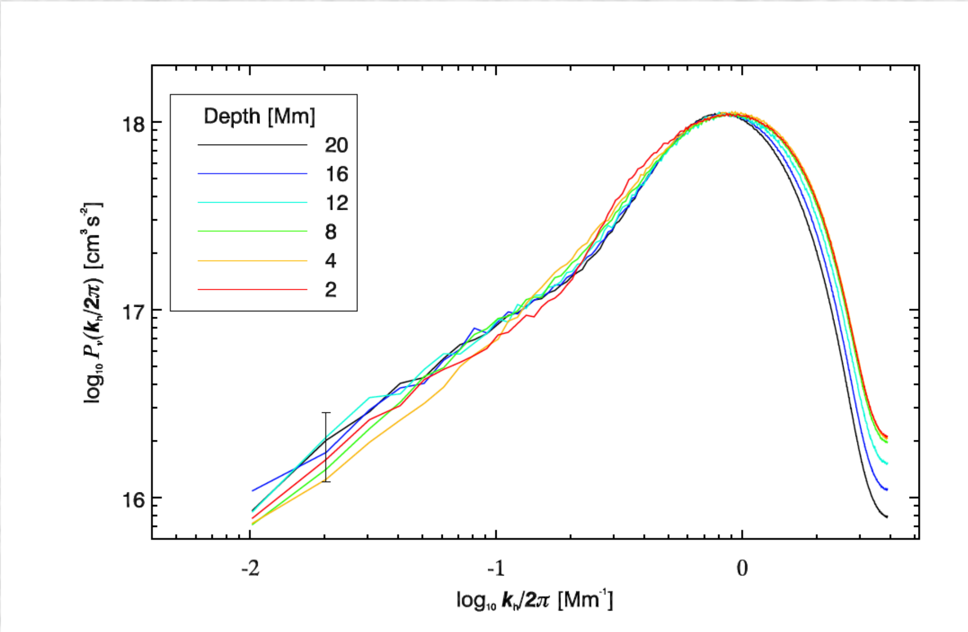




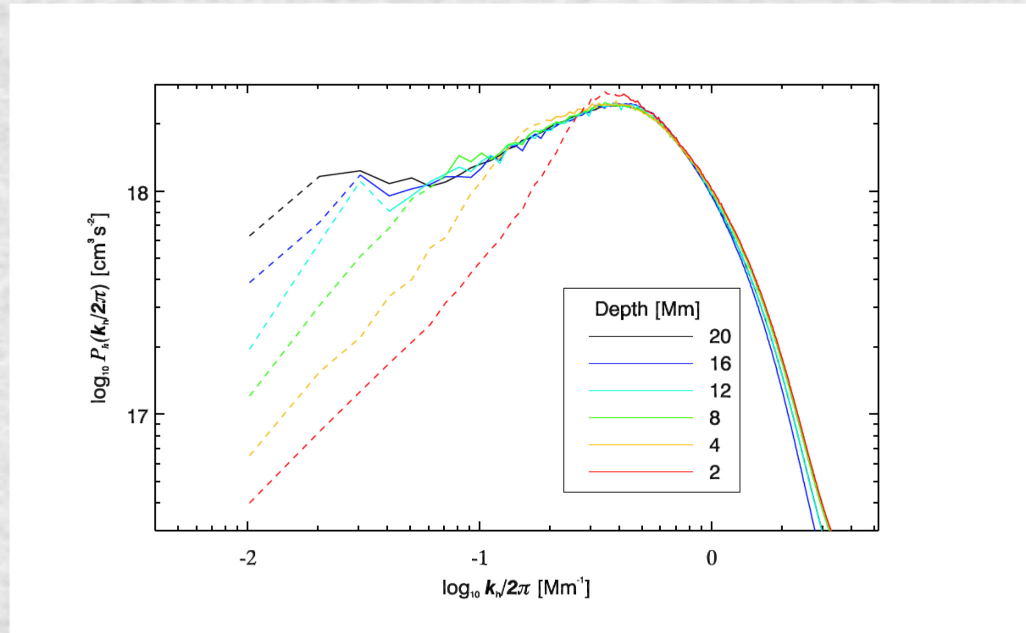
Lord et al 2013, ApJ 793, 24

Vertical velocity spectrum:

Photospheric Spectra



Horizontal velocity spectrum:

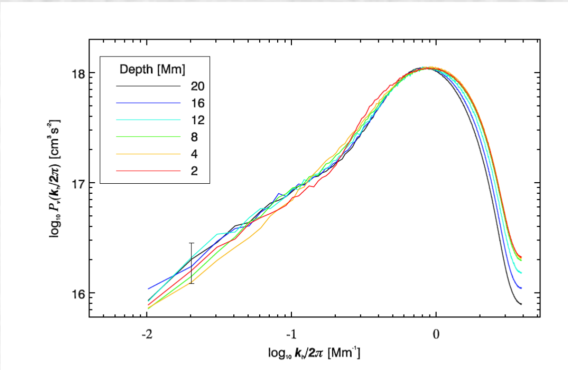


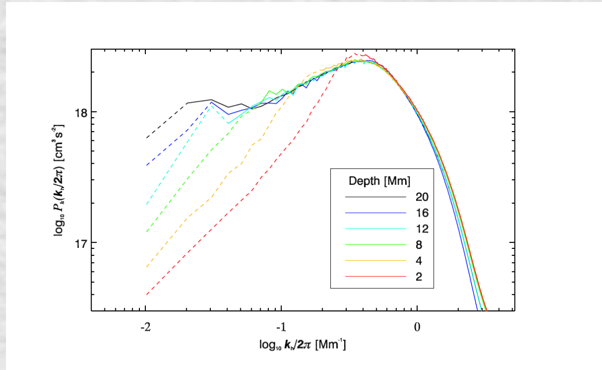
Vertical velocity spectrum quite insensitive to the depth of the domain

The horizontal velocity spectrum in the photosphere reflects the depth of the domain and the amplitude of the the vertical velocity at depth:

- Spectrum turns over for scales larger than 4 time the density scale height at the bottom of the domain
- Large scales driven deep, small scales driven shallow
 (mixing-length like dependence of integral scale on local scale height)

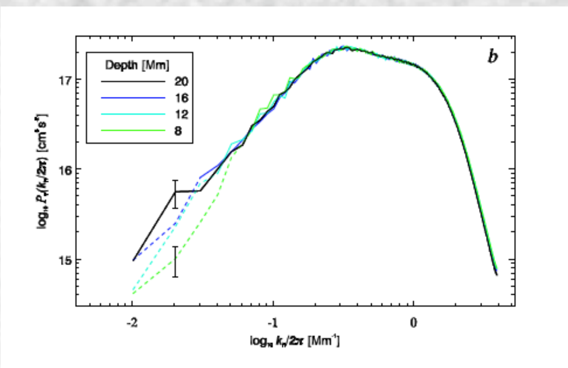
Vertical Velocity   Horizontal

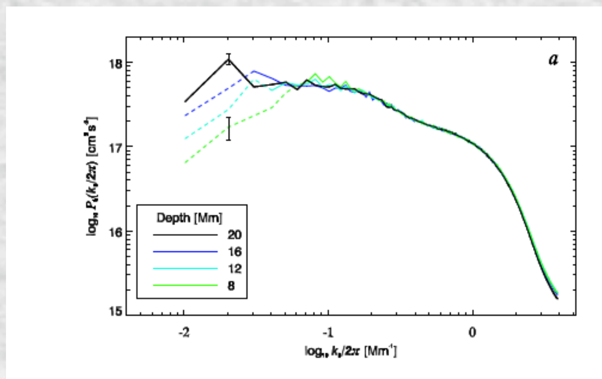




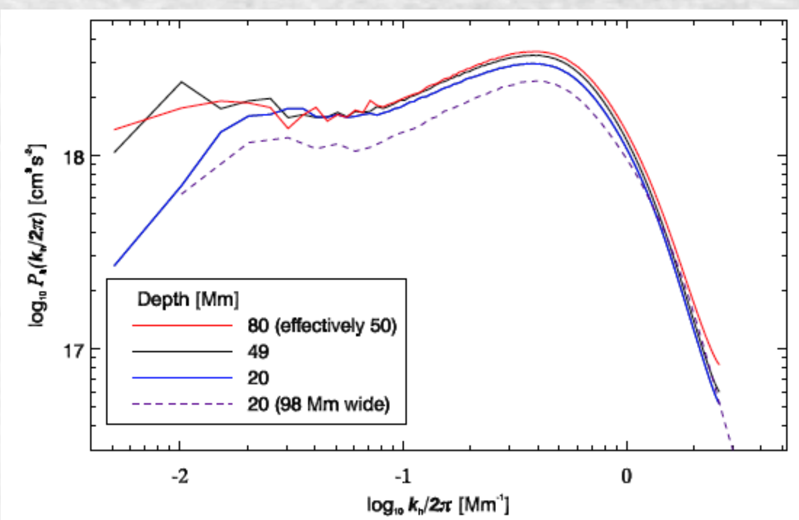
98 x 98 Mm^2 Domain

Photosphere





1.3 Mm below the surface



197x197 Mm^2 Domain

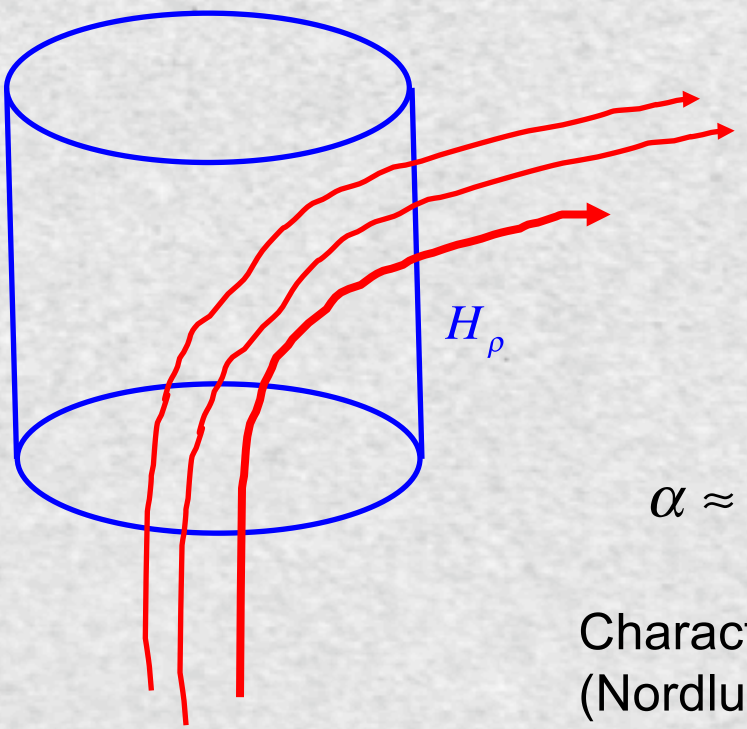
Photosphere
(effectively 50 because artificial flux in lower 30 Mm, 98Mm wide simulation has twice the vertical resolution and this plot is on a geometric height with minimum mean tau>1, thus different height)

Semi-analytic model for the photospheric horizontal velocity power spectrum:

- Model vertical velocity spectrum
- Derive horizontal velocity spectrum from vertical using continuity
- use full 3D MHD model to verify

Vertical velocity spectrum:

- Homogeneous isotropic turbulence at small scales
- Decaying large scale modal power with height



To maintain mean stratification 1 – 1/*e* of the mass must overturn over one density scale height so that density drops by factor of 1/*e*.

$$2\pi r H_\rho \rho u_h \sim \pi r^2 \rho u_z$$

$$r \sim 2\alpha H_\rho \frac{u_h}{u_z}$$

$$\alpha \approx \frac{u_h}{u_z} \approx 1 \qquad d = 2r \qquad \lambda_h \approx 4H_\rho$$

Characteristic horizontal scale that feels stratification (Nordlund et al. 2009)

Integral scale of convection (Stein et al. 2009)

Mass continuity to get horizontal velocity spectrum from vertical velocity spectrum:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = \cancel{\frac{\partial \rho}{\partial t}} + \rho\left(\frac{\partial \mathrm{u}_x}{\partial x} + \frac{\partial \mathrm{u}_y}{\partial y} + \frac{\partial \mathrm{u}_z}{\partial z}\right) + \mathrm{u}_x \cancel{\frac{\partial \rho}{\partial x}} + \mathrm{u}_y \cancel{\frac{\partial \rho}{\partial y}} + \mathrm{u}_z \frac{\partial \rho}{\partial z} = 0$$

Anelastic-like balance: $\nabla_h \cdot \mathbf{u}_h = -\frac{\partial \mathrm{u}_z}{\partial z} - \frac{\mathrm{u}_z}{H_\rho}$ or $i\mathbf{k}_h \cdot \tilde{\mathbf{u}}_h = -\frac{\partial \tilde{\mathrm{u}}_z}{\partial z} - \frac{\tilde{\mathrm{u}}_z}{H_\rho}$

$$\lambda_h \approx 4 H_\rho$$

Small scale motions:

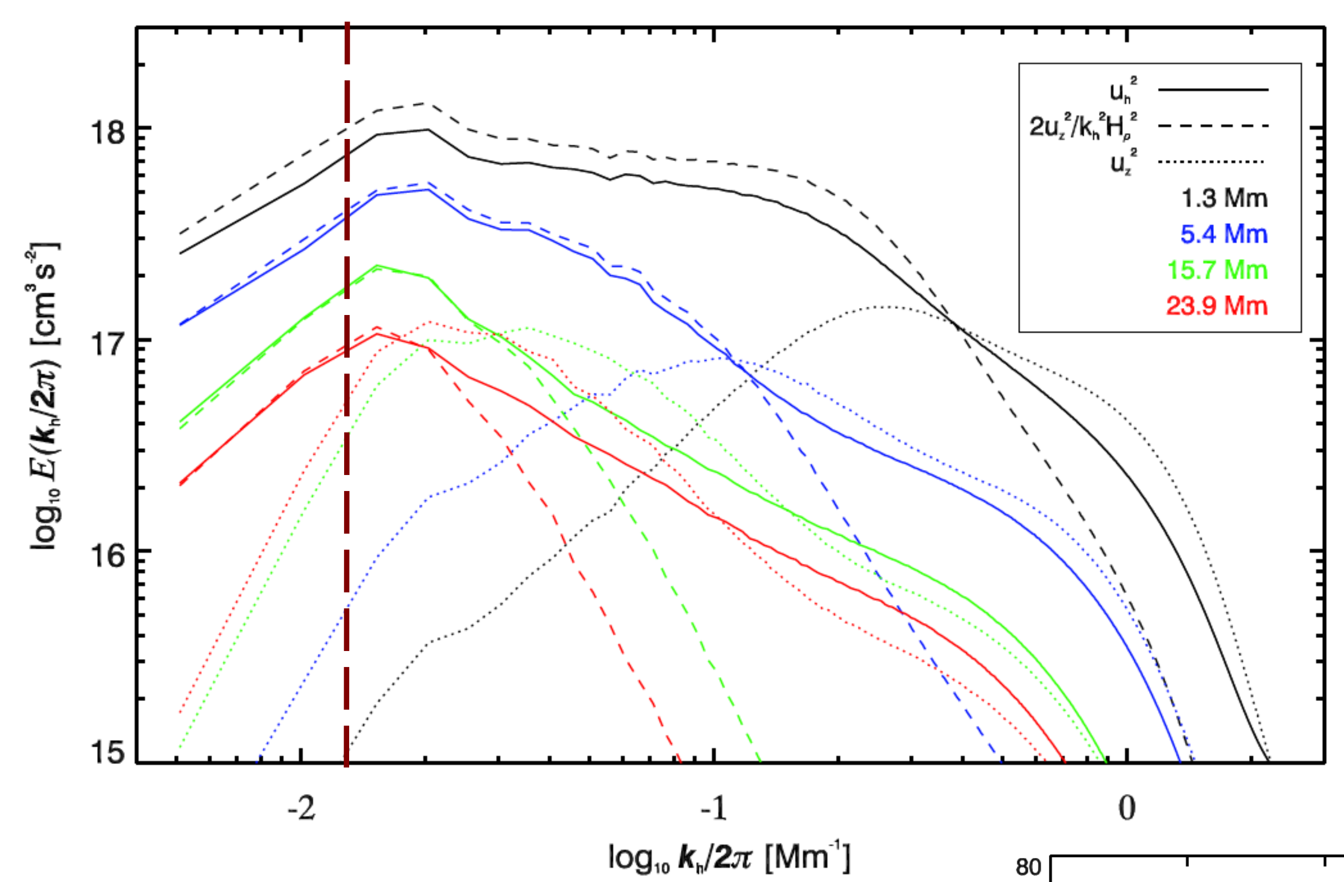
- $\frac{\partial \tilde{u}_z}{\partial z} >> \frac{\tilde{u}_z}{H_\rho}$
- Homogeneous and isotropic
- Incompressible

$$\tilde{\mathbf{u}}_h \cdot \tilde{\mathbf{u}}_h^* = \tilde{u}_z \tilde{u}_z^*$$

Large scale motions:

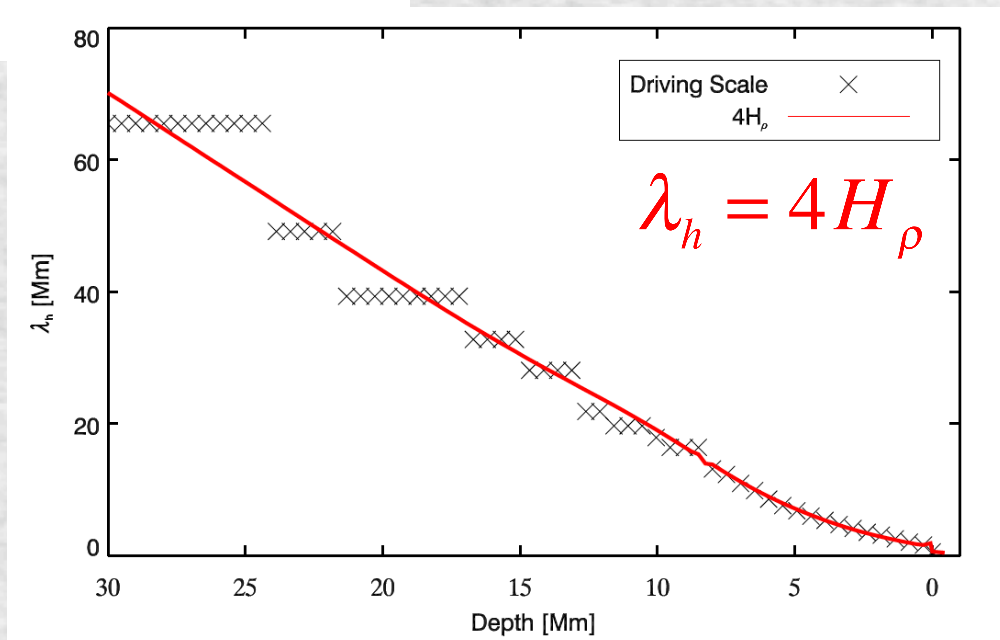
- $\frac{\partial \tilde{u}_z}{\partial z} << \frac{\tilde{u}_z}{H_\rho}$
- Cross terms from squaring left side are measured to be small in stratified simulations

$$\tilde{\mathbf{u}}_h \cdot \tilde{\mathbf{u}}_h^* = \frac{2}{k_h^2 H_\rho^2} \tilde{u}_z \tilde{u}_z^*$$



Horizontal velocity spectrum derivable from vertical velocity at each depth using the two continuity arguments.

196x196x49Mm



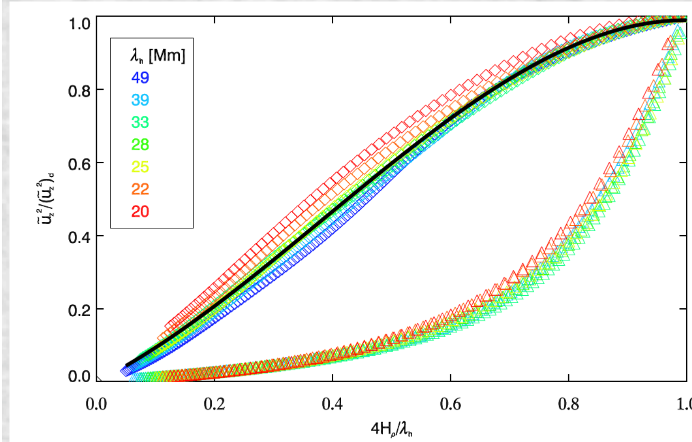
1. Construct a mixing length model of the solar convection zone integrating from the photosphere downward to 200Mm, the approximate depth of the solar convection zone.

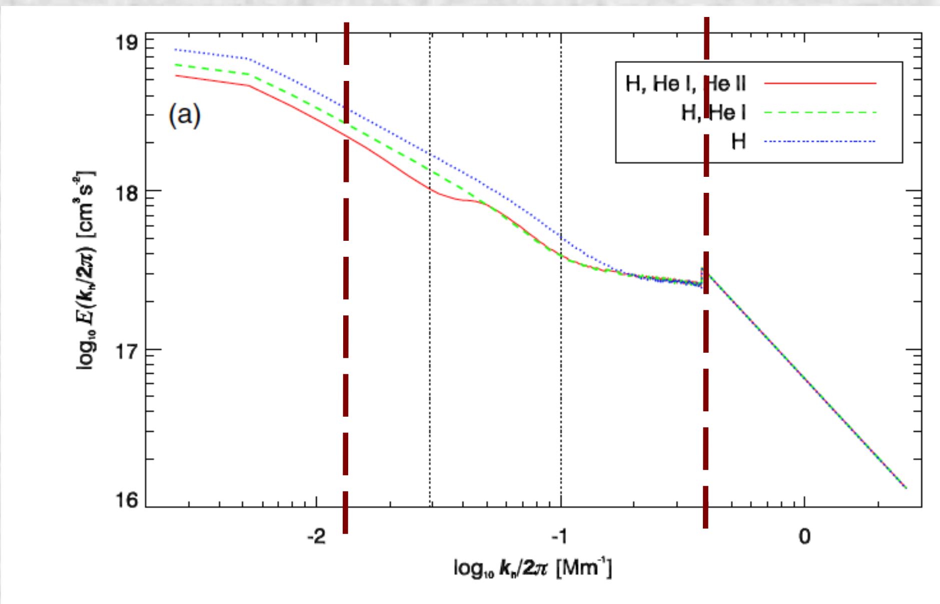
2. Determine the wavelength of the largest scale mode allowed at the bottom of the model atmosphere, $\lambda = 4H_\rho$, and use this as the lowest wavenumeber mode in a $k^{-5/3}$ spectrum. 380Mm is largest mode.

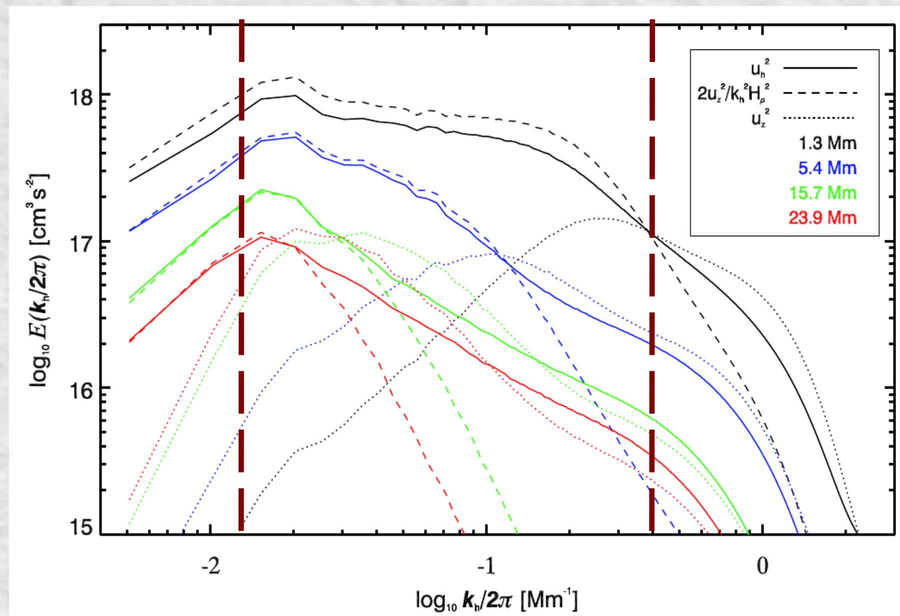
3. Move one step up in the atmosphere (a grid spacing of 64km is used to again match the hydrodynamic simulations). Decay modes with wavelengths longer than $4H_\rho$

4. Repeat Step 3 until the top of the model atmosphere is reached.

sfc ← deep

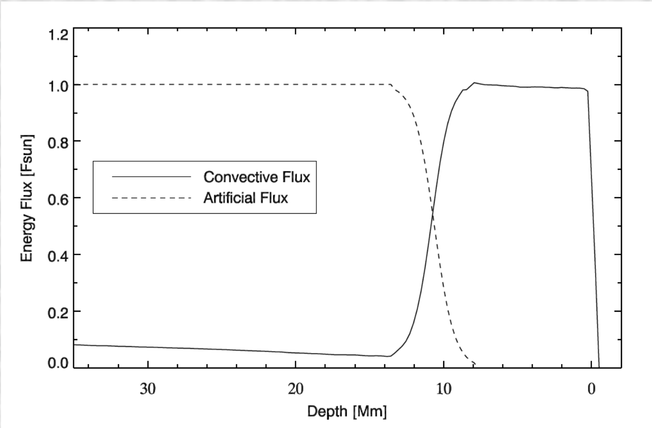




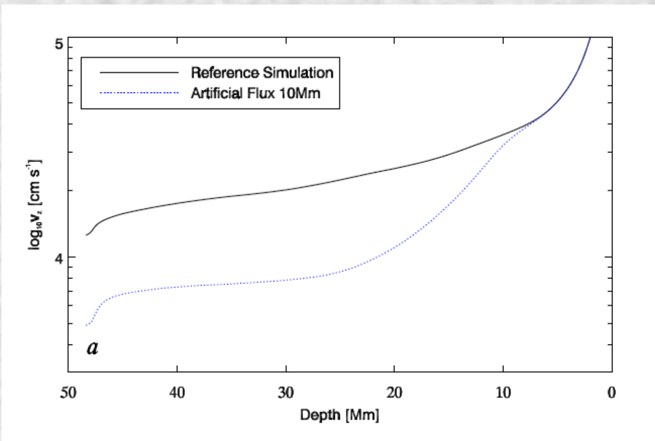


Photospheric spectrum of horizontal motions reflects the magnitude of the buoyancy driving of flows with depth.

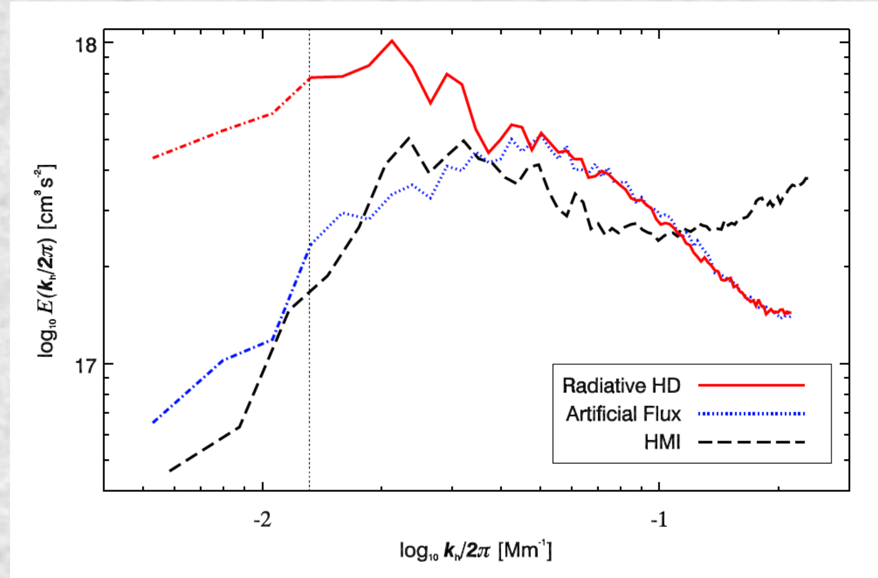
Radiative hydrodynamic (MURaM) Experiment:



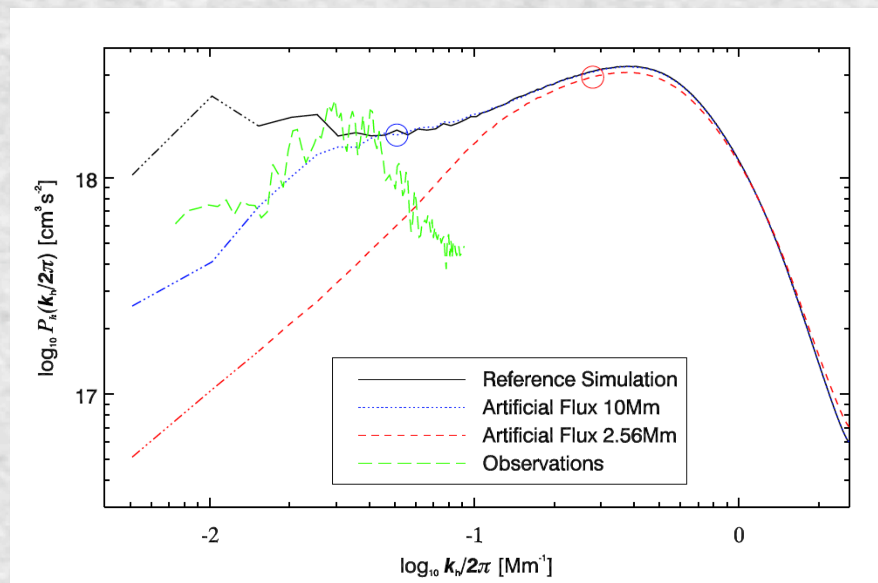
Artificial flux profile



rms velocity amplitude reduced by factor of 2.5 at depth



Identical structure tracking on data and observations



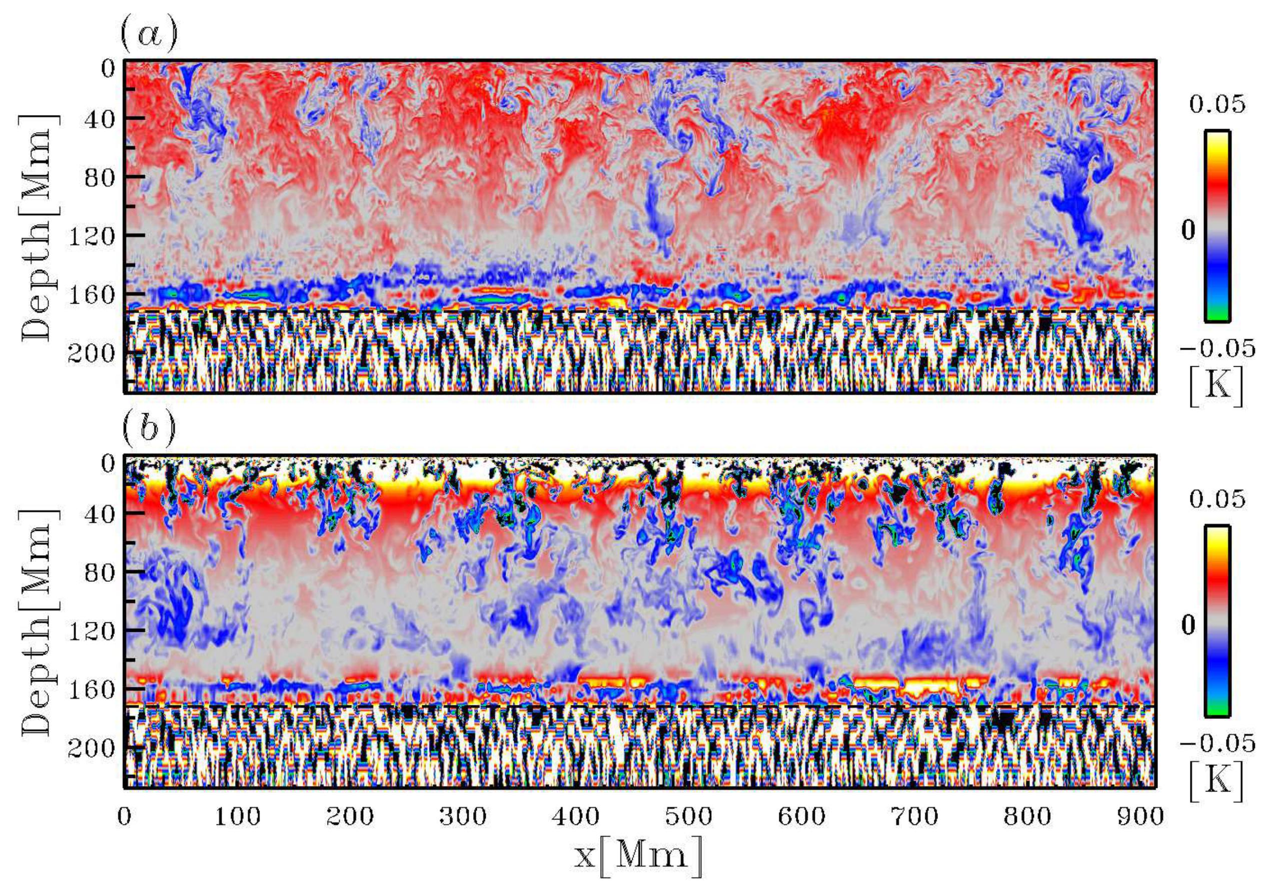
Turn spectrum over at that scale that corresponds to 4H_rho of convective layer depth

Supergranulation:

- The largest buoyantly driven mode of convection on the Sun

(only the upper 10 – 30 Mm of the solar convection zone is significantly superadiabatic)

What determines the depth over which the solar convection zone is superadiabatic?



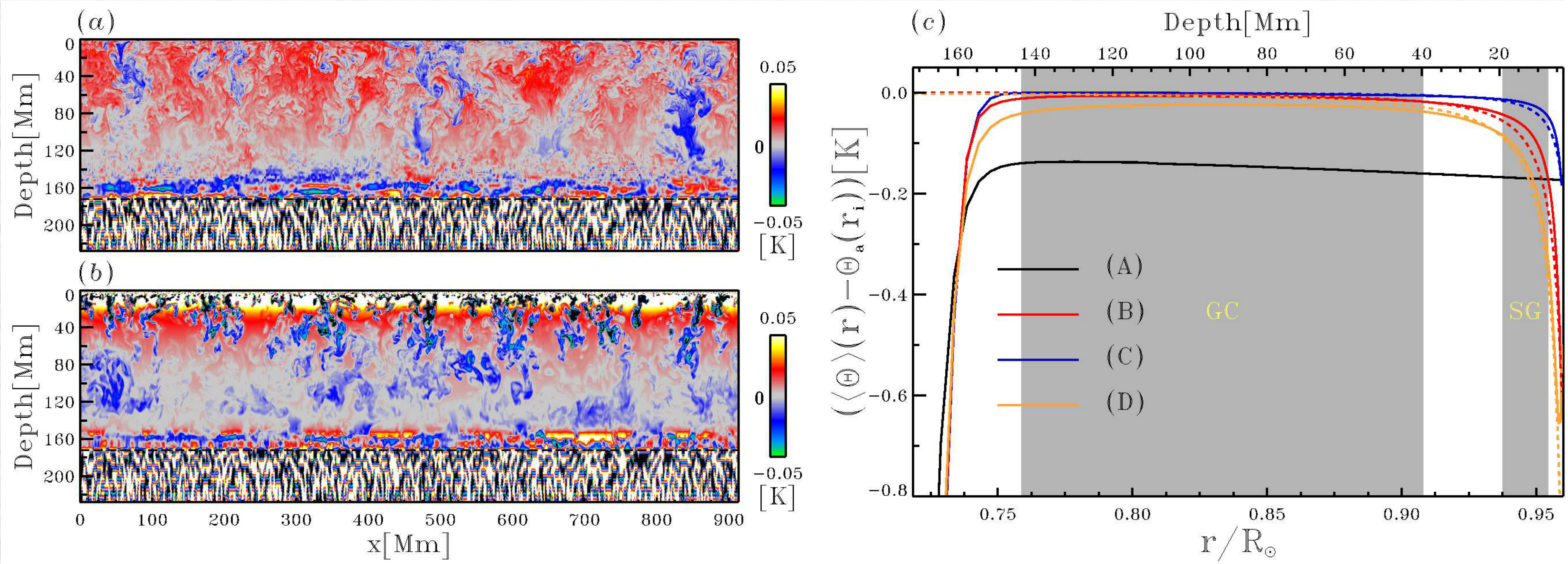
Surface driving (cooling layer):
3.5 Mm, ~5 density scale heights
H_rho = 0.36 – 2.6 Mm

Superadiabatic mean

Adiabatic interior

Stable layer

EULAG:
Anelastic
$r_b = 0.63\ R_\odot$, $r_t = 0.965\ R_\odot$
with physical dimensions
910.53Mm x 910.53Mm x 227.63Mm
on a vertically nonuniform grid of 1024^2 x 256 points

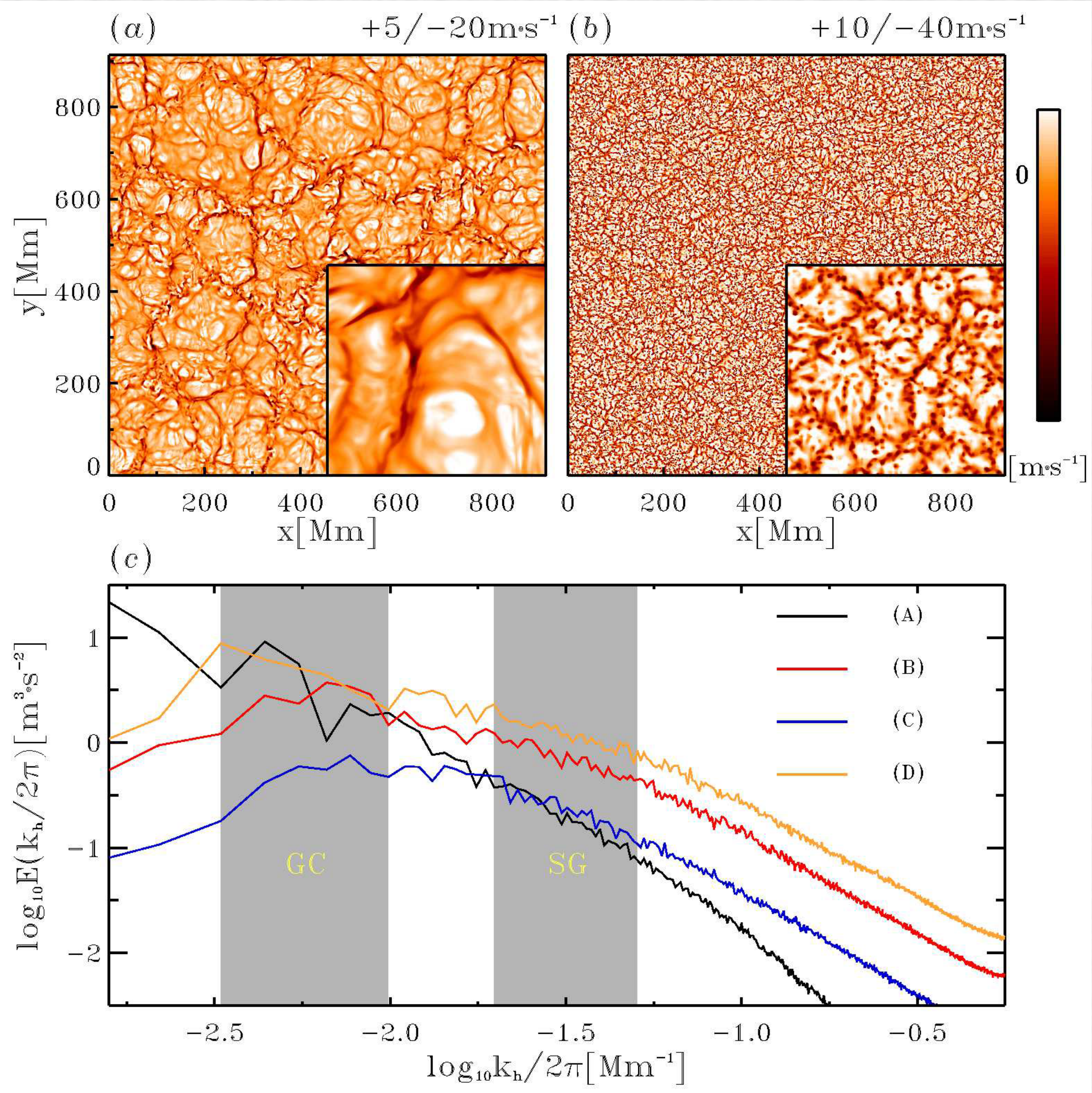


The depth of the superadiabatic region below the cooling layer depends only on the density and the entropy of the descending downflows and their geometric contribution to the mean (effective filling factor)

Dashed curves in Figure:

$$ds = c_p \, d\ln\theta \qquad \langle\theta(r)\rangle - \theta_a(r) \approx f(r)\theta_d \qquad f(r) \equiv f_d \frac{\rho_0(r_s)}{\rho_0(r)}$$

The depth of that layer in turn determines the spectrum of the horizontal motions



Snapshots of the horizontal velocity and horizontal velocity power spectra at 5 Mm

The Rossby number problem for global differential rotation and the solar supergranulation problem perhaps have the same solution:

Super granulation may be simply the largest buoyantly driven mode of convection on the Sun, with power decreasing to lower wavenumbers, because the deep solar convection zone more closely isentropic than models allow.

This implies that solar convection many be extremely non local with adiabatic motions linking the radiatively cooled photosphere above with the radiatively heated lower convection zone below across an isentropic interior with motions in the deep interior mainly the result of mass conservation, with very weak bouyancy driving.

The elevated superadiabatic gradient in simulations may result from horizontal diffusion of the downflow temperature perturbations that produces a net cooling that increases slower than the mean density. Such diffusion cools the upper layers compared to the lower and steepens the mean gradient. This should be checked.