„History of solar activity recorded in polar ice“
Dübendorf/Zürich, 14-15 November 2016

What can we learn from liquid metal experiments on dynamo action and magnetically triggered instabilities?

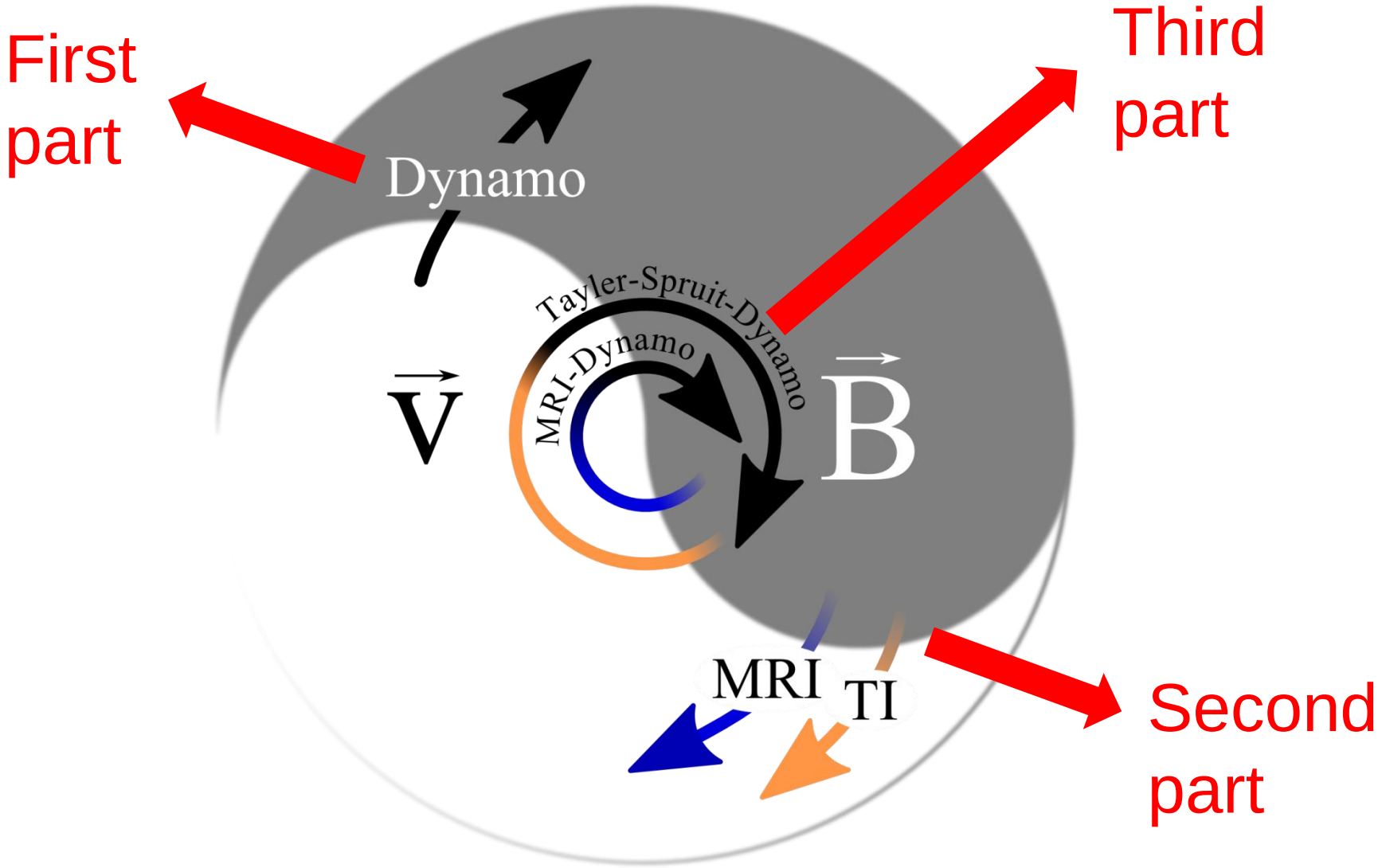
Frank Stefani

with thanks to

**G. Gerbeth, A. Giesecke, Th. Gundrum, G. Mamatsashvili ,
Ch. Steglich, M. Seilmayer, N. Weber, T. Weier (Dresden)
G. Rüdiger, M. Gellert (Potsdam),
R. Hollerbach (Leeds), A. Gailitis (Riga),
E. Kaplan (Grenoble), O. Kirillov (Moscow)**

HZDR
HELMHOLTZ
ZENTRUM DRESDEN
ROSSENDORF

The Yin-Yang of cosmical MHD

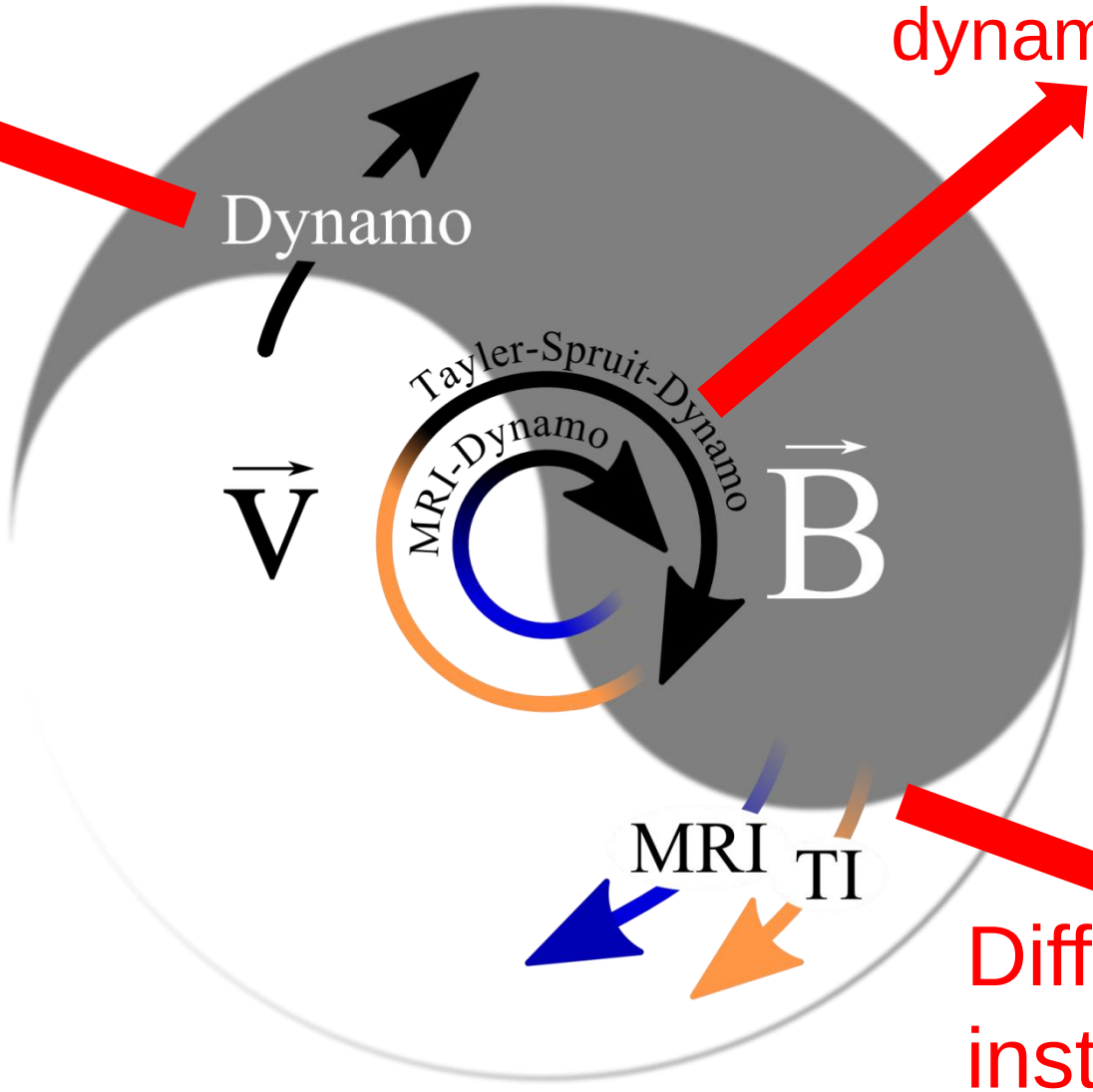


The Yin-Yang of cosmical MHD

Dynamos with magnetic materials

Reversal theory

Synchronized dynamos



Diffusive instabilities

THEORY

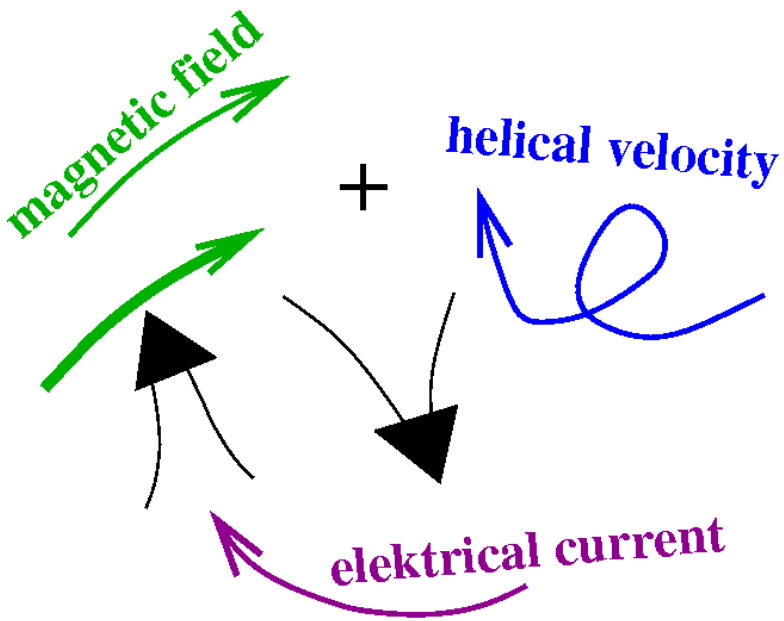
Motivation

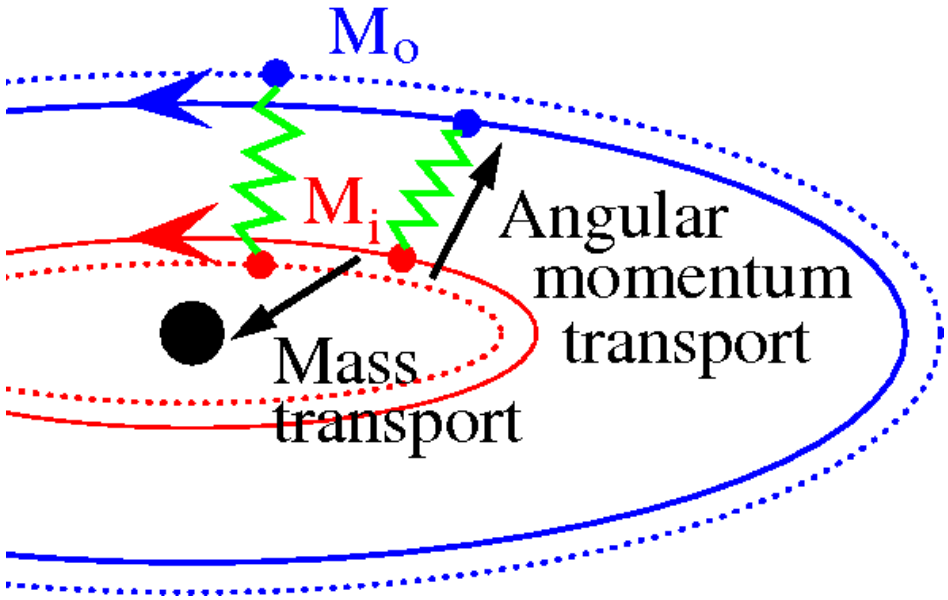
Homogeneous dynamo effect:

Self-excitation of magnetic fields in sufficiently strong, helical flows of conducting fluids

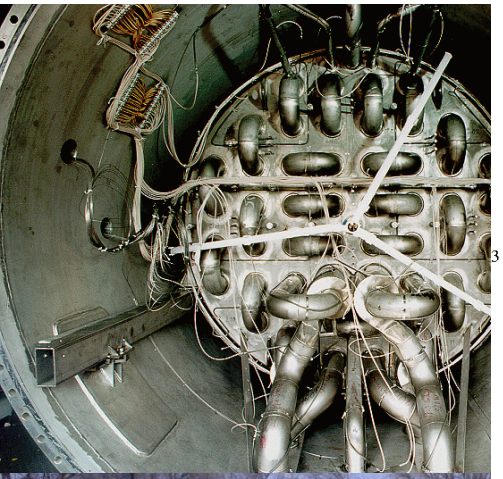
Magnetorotational instability (MRI):

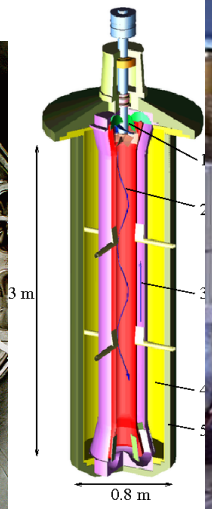
Magnetic fields act like springs and trigger angular momentum transport in accretion disks around protostars and black holes

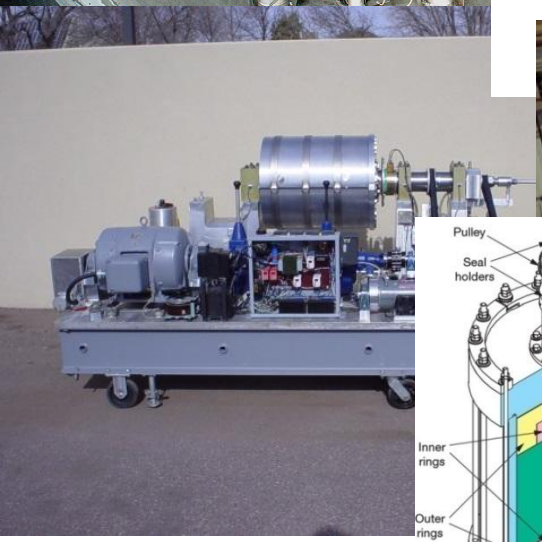


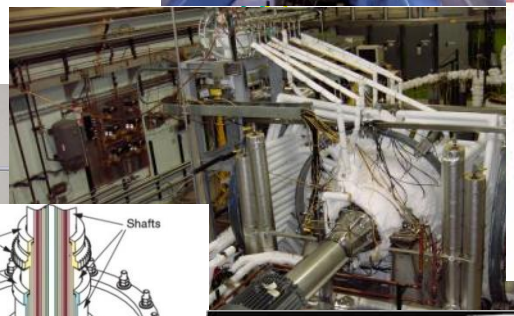


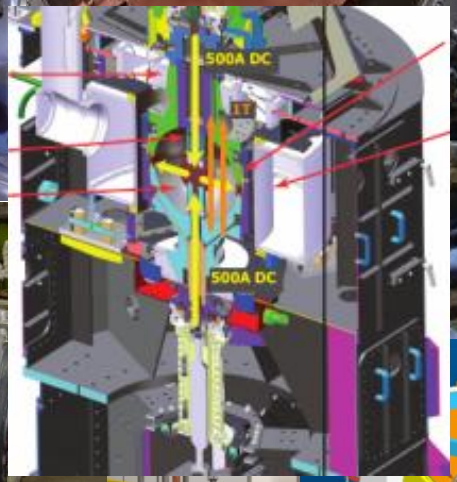
Previous, present, and future experiments

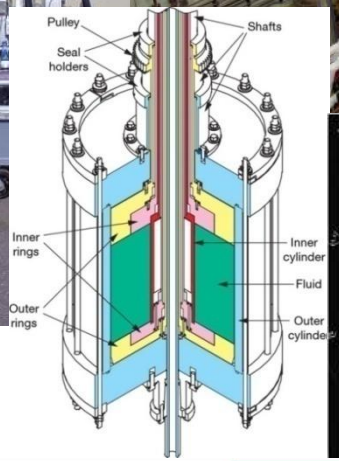


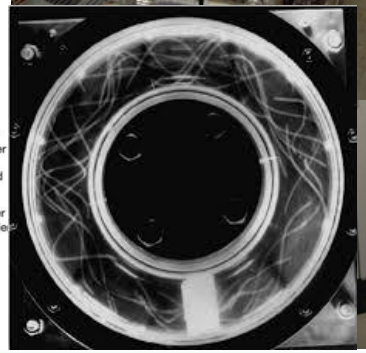












Prospects: DRESDYN

The *DRE*sden Sodium facility for *DYN*amo and *thermohydraulic studies* → **platform for geo- and astrophysics experiments** as well as for liquid metal applications in energy related technologies.

- Precession driven dynamo experiment
- Magnetorotational instability (MRI) and Tayler instability (TI)
- Liquid metal batteries (storage of intermittent renewables)
- Measuring techniques (magnetic flow tomography etc...)
- Thermohydraulics of sodium fast reactors
- Large Rayleigh-Bénard (rotating?, magnetic field?) ???

Stefani et al.: Magnetohydrodynamics 48 (2012), 103;
Magnetohydrodynamics (2015), 51 (2015), 275

DRESDYN: General features

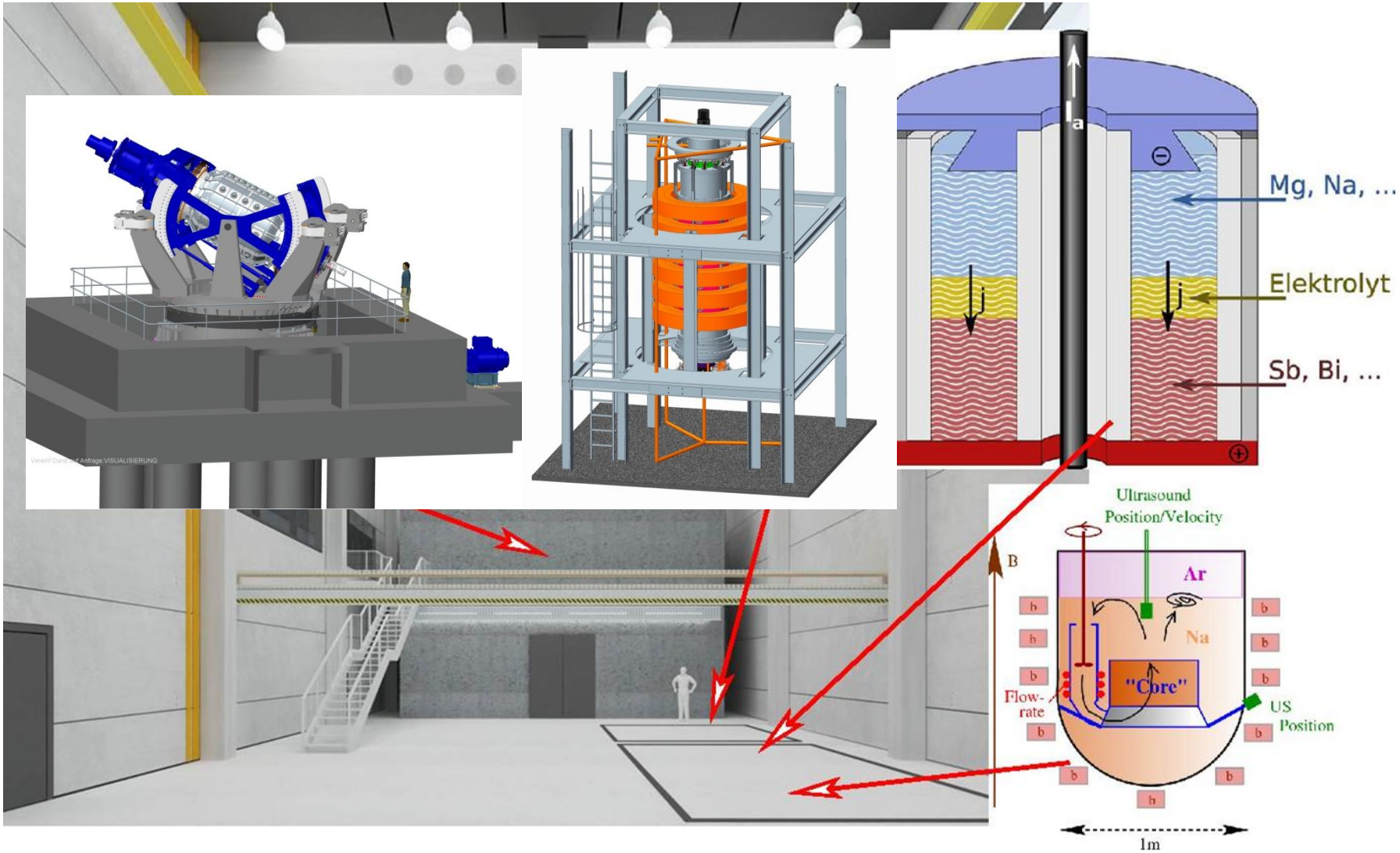
- New building ~500 m^2
- Total sodium inventory: 12 tons
- Large experimental hall for MRI/TI experiment, sodium loop, X-ray lab, liquid metal batteries, Rayleigh-Bénard ?
- Precession driven dynamo experiment with separate strong basement and containment for Argon flooding

DRESDYN building as of January 2016: READY

DRESDYN: General features

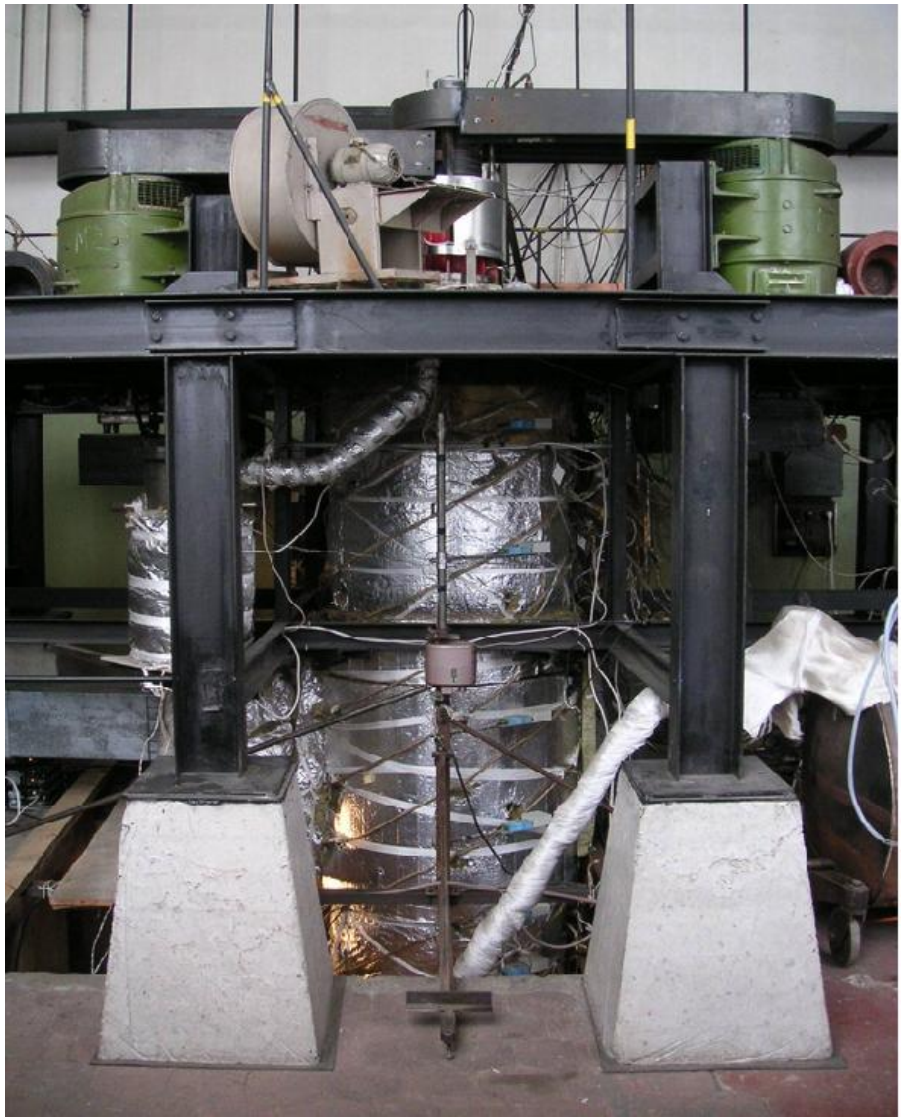
- First sodium tank (of 4) has arrived (September 2016)

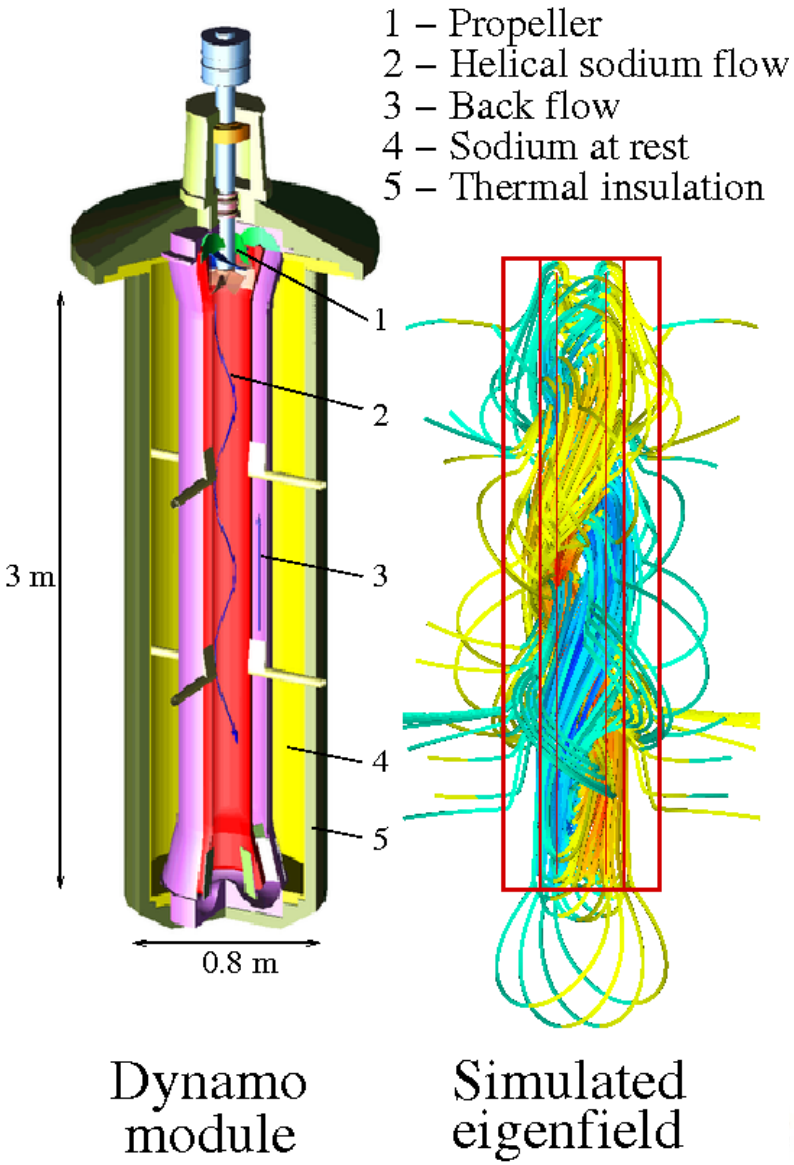
DRESDYN: General scheme



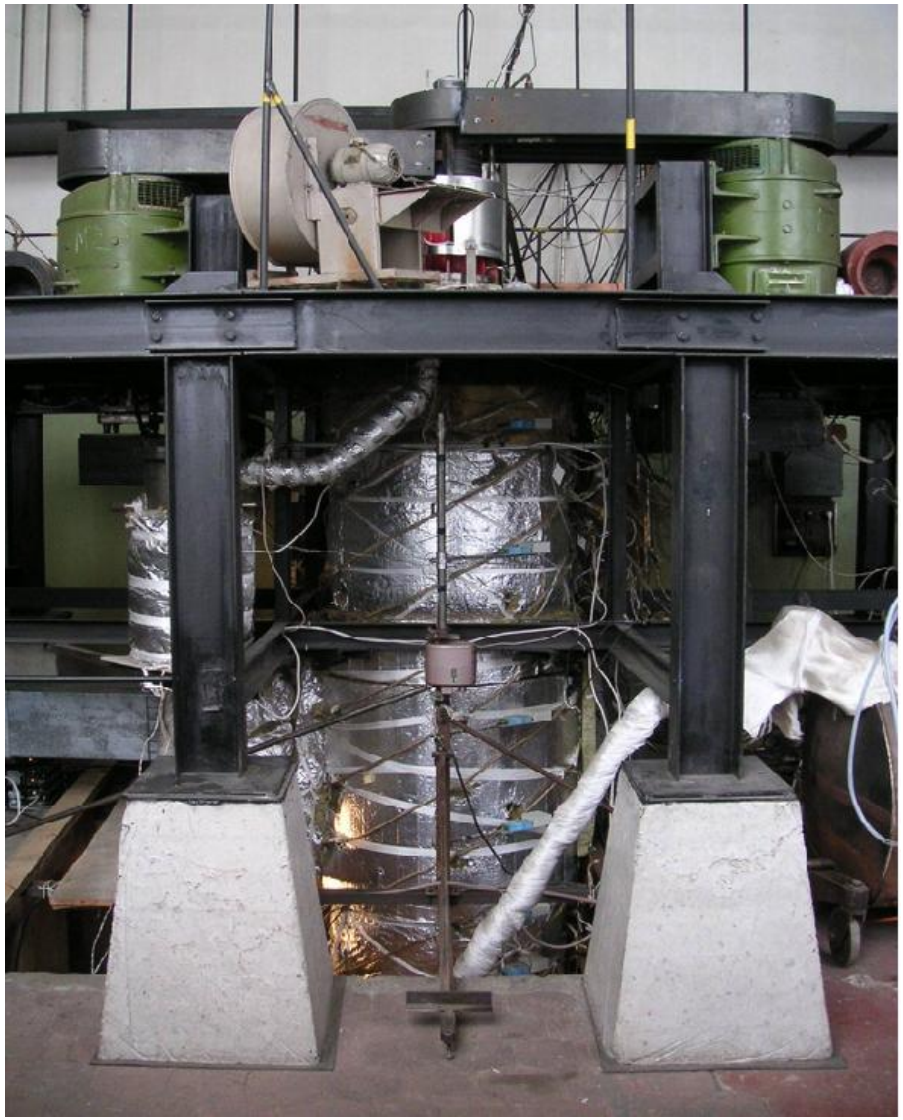
Dynamos

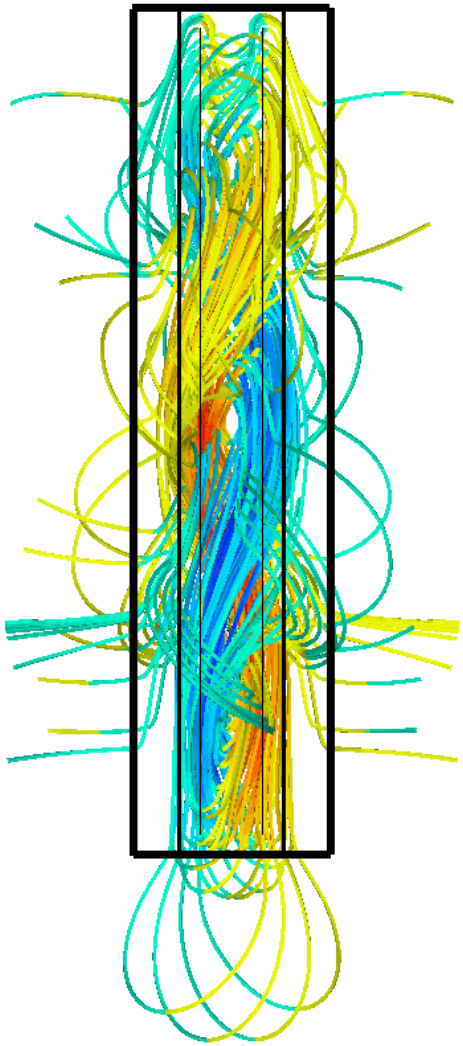
Riga dynamo experiment





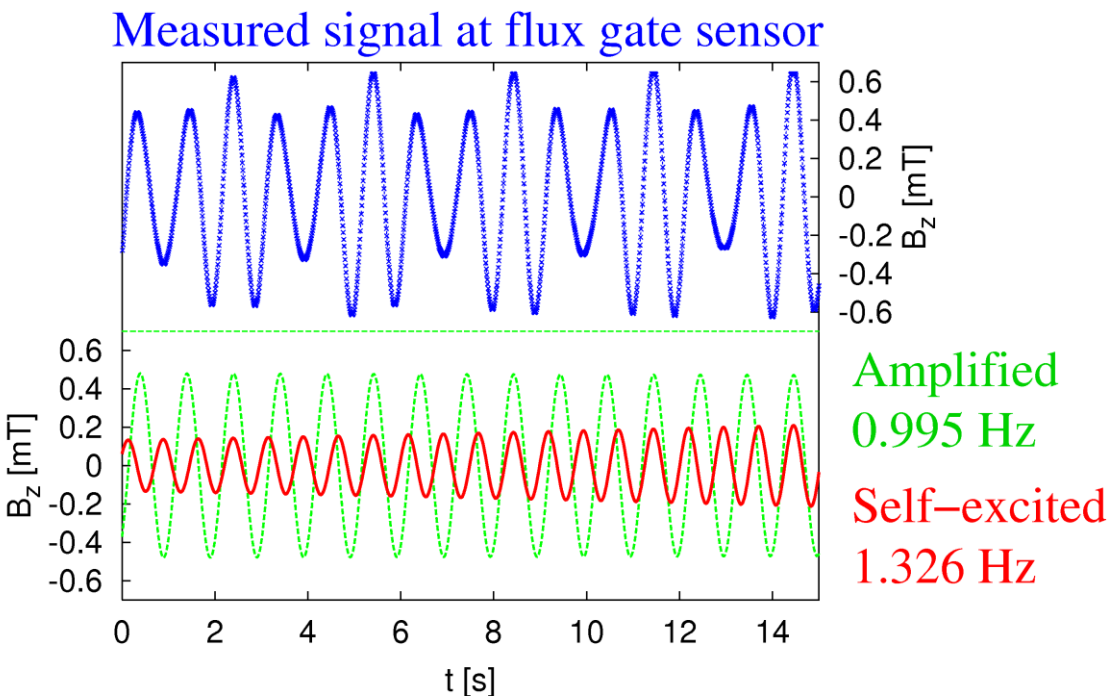
Riga dynamo experiment

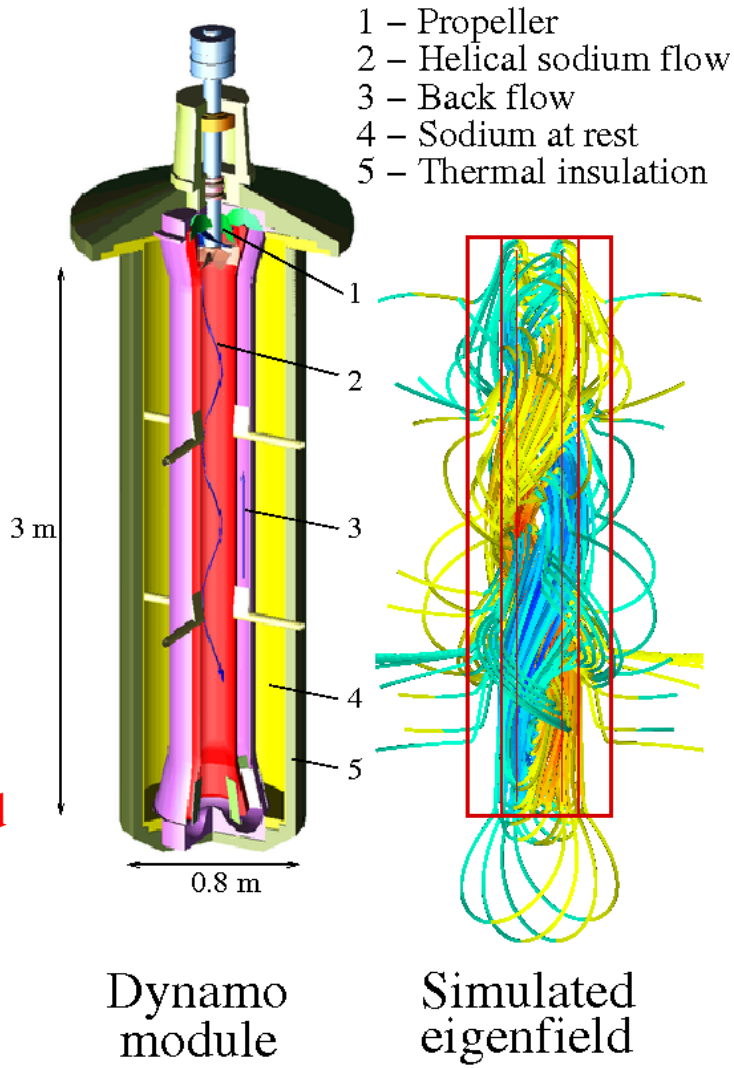




Riga dynamo experiment

First experimental realization of magnetic field self-excitation in a liquid metal flow (11 November 1999)

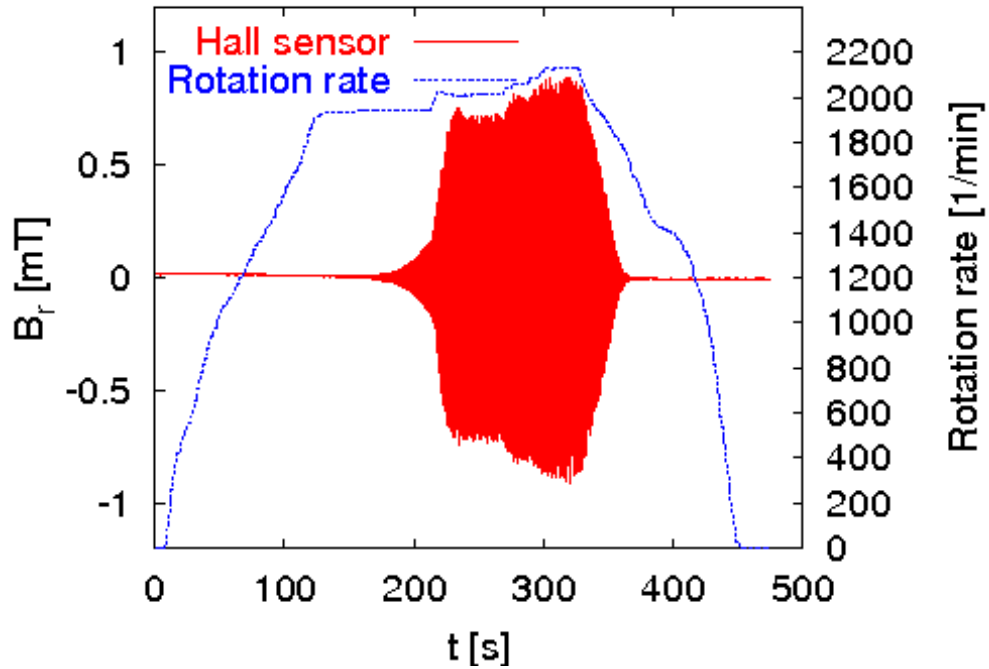


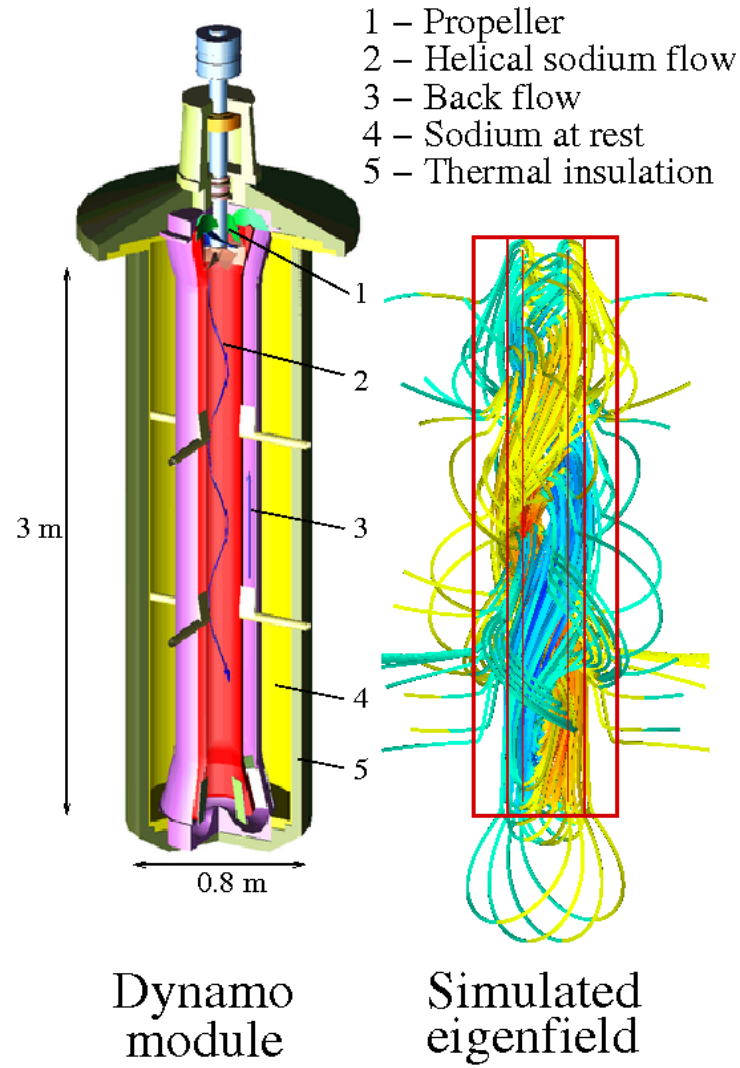


Gailitis et al., Phys. Rev. Lett. 84 (2000) 4365; Phys. Rev. Lett. 86 (2001) 3024; Rev. Mod. Phys. 74 (2002) 973 ; Phys. Plasmas 11 (2004) 2838; Compt. Rend. Phys. 9 (2008), 721

Riga dynamo experiment

From the kinematic to the saturated regime (July 2000)

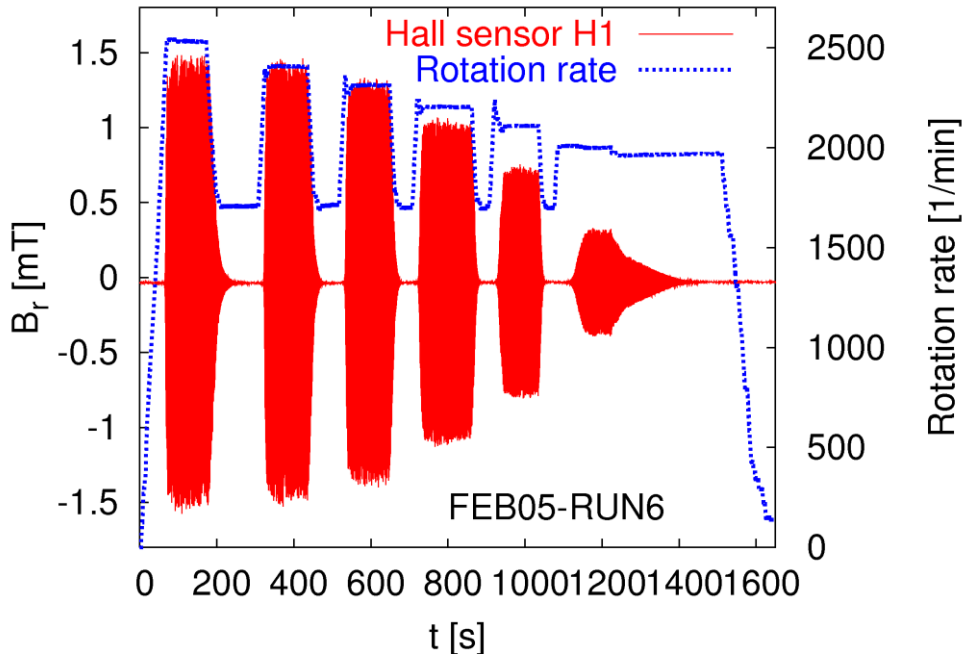


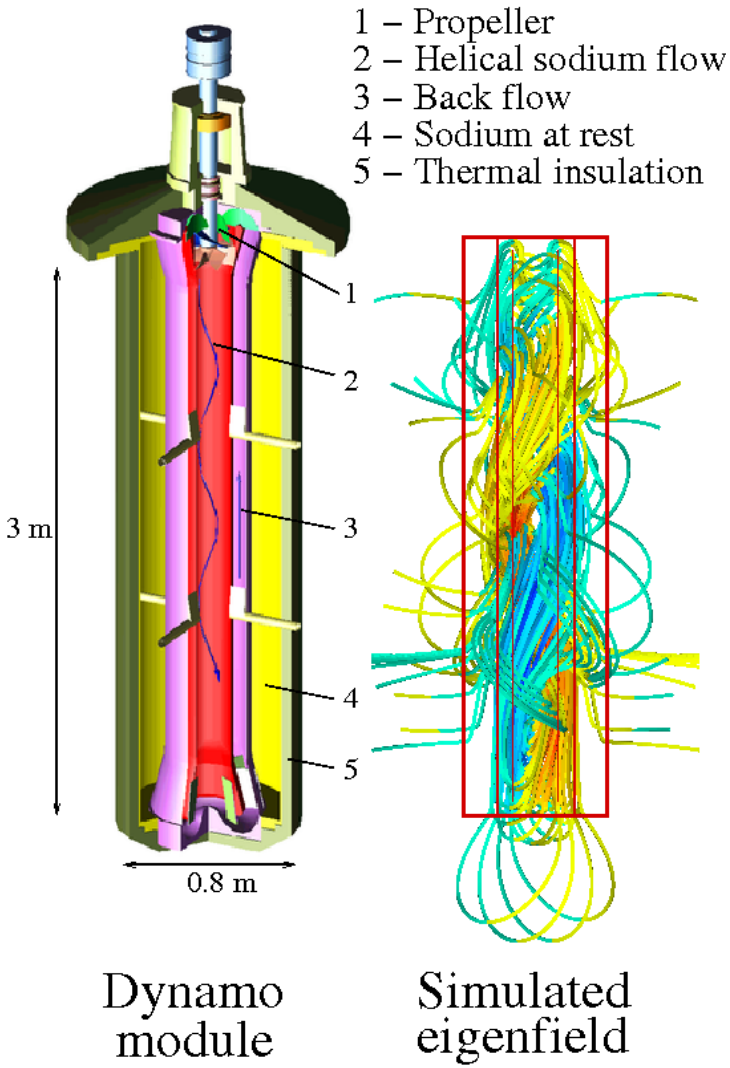


Gailitis et al., Phys. Rev. Lett. 84 (2000) 4365; Phys. Rev. Lett. 86 (2001) 3024; Rev. Mod. Phys. 74 (2002) 973 ; Phys. Plasmas 11 (2004) 2838; Compt. Rend. Phys. 9 (2008), 721

Riga dynamo experiment

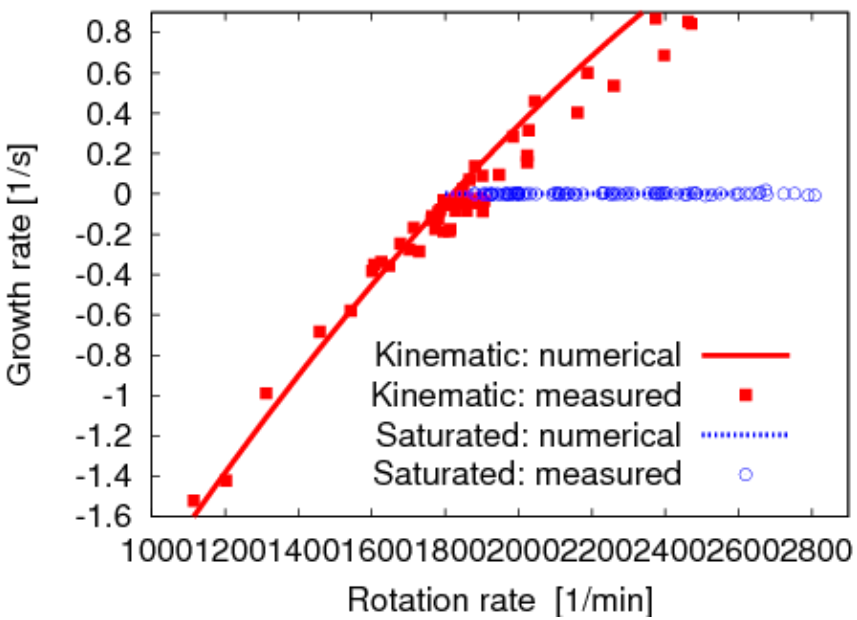
Switching the dynamo on and off
(February 2005)

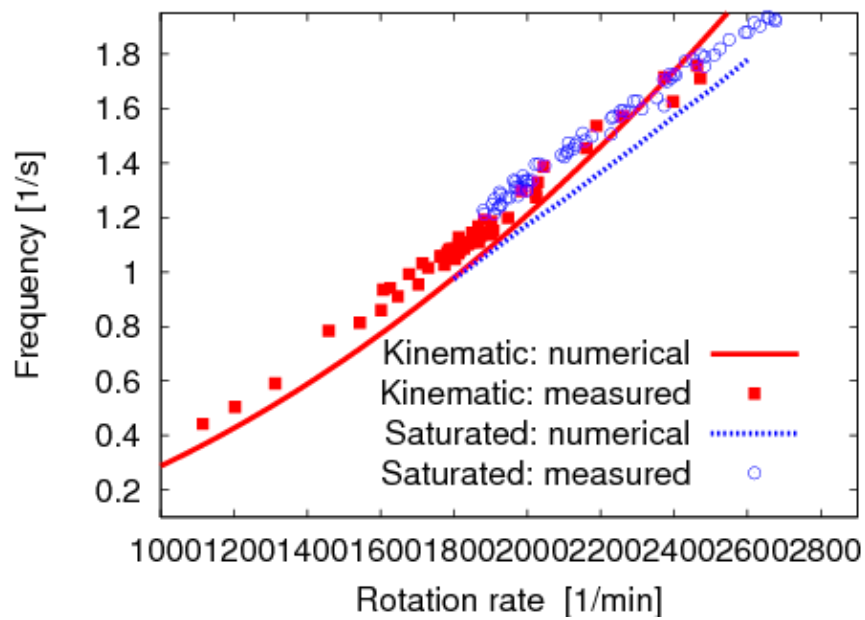




Gailitis et al., Phys. Rev. Lett. 84 (2000) 4365; Phys. Rev. Lett. 86 (2001) 3024; Rev. Mod. Phys. 74 (2002) 973 ; Phys. Plasmas 11 (2004) 2838; Compt. Rend. Phys. 9 (2008), 721

Riga dynamo experiment: Growth rates and frequencies





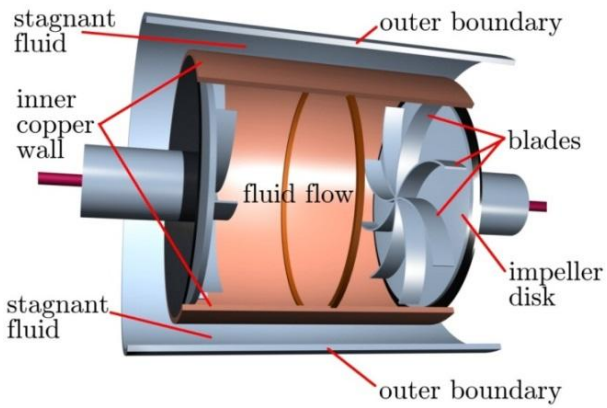
Numerical predictions (with correct vacuum boundary conditions) of the kinematic dynamo were accurate to some 5-10 per cent

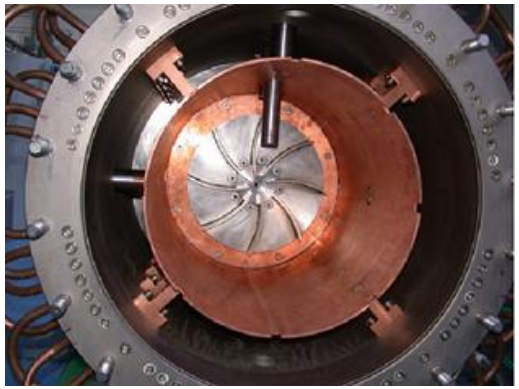
Simplified back-rection model (Lorentz forces acting along streamlines) gives very reasonable field amplitudes and structures in the saturation regime

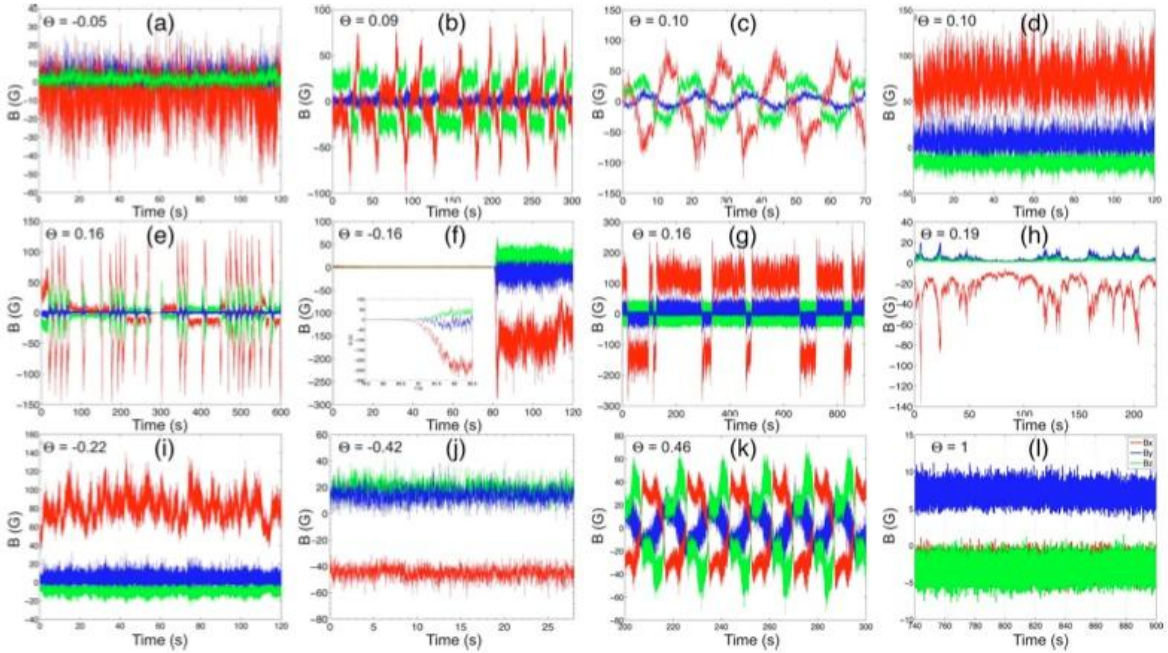
Gailitis et al., C. R. Physique 9 (2008), 721

von-Karman-Sodium (VKS) Experiment

VKS has shown self-excitation and a wealth of wonderful dynamical effects, including oscillations, reversals, burst, localized fields….

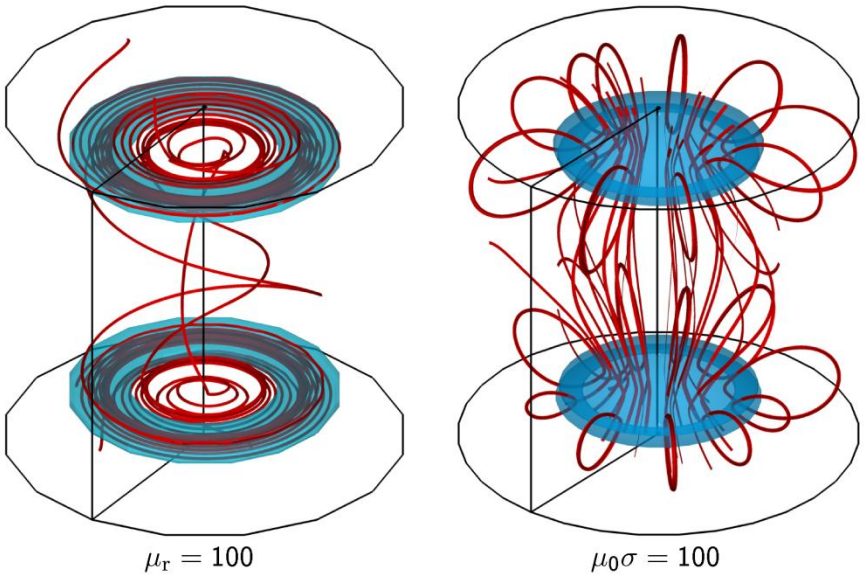


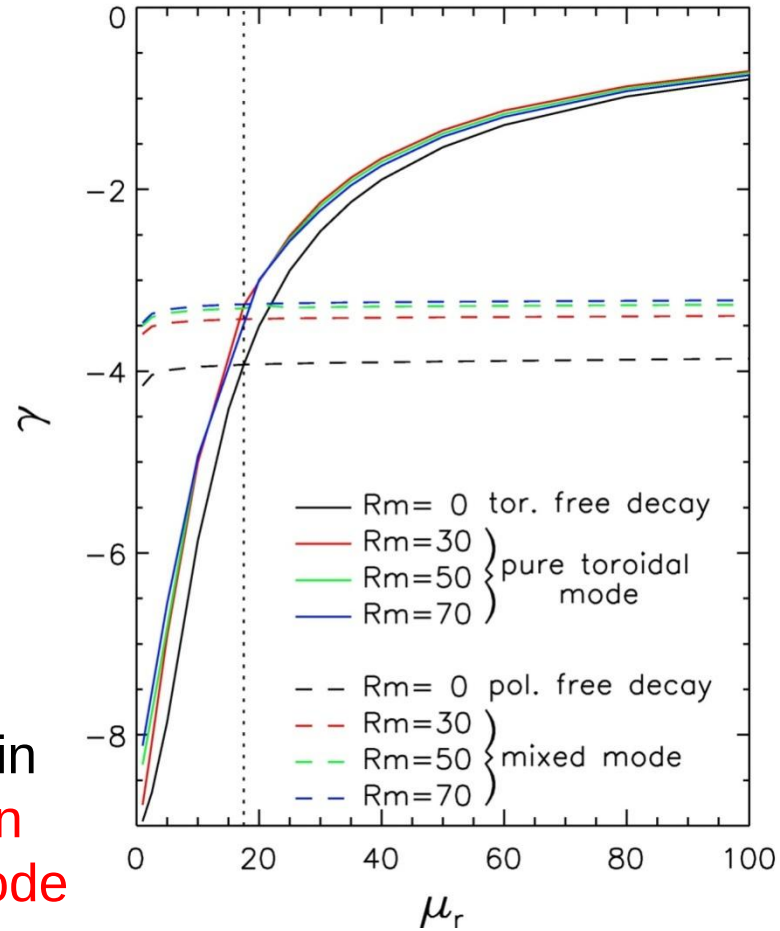




Monchaux et al., Phys. Fluids 21, 035108 (2009)

VKS-Dynamo: Role of high μ impellers?



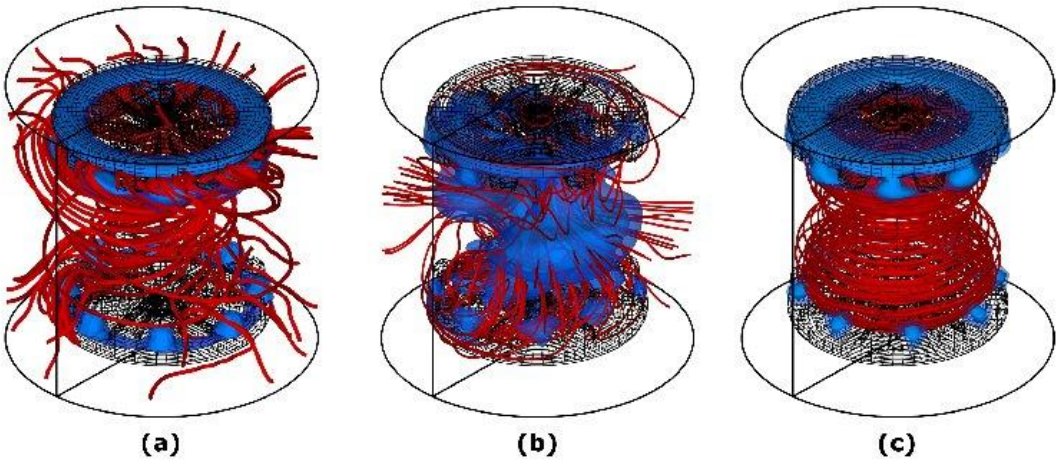


Crucial role of high μ_{rel} impellers, even in the free-decay case: Transition between dominant poloidal and toroidal eigenmode at μ_{rel} ~20

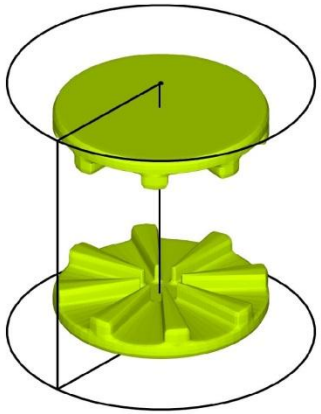
Giesecke et al., GAFD 104 (2010); New J. Phys. 14 (2012)

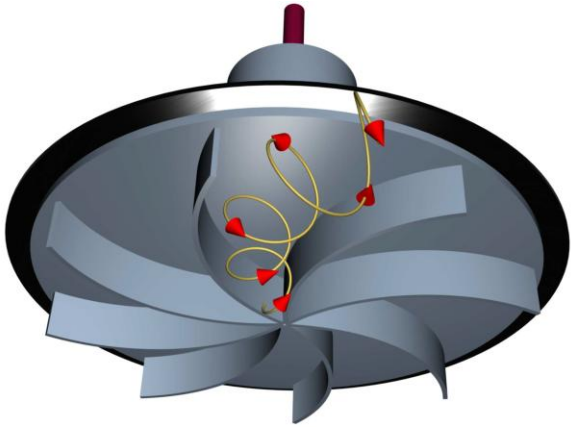
THEORY

VKS-Dynamo: Role of high μ impellers?



Rm=30, α=−1.5	Rm=70, α=0	Rm=30, α=0
growing m=0	growing m=1	decaying m=0



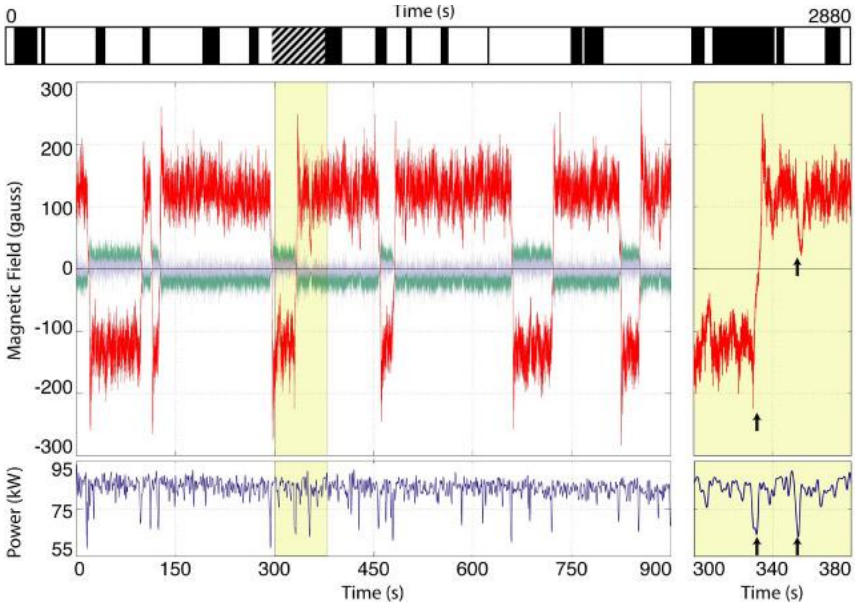


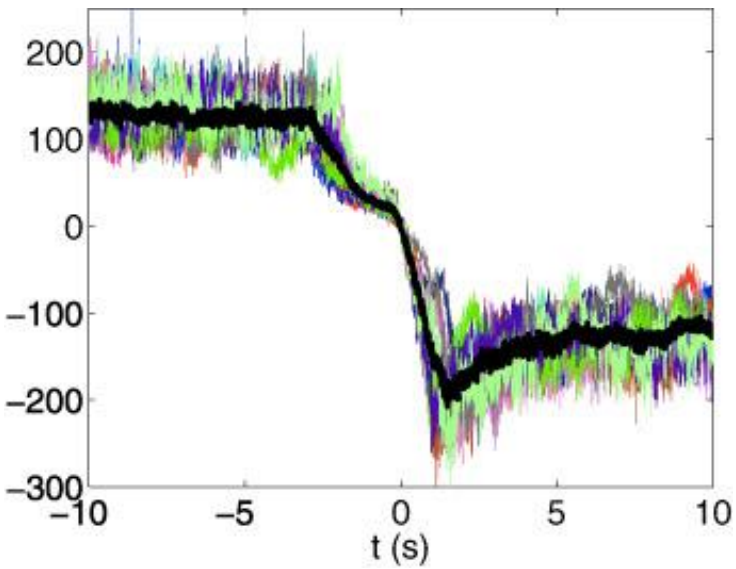
…on the basis of the dominant toroidal mode, some small-scale helicity between the blades (α-effect) is sufficient to ignite the dynamo

Giesecke et al., Phys. Rev. Lett. 104 (2010)

THEORY

Reversals of the geomagnetic field and the VKS dynamo field

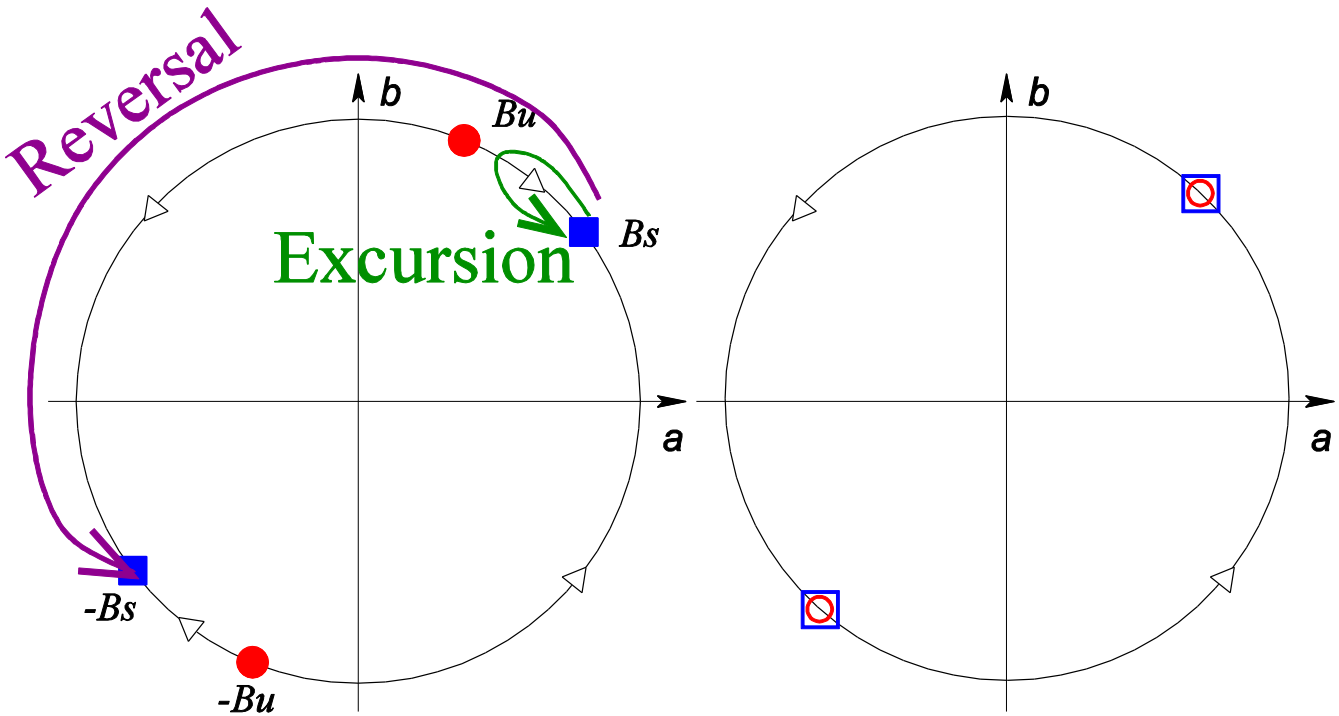




Berhanu et al., EPL 77 (2007), 59001

THEORY

Reversals of the geomagnetic field and the VKS dynamo field



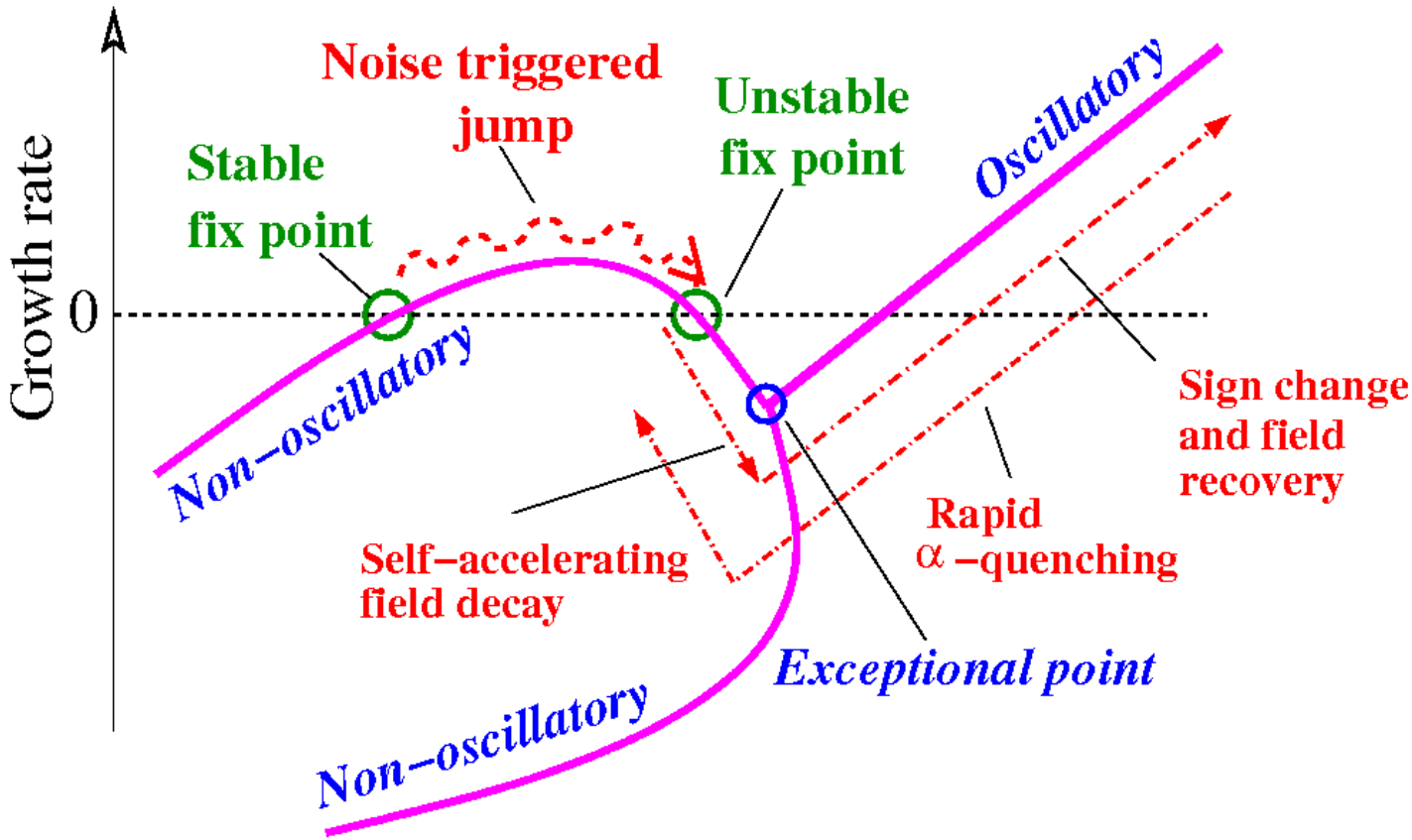
Dynamical systems approach: Saddle-node bifurcation

$$\frac{d\Theta}{dt} = \alpha_0 + \alpha_1 \sin(2\Theta) + \text{noise}$$

Petrelis et al., PRL 102 (2009), 144503

THEORY

Reversals of the geomagnetic field and the VKS dynamo field

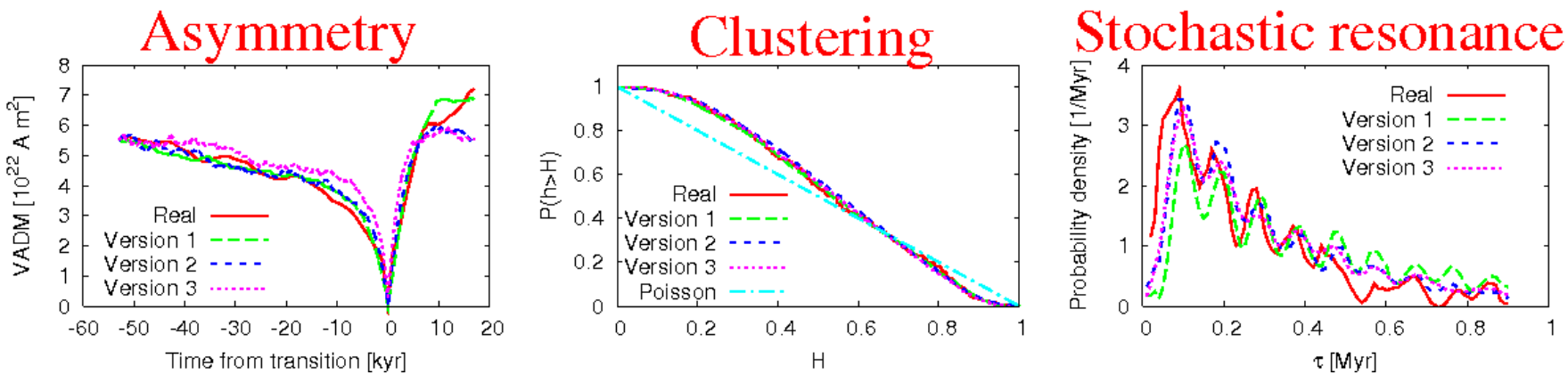


Spectral properties of a (spherically symmetric) α^2 dynamo

Stefani et al., PRL 94 (2005) 184506; Earth Planet. Sci.Lett. 243 (2006), 828; GAFD 101 (2007) 227; Fischer et al., Inverse Probl. 25 (2008) 065011

THEORY

Reversals of the geomagnetic field and the VKS dynamo field



Simple model explains many features of reversals, and can be used to constrain basic parameters of the geodynamo. Best results for:

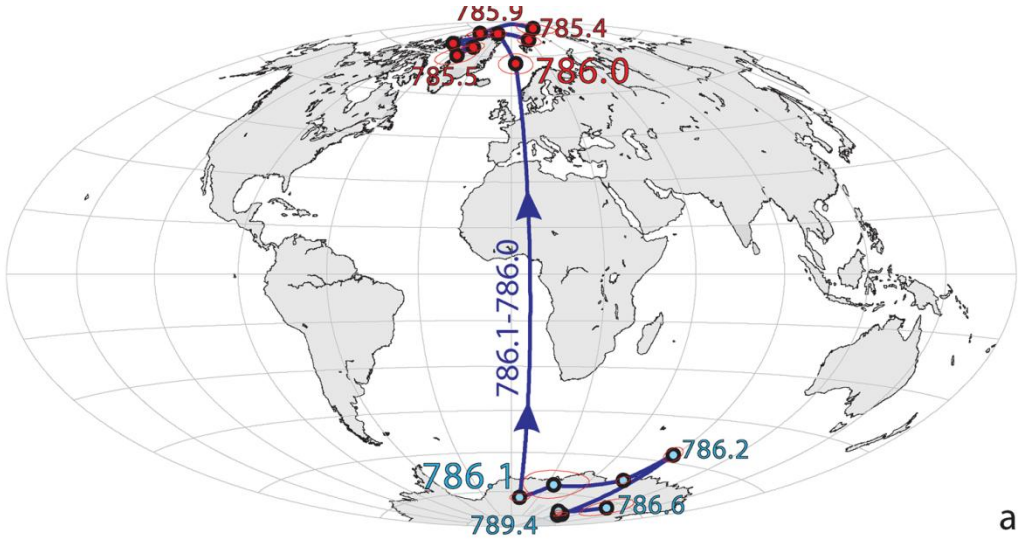
- Supercriticality of the dynamo: Factor 10
- Relative strength of periodic forcing: 10 per cent
- Diffusion time: 64 kyr, i.e reduction by a factor 3.5 compared to 225 kyr from molecular conductivity. This is in rough agreement with numerical simulations (Schrinner et al. 2007) and β effect measured in Perm .

Fischer et al., Inverse Probl. 25 (2008) 065011

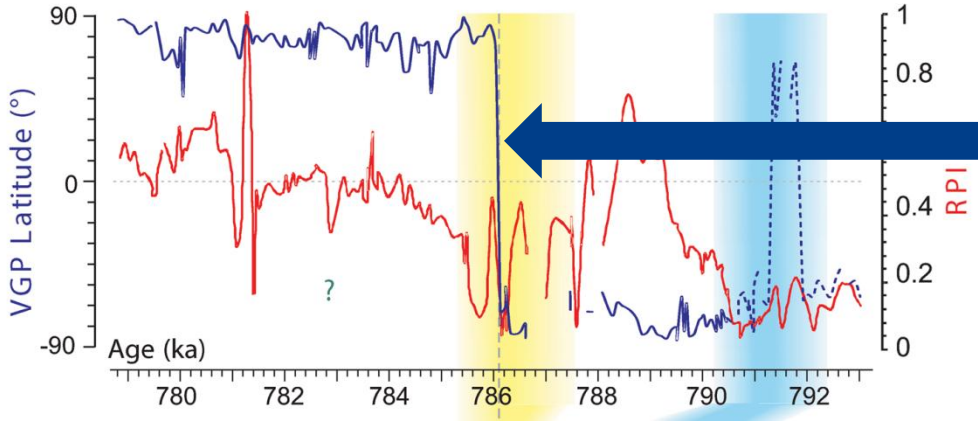
THEORY

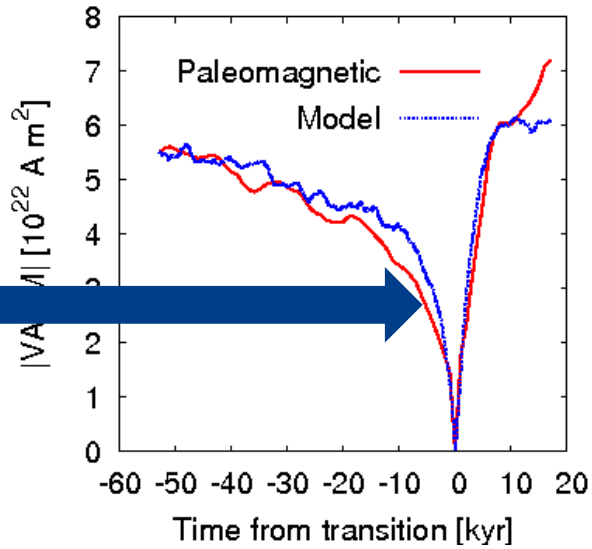
Reversals of the geomagnetic field and the VKS dynamo field

Further dynamo constraints may follow from the recent observation of very fast reversals



Sagnotti et al., Geophys.J.Int. 199 (2014), 1110





THEORY

Summary on Reversals

- Reversals are (very likely...) noise triggered relaxation oscillations in the vicinity of exceptional points of the spectrum of the dynamo operator ⟷ in the parlance of dynamical systems, this corresponds to a saddle-node bifurcation

- Highly supercritical dynamos tend to self-tune into a reversal-prone state (”edge of chaos”, ”self-organized criticality” ???)

- Asymmetry, clustering, and stochastic resonance can be used as proxies for constraining essential parameters of the geodynamo.

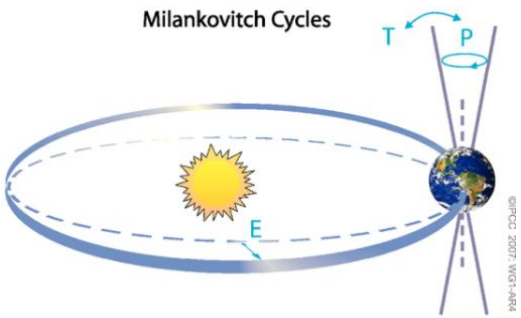
- The estimated conductivity reduction, if anisotropic, would be important for dynamo simulations (selection of axial or equatorial dipole, etc.)

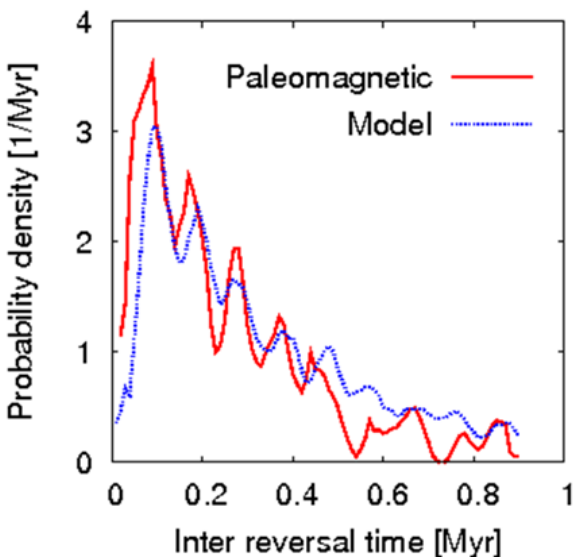
Fischer et al., Inverse Probl. 25 (2008) 065011

THEORY

The DRESDYN precession dynamo: Geo/astrophysical motivation

Strong indication for influence of variations of Earth‘s orbit parameters on the geodynamo





Probability density of inter-reversal times shows maxima at multiples of the Milankovic cycle of Earth‘s orbit eccentricity (95 ka) ⟷ climate??

Consolini and De Michelis, Phys. Rev. Lett. 90 (2003), 058503

Recent discussion of the lunar dynamo in terms of precession or impacts

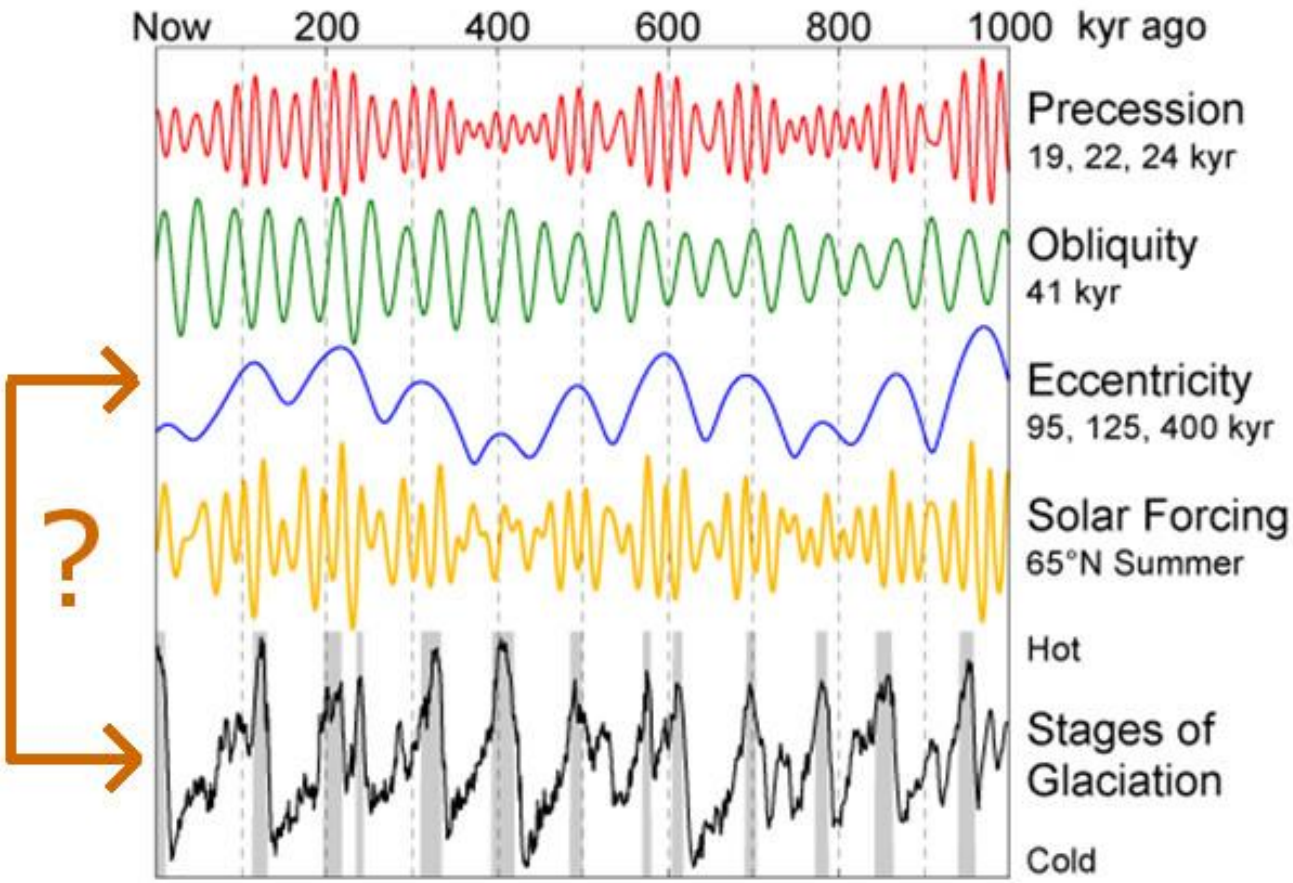
Dwyer et al., Nature 479 (2011), 212; Le Bars et al., Nature 479 (2011), 215

Evidence for ancient core dynamo in asteroid Vesta

Fu et al., Science 338 (2012), 238

The DRESDYN precession dynamo: Geophysical motivation

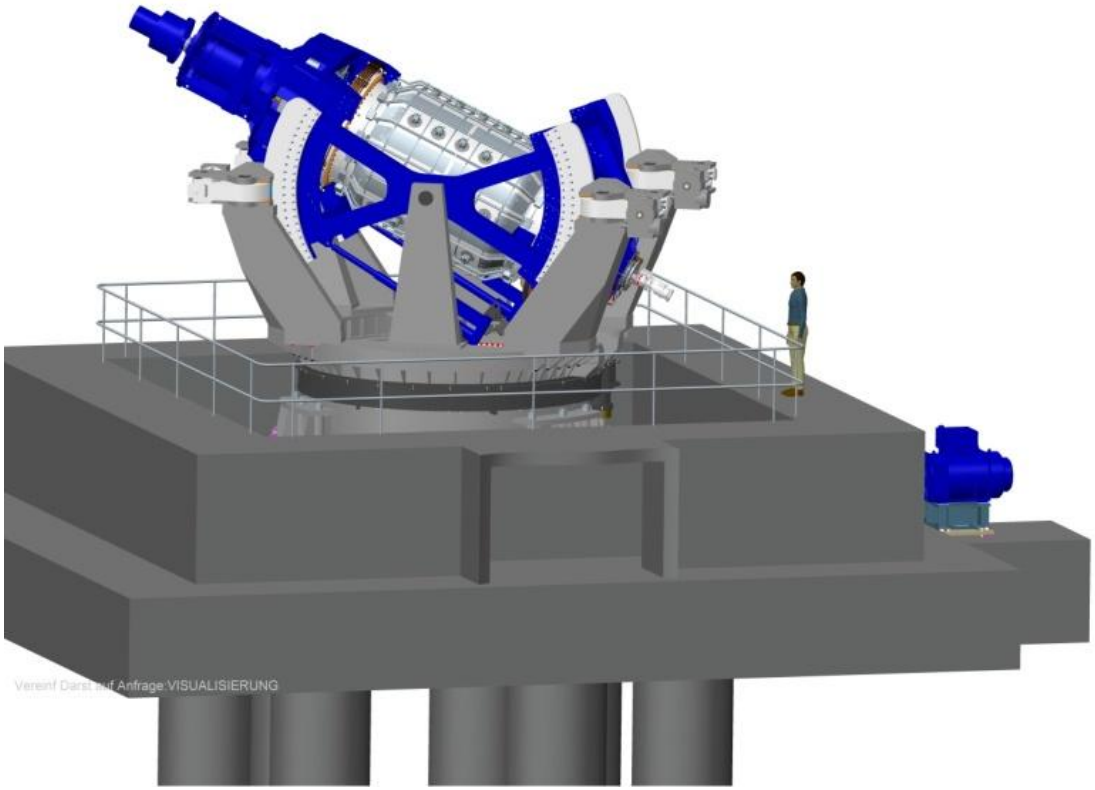
Most interesting connection between geomagnetic field and climate (sequence of ice ages)



Precession driven dynamo: Experiment is designed and (partly) under construction

Key parameters:

- 2 m diameter, 2 m height, 8 m^3 liquid sodium
- Cylinder rotation: 10 Hz (will need some 800 kW motor power)
- Turntable rotation: 1 Hz
- Magnetic Reynolds number ~ 700
- Gyroscopic torque onto the basement: 8 MNm !



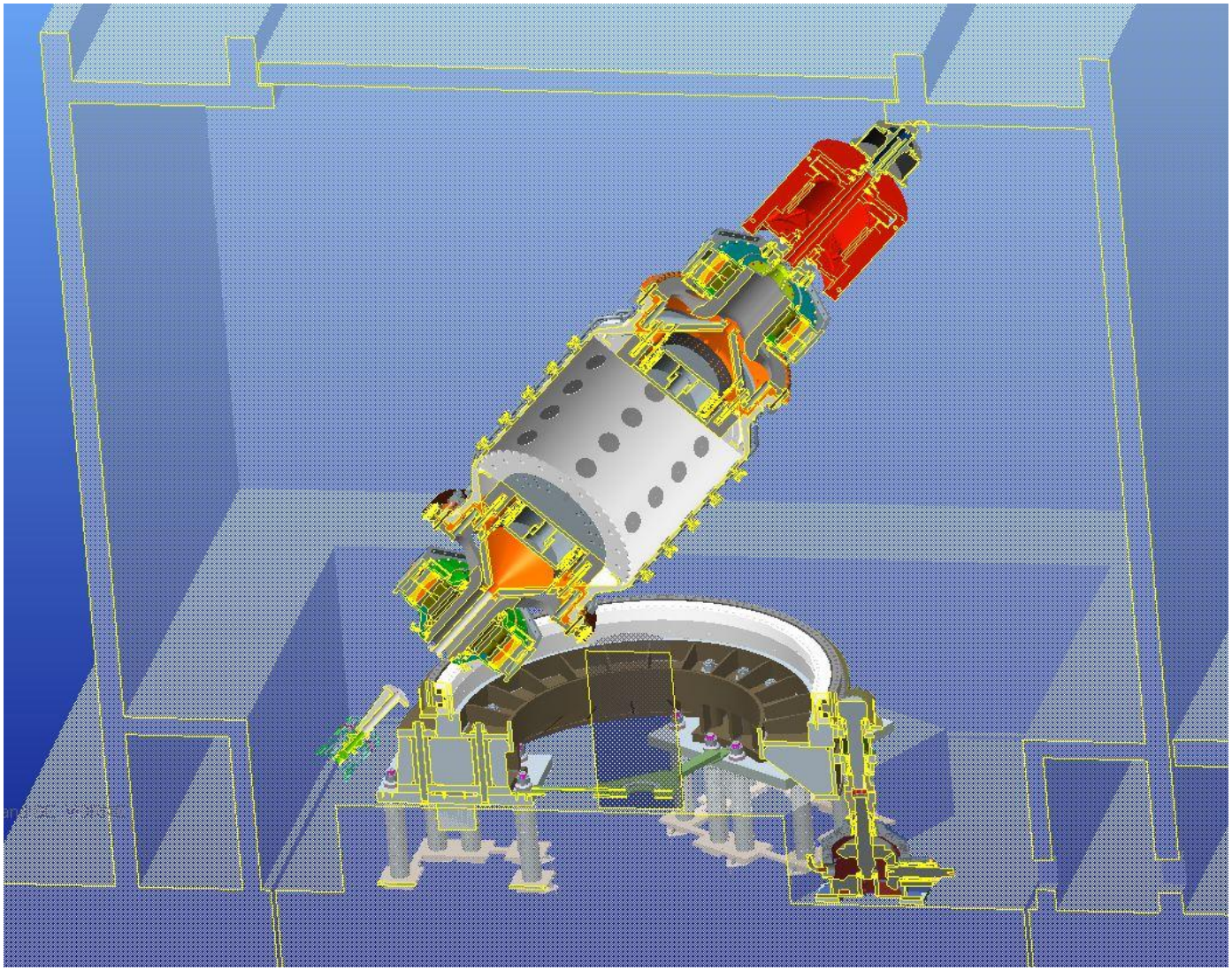
“Fundamental” problems due to huge gyroscopic torque

April 2013: drilling 7 holes (22 m deep)

July 2013: Constructing the ferroconcrete basement

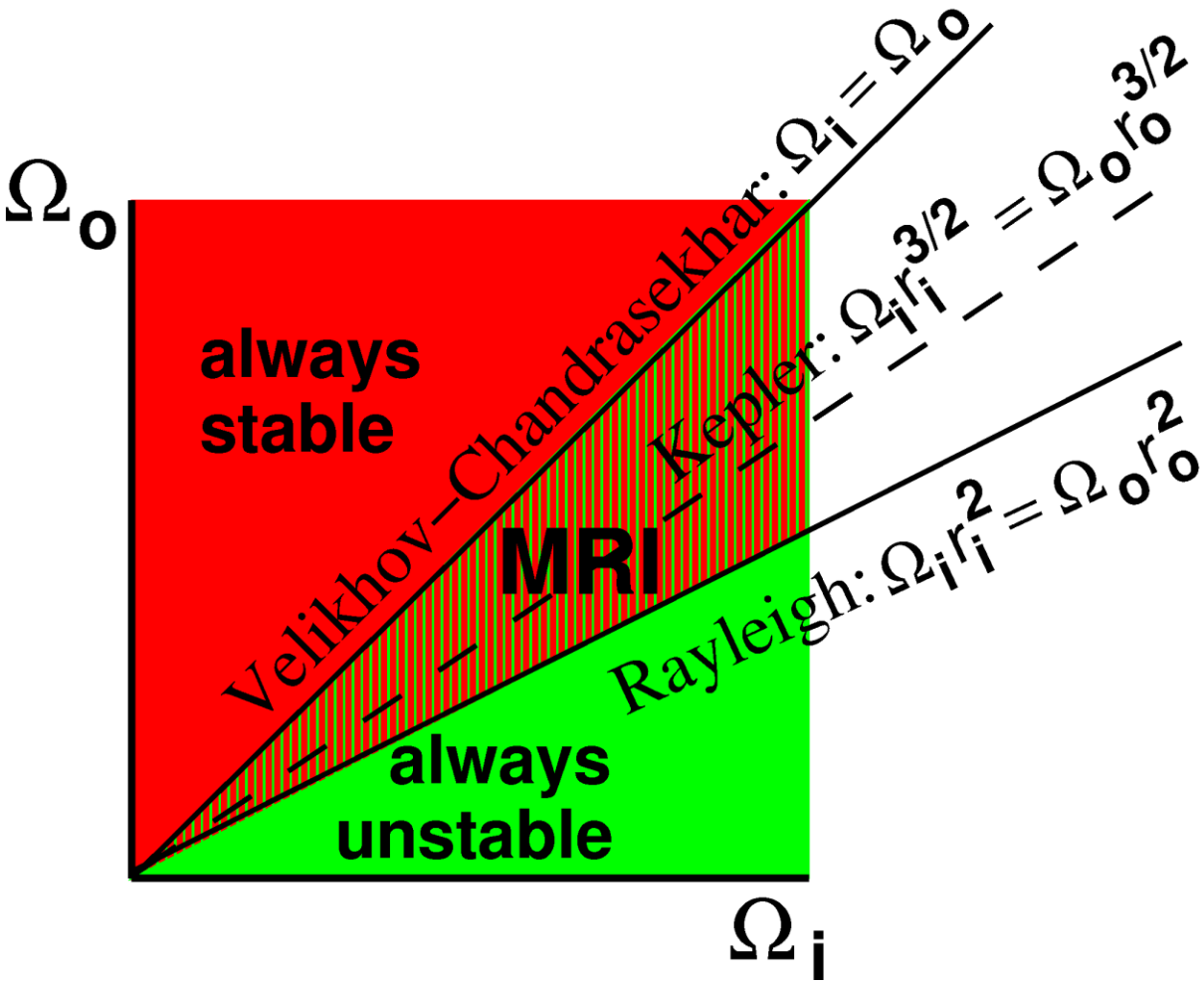
May 2015: The tripod for the dynamo within the containment (with stainless steel “wallpaper”)

Precession driven dynamo: Situation in the containment



Magnetorotational and Tayler instability

History of MRI: Magnetized Tayler-Couette flow



E.P. Velikhov: Sov. Phys. JETP 9 (1959), 995

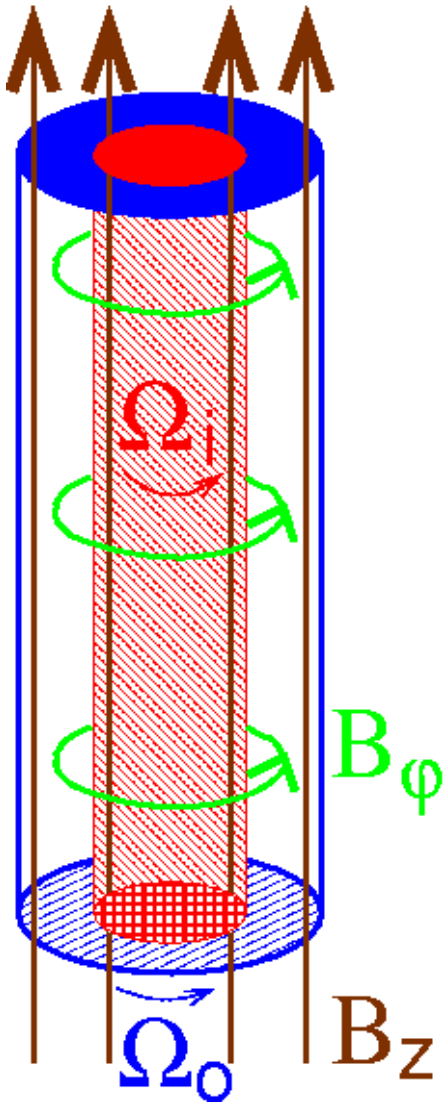
Standard MRI and helical MRI

- Standard MRI (with purely axial field) scales with Lundquist (S) und magnetic Reynolds (Rm)
- Experiments on SMRI with large Rm in Maryland and Princeton → DRESDYN
- Helical MRI: B_z replaced by B_z+B_φ: scales with Hartmann (Ha) and Reynolds (Re)

Hollerbach and Rüdiger: Phys. Rev. Lett. 95 (2005), 124501

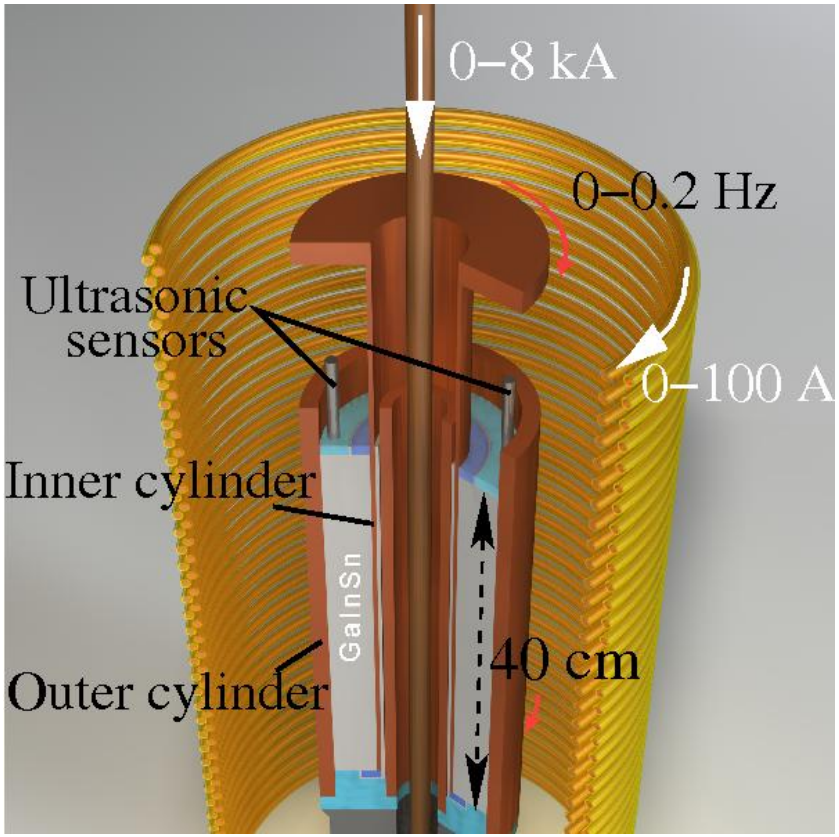
- Re_{crit}: 10^3 instead of 10^6
- $Ha_{crit:}$ 30 instead of 1000

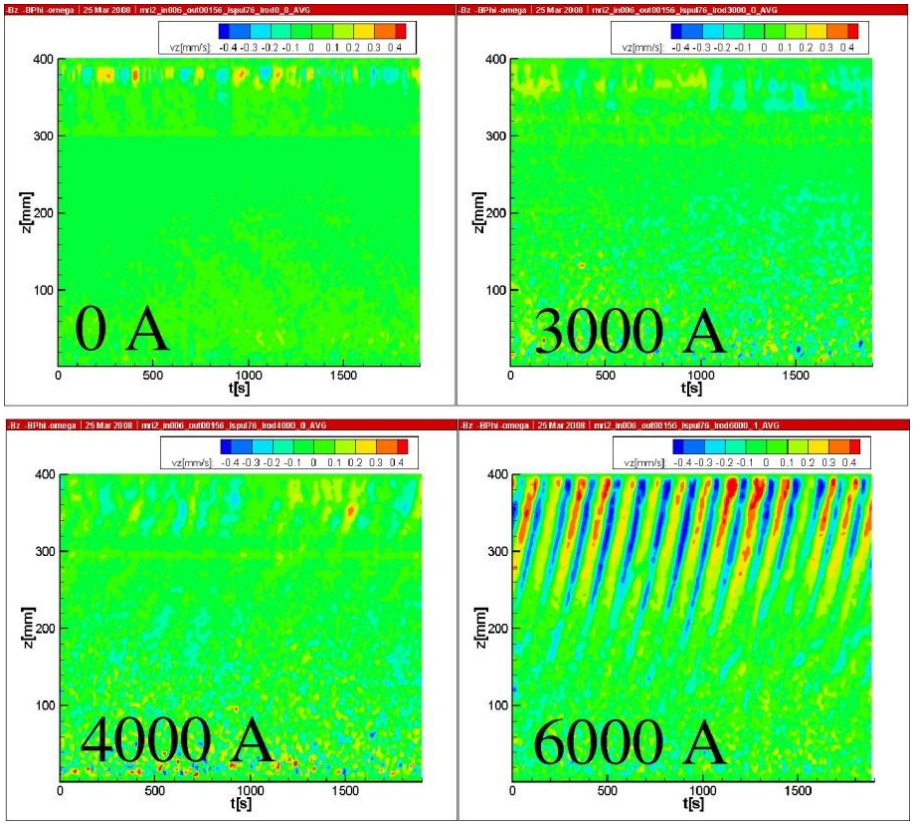
- → Potsdam ROssendorf Magnetic InStability Experiment (PROMISE)
- Drawback: does not work for Kepler (yet)!



Helical magnetorotational instability (HMRI)

2006: First experimental evidence of HMRI



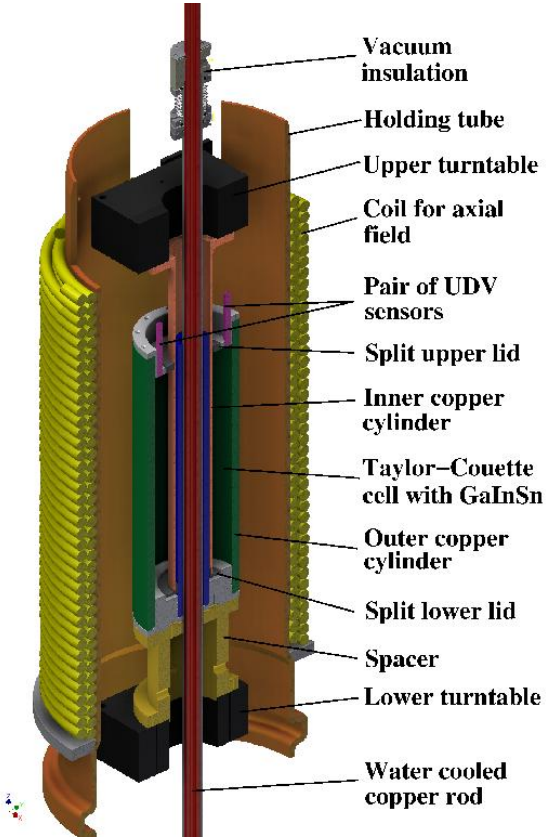


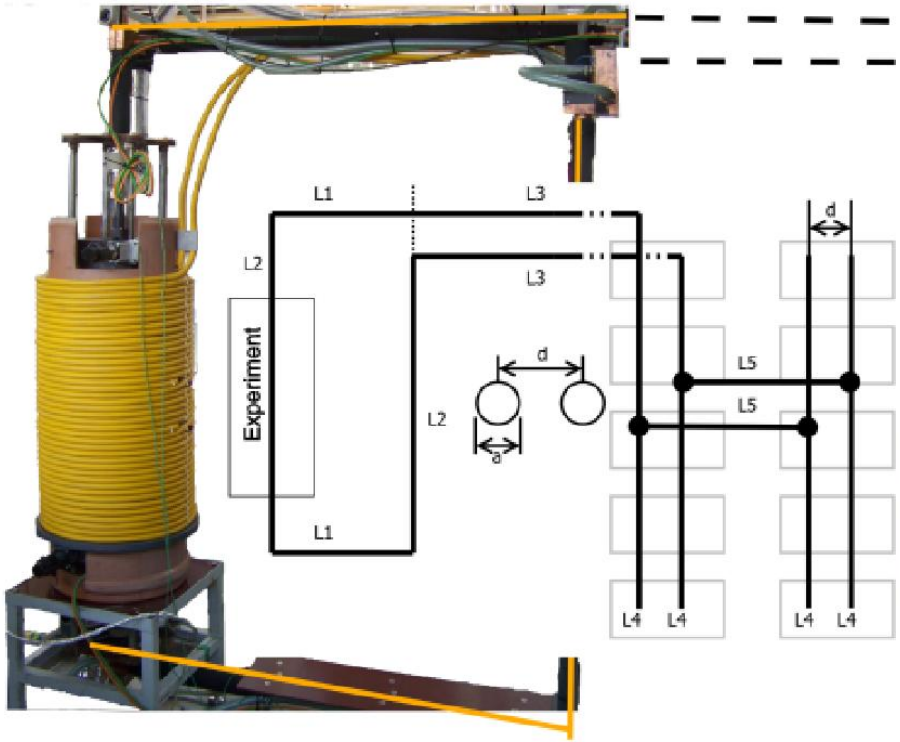
Stefani et al., Phys. Rev. Lett. 97 (2006), 184502; New J. Phys. 9 (2007), 295; Phys. Rev. E 80 (2009), 066303

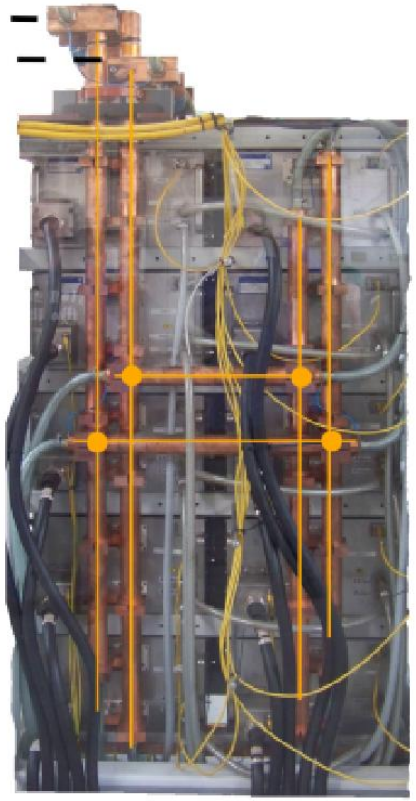
Azimuthal MRI (AMRI): m=1 mode under influence of dominant B_φ

Hollerbach, Teeluck, Rüdiger:
Phys. Rev. Lett. 104 (2010), 044502

New power supply for 20 kA

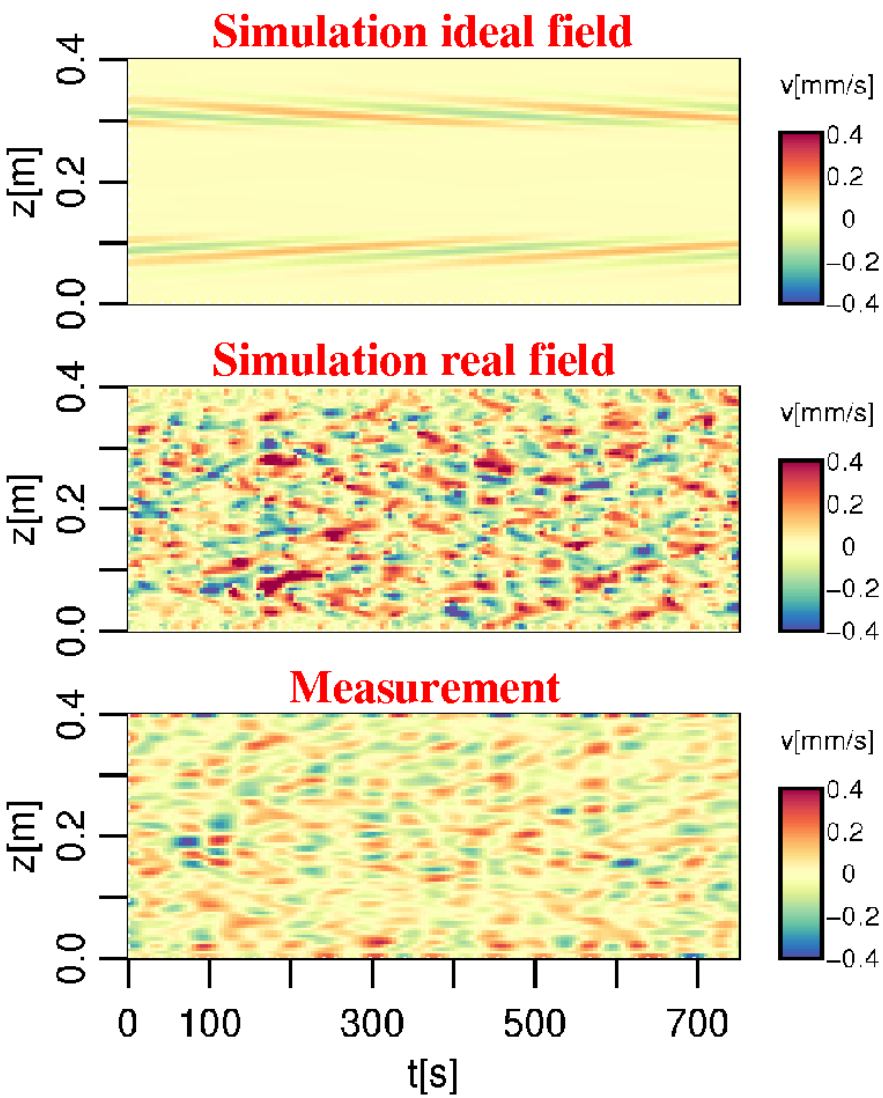


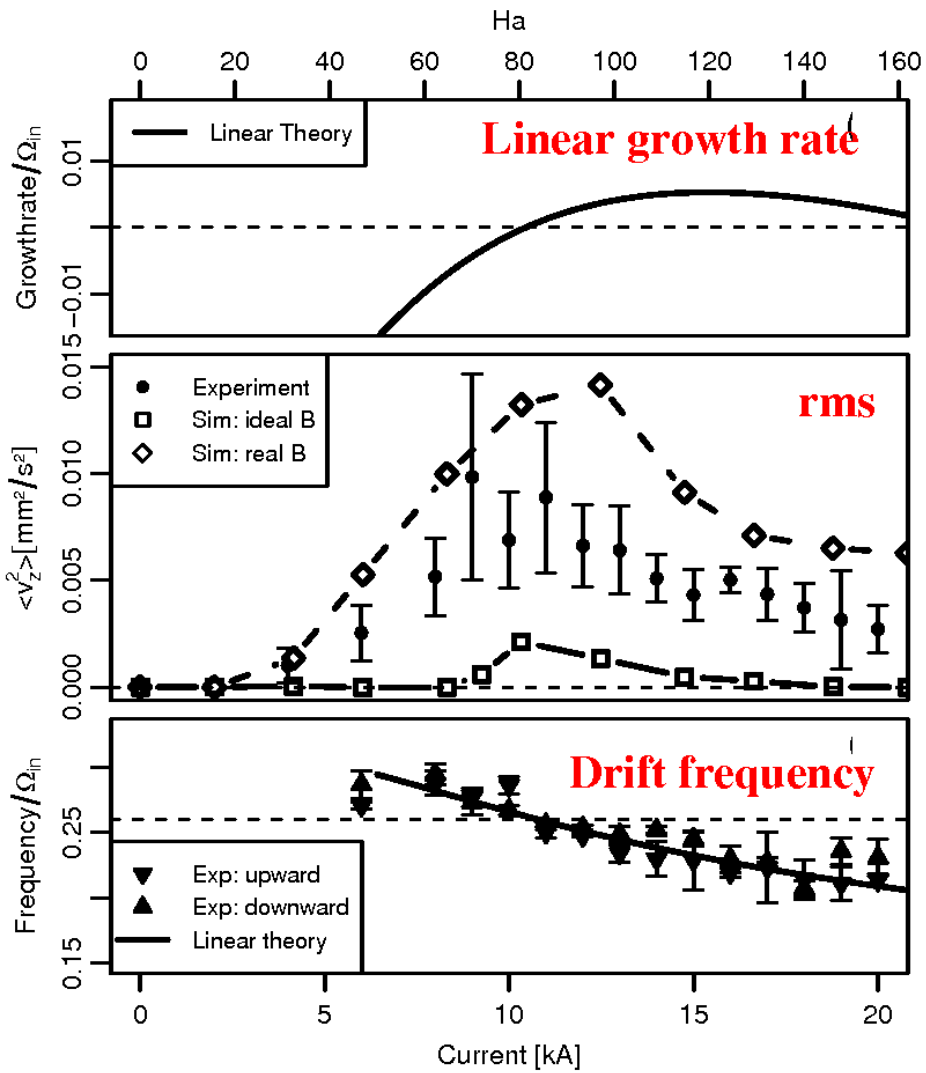




Very important: Numerical simulation of the real geometry, including the slight symmetry breaking of the applied magnetic field

Evidence for AMRI: m=1 mode under influence of (nearly) pure B_φ

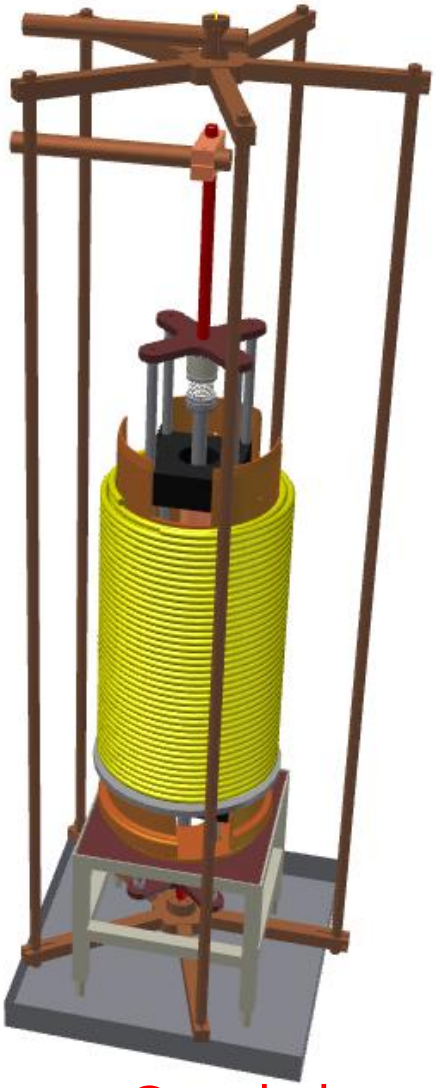




Seilmayer et al., Phys. Rev. Lett. 113 (2014), 024505

PROMISE 3: Improves the symmetry of the applied field

The symmetry of the field can be strongly enahnced by a cage-like structure of the return-current with n “wires”

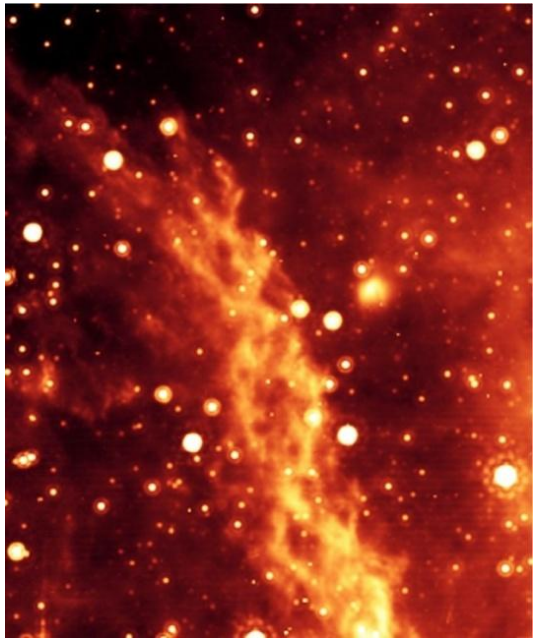


Our choice: n=5

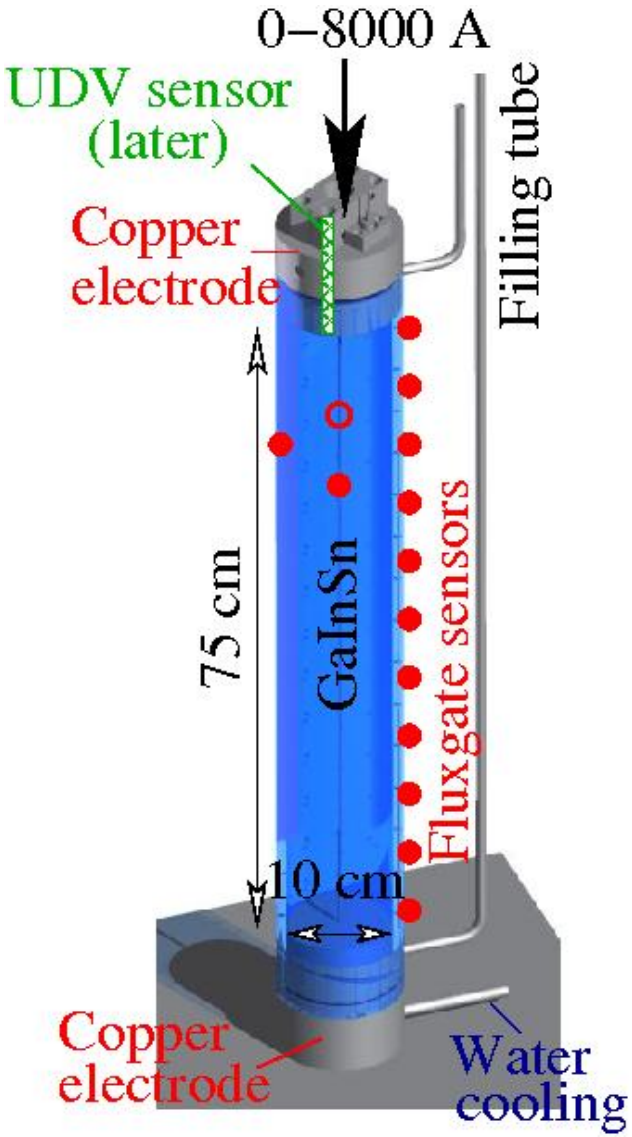
Kink-type Tayler instability (TI)

Astrophysical motivation:

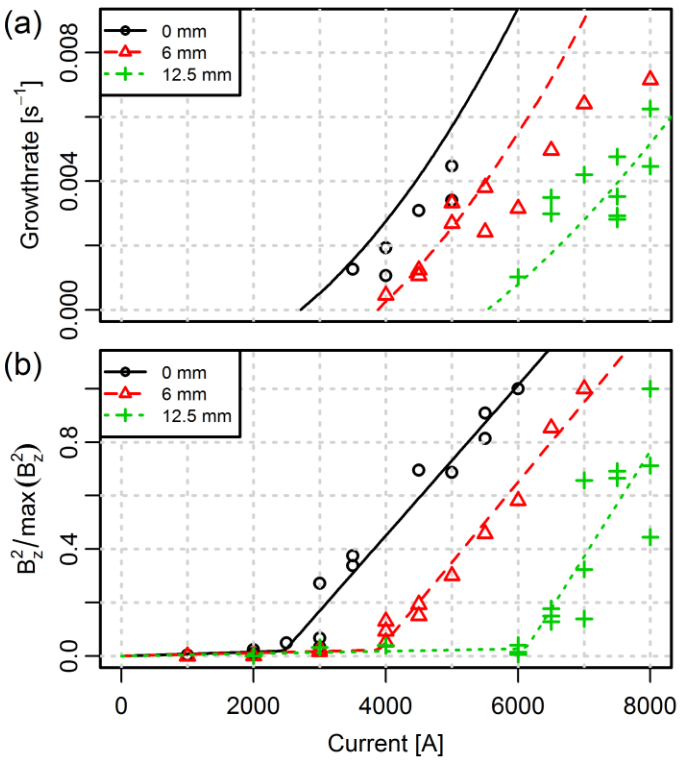
- Alternative mechanism of solar dynamo (Tayler-Spruit dynamo)
- Braking of neutron stars
- Structure formation in cosmic jets

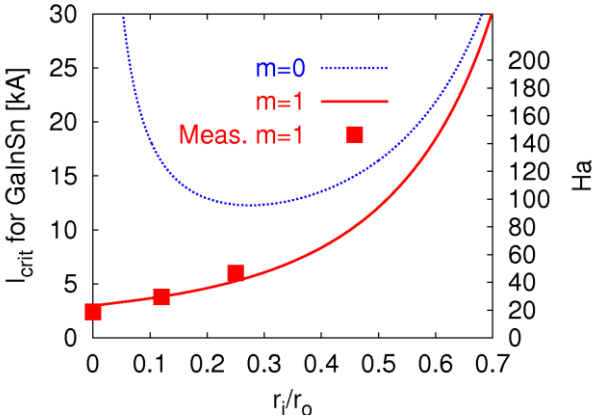


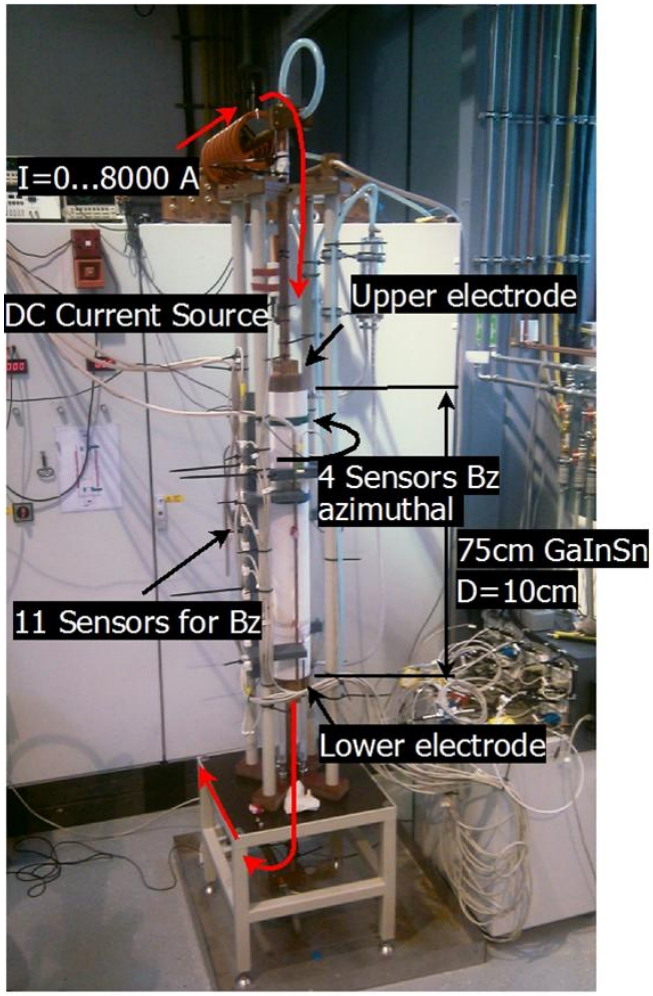
Seilmayer et al., Phys. Rev. Lett. 108 (2012), 244501



Results of TI experiment for different r_i, and comparison with numerics







Seilmayer et al., Phys. Rev. Lett. 108 (2012), 244501

Combined MRI/TI experiment planned in the framework of DRESDYN

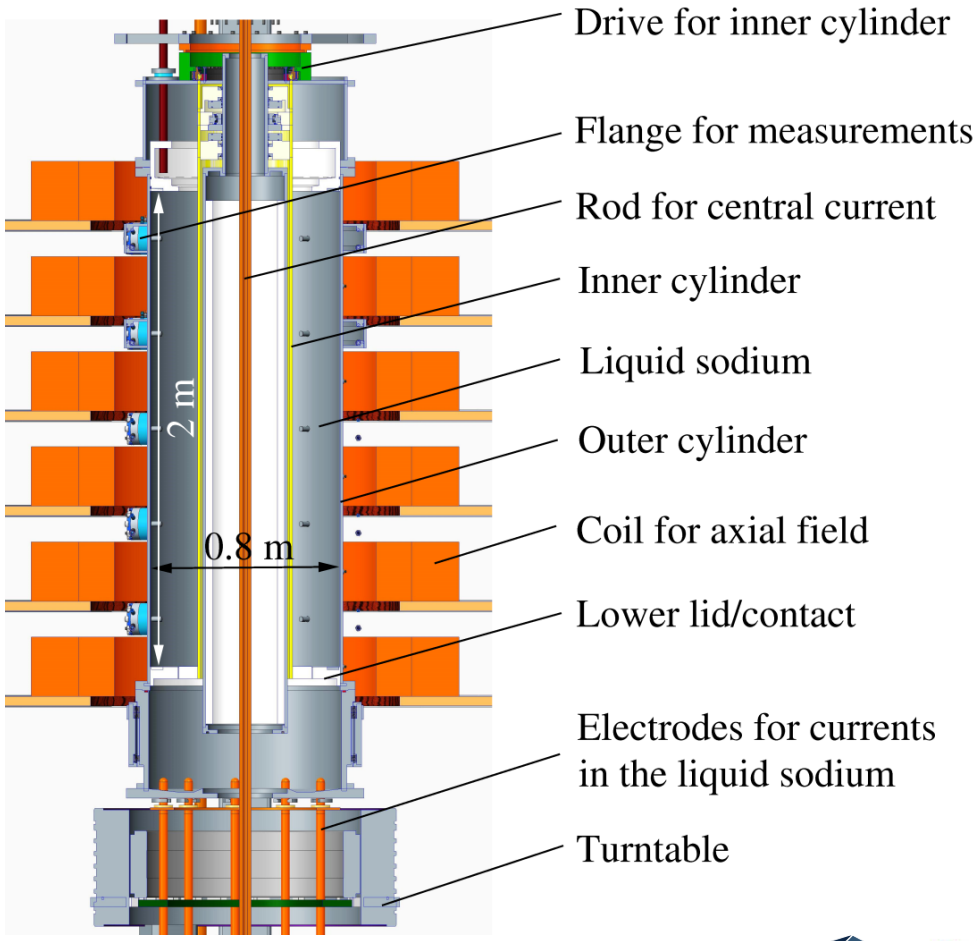
...will (hopefully) allow us to study helical MRI, azimuthal MRI, standard MRI, and their combinations with Tayler instability

- R_{in}=0.2 m
- R_{out}=0.4 m
- H=2 m

- f_{in}=20 Hz
- f_{out}=6 Hz

- B_z=120 mT
- (will need some 200 kW)

- Rm=40
- Lundquist=8



Diffusive instabilities

"Much as in the mid-nineteenth century, the point was missed ***that for fluid equations of the Navier-Stokes type the ideal limit with zero dissipation coefficients has essentially nothing to do with the case of small but finite dissipation coefficients"***

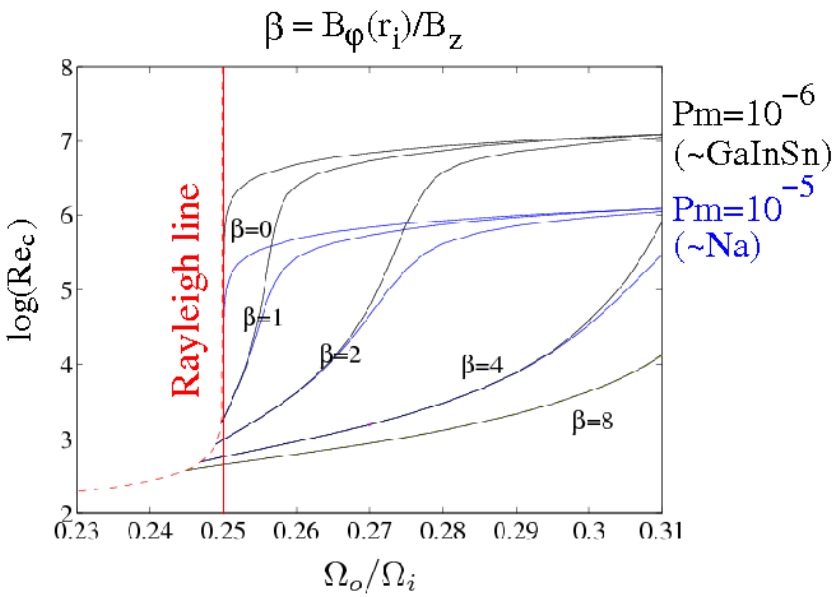
D. Montgomery: *Hartmann, Lundquist, and Reynolds: The role of dimensionless numbers in non-linear magnetofluid behavior.* Plasma Phys. Control. Fusion 35 (1993) B105-B113

THEORY

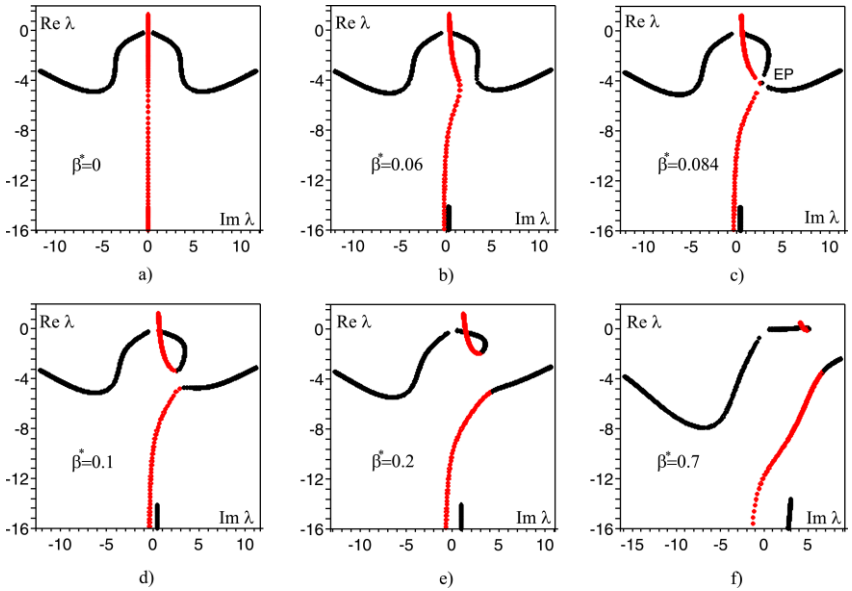
Relation between standard MRI (SMRI) and helical MRI (HMRI)

Apparent paradox: Transition between SMRI and HMRI is continuous and monotonic, although they represent different branches of the dispersion relation

Solution of the riddle: At small Pm the inertial mode and the slow magneto-Coriolis coalesce at an exceptional point and exchange the unstable branches



R. Hollerbach 2008



Kirillov and Stefani, Astrophys. J. 712 (2010), 52

Hollerbach and Rüdiger, PRL 95 (2005), 124501

THEORY

MRI and TI: Viscous, resistive MHD with azimuthal wavenumber m

Navier-Stokes equation:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\frac{\nabla P}{\rho} + \frac{\mathbf{B}\cdot\nabla\mathbf{B}}{\mu_0\rho} + \nu\nabla^2\mathbf{u}$$

Induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{B} = \mathbf{B}\cdot\nabla\mathbf{u} + \eta\nabla^2\mathbf{B}$$

Viscous, resistive, and two Alfvén frequencies for axial and azimuthal field

$$\omega_\nu = \nu|\mathbf{k}|^2, \quad \omega_\eta = \eta|\mathbf{k}|^2, \quad \omega_{A_z} = \frac{k_z B_z^0}{\sqrt{\rho\mu_0}}, \quad \omega_{A_\phi} = \frac{B_\phi^0}{r\sqrt{\rho\mu_0}}$$

Dimensionless: Magnetic Prandtl, Field ratio, Reynolds, Hartmann, n

$$\mathrm{Pm} = \frac{\omega_\nu}{\omega_\eta}, \quad \beta = \alpha\frac{\omega_{A_\phi}}{\omega_{A_z}}, \quad \mathrm{Re} = \alpha\frac{\Omega}{\omega_\nu}, \quad \mathrm{Ha} = \frac{\omega_{A_z}}{\sqrt{\omega_\nu\omega_\eta}}, \quad n = \frac{m}{\alpha}$$

with $\alpha = k_z|\mathbf{k}|^{-1}, |\mathbf{k}|^2 = k_r^2 + k_z^2$

THEORY

Theory: Short wavelength approximation (WKB):

Hydrodynamic and magnetic Rossby numbers:

$$\mathrm{Ro} = \frac{r}{2\Omega}\frac{\partial\Omega}{\partial r}, \quad \mathrm{Rb} = \frac{r}{2\omega_{A_\phi}}\frac{\partial\omega_{A_\phi}}{\partial r}$$

Secular equation: $p(\lambda) = \det(\mathbf{H} - \lambda\mathbf{E}) = 0$ with

$$\mathbf{H} = \begin{pmatrix} -in\,\mathrm{Re}-1 & 2\alpha\,\mathrm{Re} & \frac{i\mathrm{Ha}(1+n\beta)}{\sqrt{\mathrm{Pm}}} & -\frac{2\alpha\beta\mathrm{Ha}}{\sqrt{Pm}} \\ -\frac{2\,\mathrm{Re}(1+\mathrm{Ro})}{\alpha} & -in\,\mathrm{Re}-1 & \frac{2\beta\mathrm{Ha}(1+\mathrm{Rb})}{\alpha\sqrt{\mathrm{Pm}}} & \frac{i\mathrm{Ha}(1+n\beta)}{\sqrt{\mathrm{Pm}}} \\ \frac{i\mathrm{Ha}(1+n\beta)}{\sqrt{\mathrm{Pm}}} & 0 & -in\,\mathrm{Re}-\frac{1}{\mathrm{Pm}} & 0 \\ \frac{-2\beta\mathrm{HaRb}}{\alpha\sqrt{\mathrm{Pm}}} & \frac{i\mathrm{Ha}(1+n\beta)}{\sqrt{\mathrm{Pm}}} & \frac{2\,\mathrm{Re}\,\mathrm{Ro}}{\alpha} & -in\,\mathrm{Re}-\frac{1}{\mathrm{Pm}} \end{pmatrix}$$

Kirillov and Stefani, Phys. Rev. Lett. 111 (2013), 061103; JFM 760 (2014), 591

THEORY

Combining MRI and TI: A chance to destabilize Keplerian profiles?

WKB-Analysis of the complete viscous and resistive MRI/TI problem for arbitrary azimuthal modes

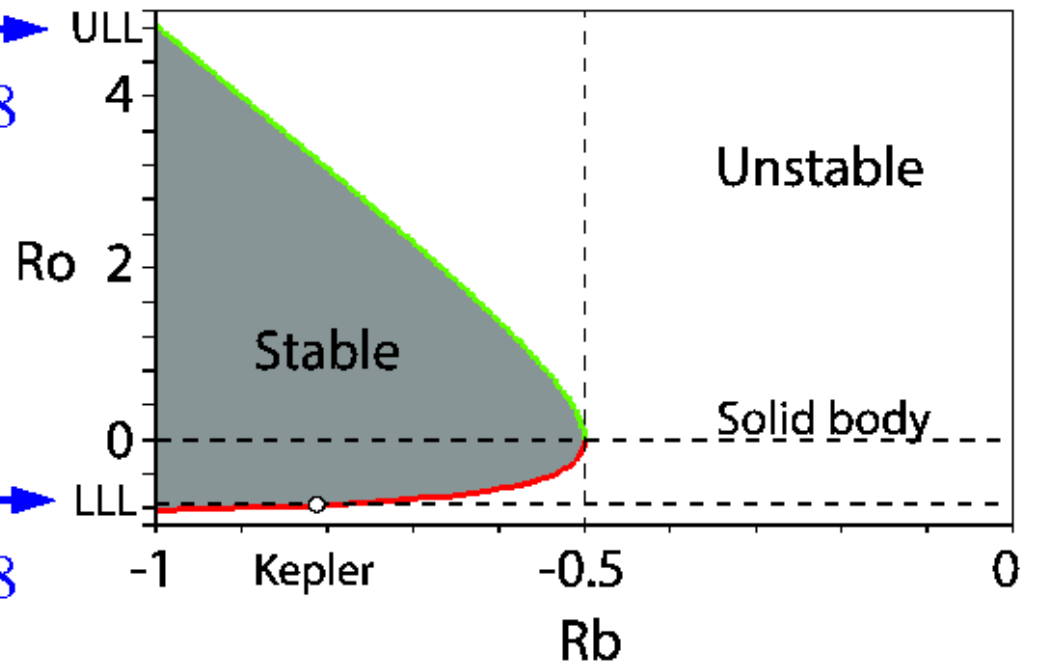
Main results:

$$Rb = -\frac{1}{8}\frac{(Ro+2)^2}{Ro+1}$$

Upper Liu limit: Ro=+4.828

Liu, Goodman, Herron, Ji, Phys. Rev. E 74 (2006), 056302

Lower Liu limit: Ro=–0.828



Steepness of rotation

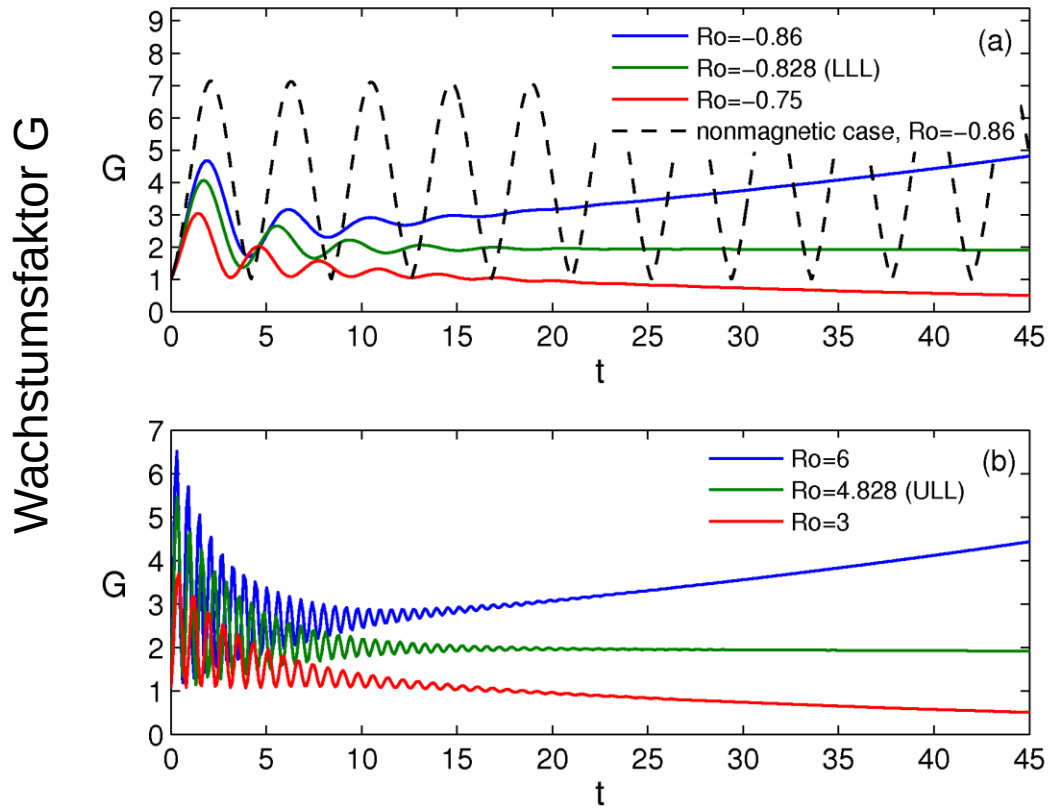
Steepness of azimuthal magnetic field

Kirillov and Stefani, Phys. Rev. Lett. 111 (2013), 061103; Fluid Dyn. Res. 46 (2014), 031403; JFM 760 (2014), 591

THEORY

Physical background of Liu limits

Transient non-modal growth, intermediate decay, and final exponetial growth of eigenfunctions of MRI



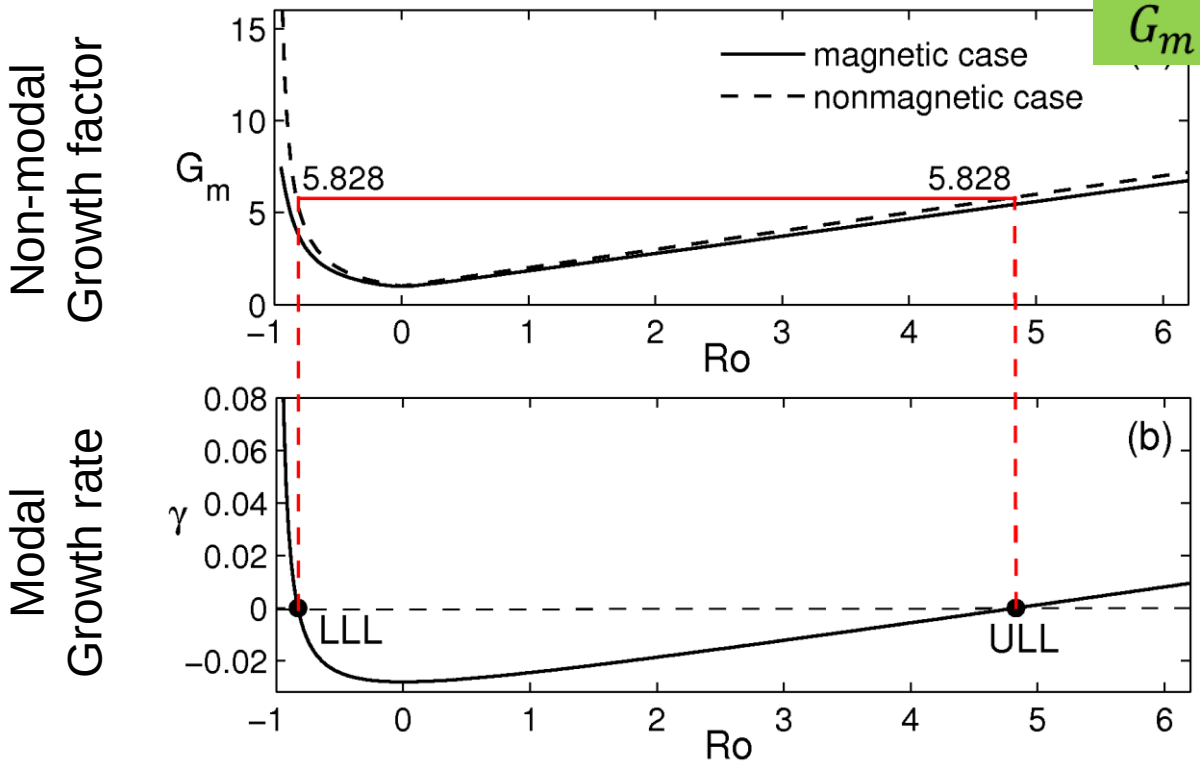
Negative shear, Ro<0

Positive shear, Ro>0

Mamatsashvili and Stefani, arXiv:1604.07205

Analytical connection between transient growth of the pure hydrodynamic case and modal growth of the HMRI

$$G_m = (1 + Ro)^{sgn(Ro)}$$



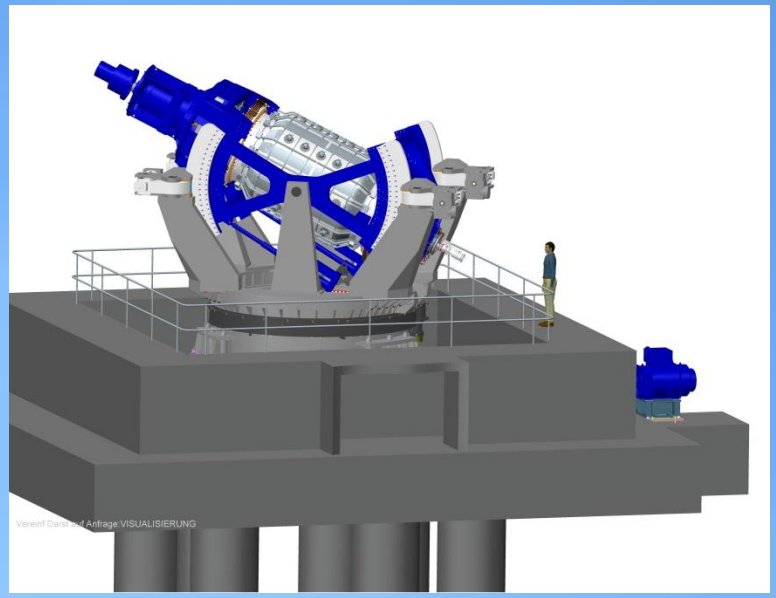
$$\gamma = \frac{Ha^2}{Re}\left[\frac{(Ro+2)^2}{8(1+Ro)} - 1\right] = \frac{Ha^2}{Re}\left[\frac{(G_m+1)^2}{8G_m} - 1\right]$$

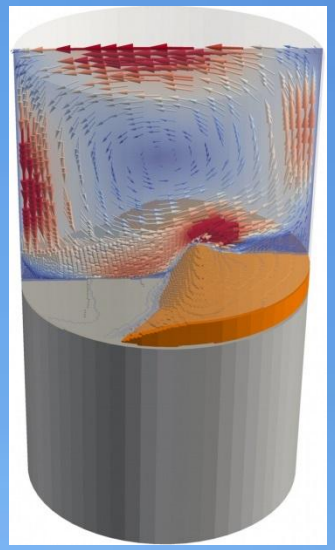
Mamatsashvili and Stefani, arXiv:1604.07205

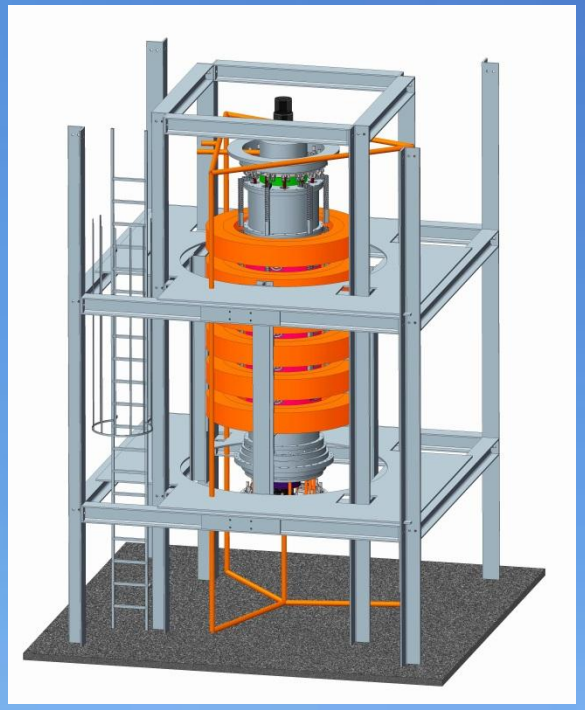
Summary on diffusive instabilities

- HMRI and AMRI are diffusive, i.e. dissipation induced instabilities
- Chandrasekhar's equipartition solution can be destabilized by AMRI
- Presence of weak internal currents can lead to destabilization of Keplerian flows
- Flows with steep positive shear can be destabilized by HMRI and AMRI
- Very close **connection of the non-normal** (transient) **growth factor of purely hydrodynamic flow** with the **normal growth rate of** (dissipation-induced**) HMRI**

THEORY







Thank you for your attention