

**KIS Thinkshop 2018**  
**Planetary-Stellar Connection: The Sun's Lesson**  
**Freiburg, May 7-9 2018**

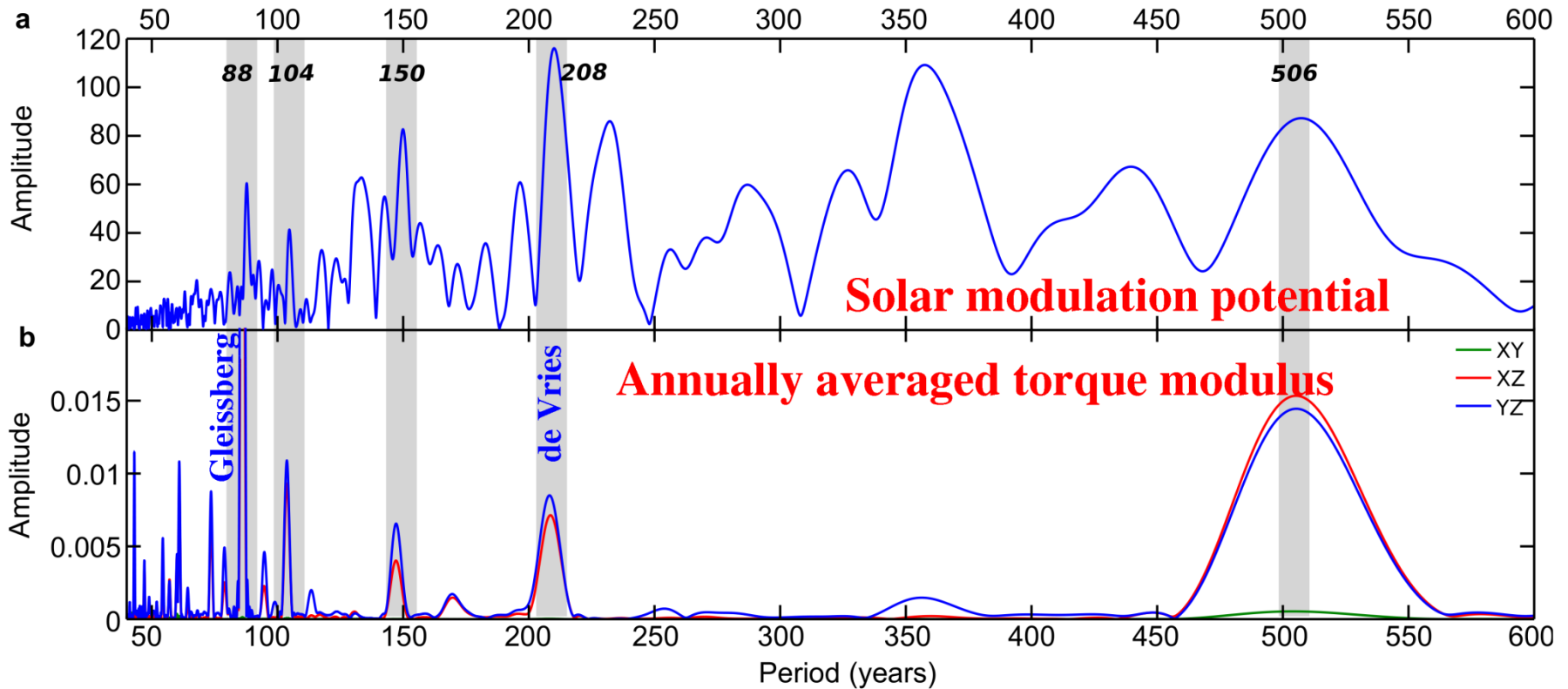
# **Tidally synchronized Babcock-Leighton and Tayler-Spruit type dynamos**

**Frank Stefani, André Giesecke, Norbert Weber, Tom Weier**

**hzdr**

 **HELMHOLTZ**  
ZENTRUM DRESDEN  
ROSSENDORF

# Planetary motion and long periods



Abreu et al., *Astron. & Astrophys.* 548 (2012), A88

## Outline

1. Planetary tides and the solar cycle
2. Solar dynamo models
3. Synchronized Babcock-Leighton type dynamos
4. Tayler-Spruit dynamo and the helicity question
5. Resonant excitation of helicity oscillations
6. A simple, and a slightly more complicated model of a synchronized dynamo
7. Summary and prospects (Lithium depletion)

# Planetary tides and the solar cycle

## Planetary tides and the solar cycle: an old idea of R. Wolf

the researches commenced in the seventh number. I shall accordingly show, by employing, on the one hand, my own observations in the year 1849 to 1858; and on the other, extracts from the observations of Schwabe in the years 1826 to 1848, that the formula

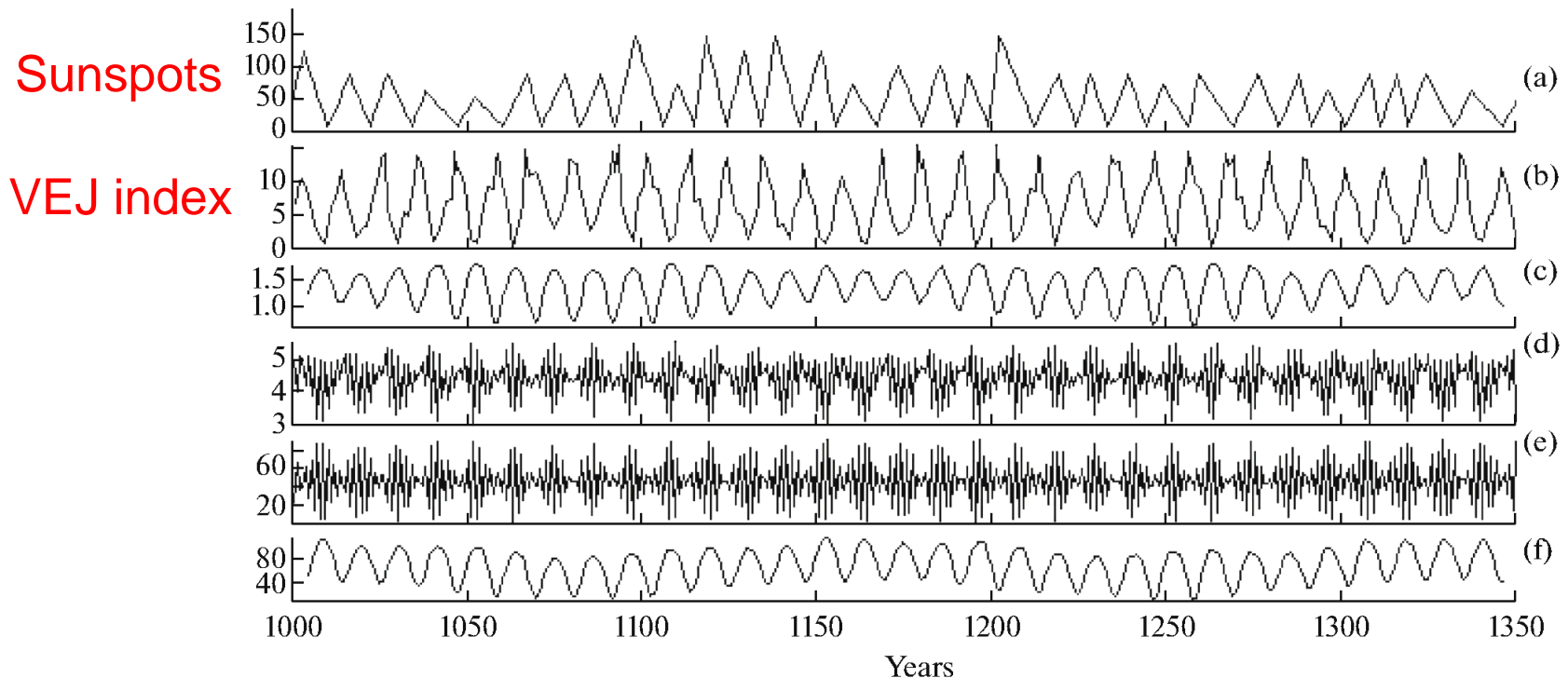
$$M = 50.31 + 3.73 \left\{ \begin{array}{l} 1.68 \sin 585^{\circ}.26 t + 1.00 \sin 360^{\circ} t + \\ 12.53 \sin 30^{\circ}.35 t + 1.12 \sin 12^{\circ}.22 t \end{array} \right\}$$

in which  $t$  denotes the number of years elapsed since a period of mean spot-frequency, gives a curve very similar to the sun-spot-curve; and therefore is very fit to be taken as the foundation of the more detailed research which I have now in hand. Now, as the coefficients of the four sines are the values which the fraction  $\frac{m}{r^2}$  assumes, when for  $m$  and  $r$  are successively substituted the masses and mean distances of Venus, Earth, Jupiter, and Saturn; and the angles of the four sines are the values of  $\frac{360^{\circ}}{t}$ , when for  $t$  are substituted the periodic times of

Wolf, R., Mon. Not. R. Astron. Soc. 19 (1859), 85

# Planetary tides and the solar cycle: Venus-Earth-Jupiter alignments

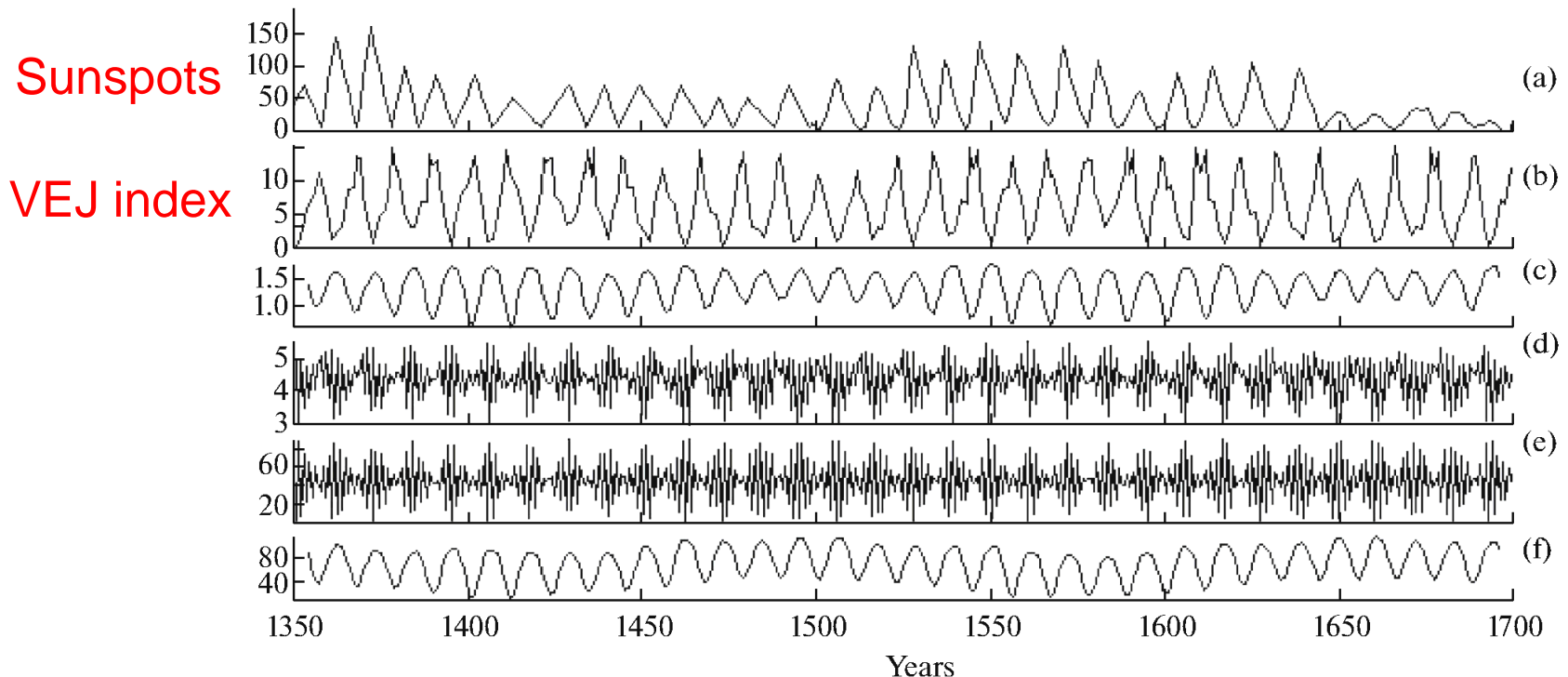
Amazing synchronization of solar cycle with the 11.07 years alignment cycle of the **Venus-Earth-Jupiter** system (despite tiny tidal forces!)



Bollinger, Proc. Okla. Acad. Sci. 33 (1952), 307; Takahashi, Solar. Phys. 3 (1968), 598; Wood, Nature 240 (1972), 91; Wilson, Pattern Recogn. Phys. 1 (2013), 147; Okhlopov, Mosc. U. Bull. Phys. B. 69 (2014), 257; **Okhlopov, Mosc. U. Bull. Phys. B. 71 (2016), 444**; Scafetta, Pattern Recogn. Phys. 2 (2014), 1

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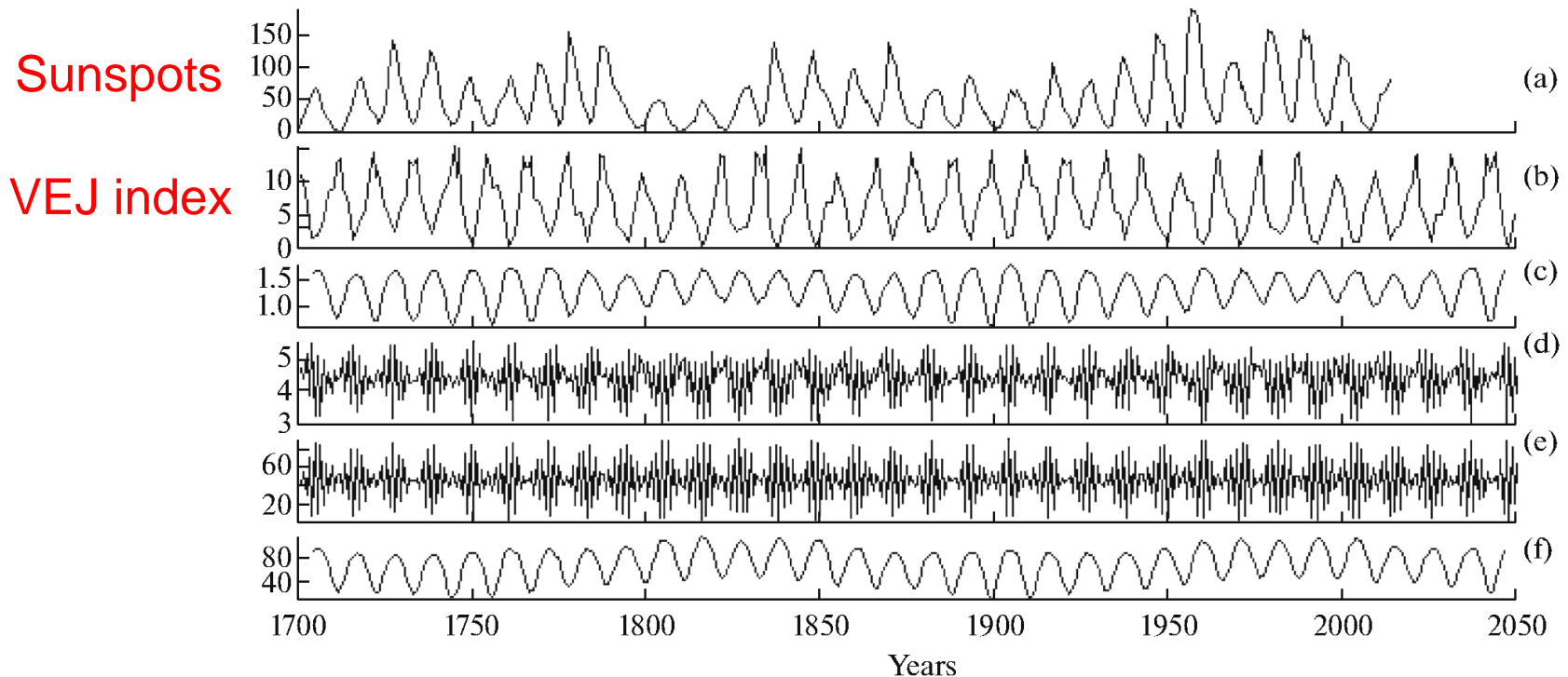
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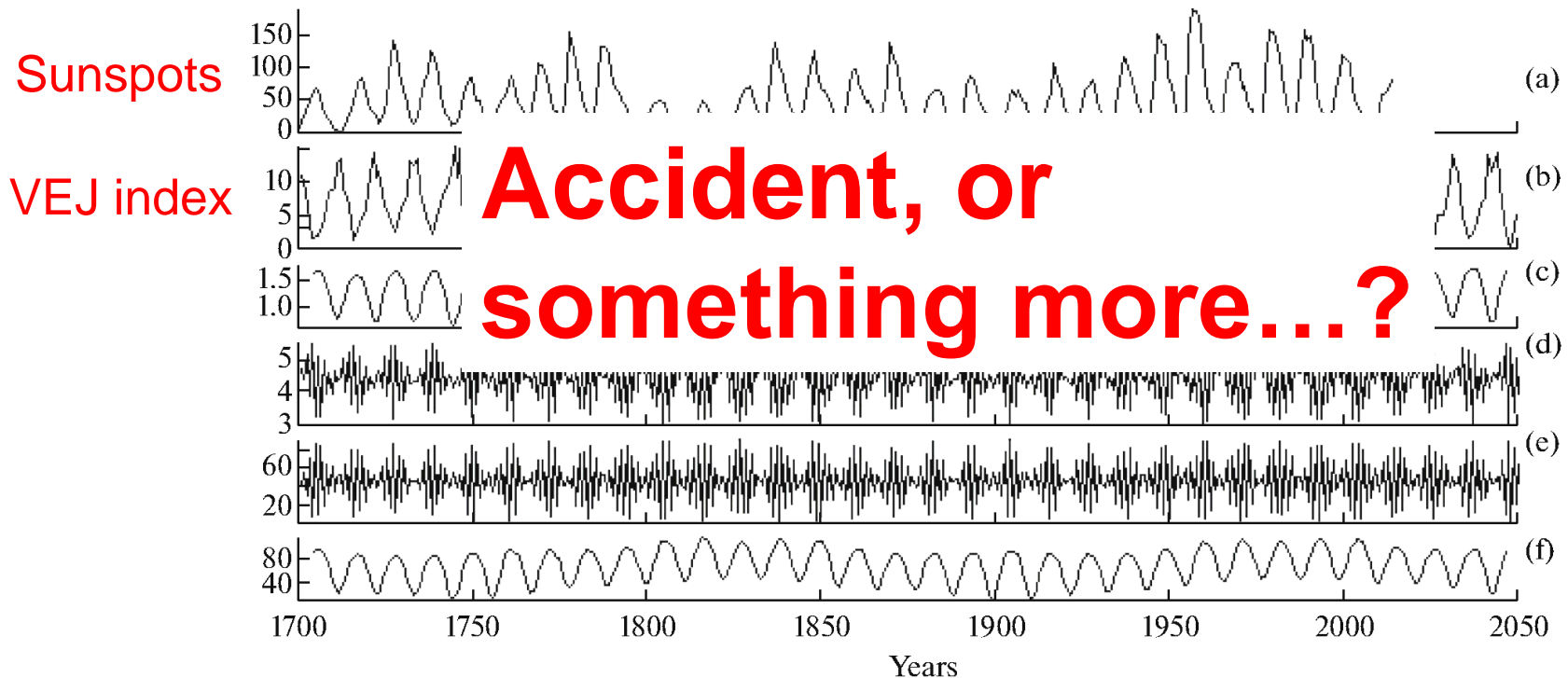


Bollinger, Proc. Okla. Acad. Sci. 33 (1952), 307; Takahashi, Solar. Phys. 3 (1968), 598; Wood, Nature 240 (1972), 91; Wilson, Pattern Recogn. Phys. 1 (2013), 147; Okhlopov, Mosc. U. Bull. Phys. B. 69 (2014), 257; **Okhlopov, Mosc. U. Bull. Phys. B. 71 (2016), 444**; Scafetta, Pattern Recogn. Phys. 2 (2014), 1



# Planetary tides and the solar cycle: Venus-Earth-Jupiter alignments

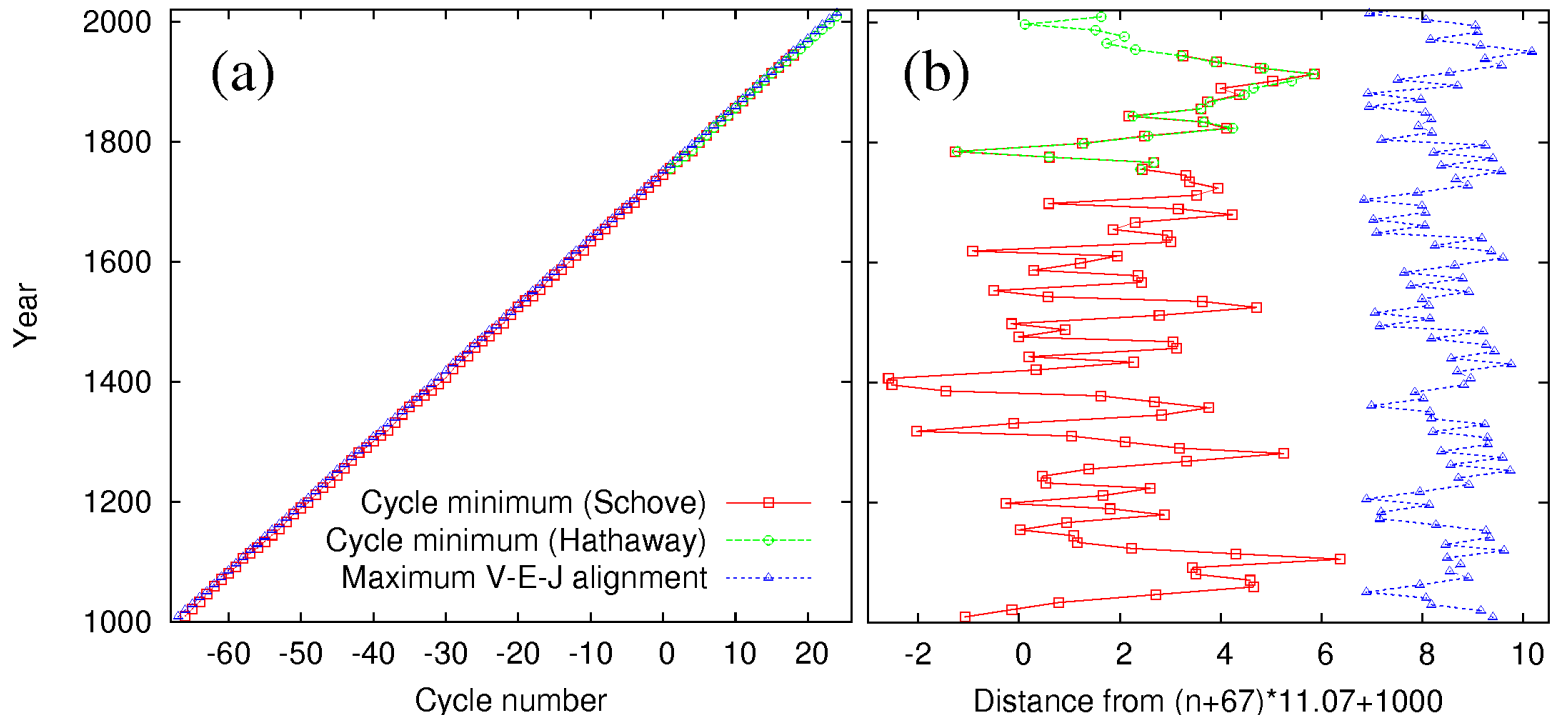
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# Planetary tides and the solar dynamo: The basic 22 years cycle

Amazing synchronization of solar cycle with the 11.07 years conjunction cycle of the **Venus-Earth-Jupiter** system (despite tiny tidal forces!)



Schöve, D.J.: J. Geophys. Res. 60 (1955), 127; Hathaway, D.H., Liv. Rev. Sol. Phys. 7 (2010), 1; Okhlopov, Mosc. U. Bull. Phys. B. 71 (2016), 444

Stefani et al., arXiv:1803.08692

# Planetary motion and the solar cycle: Dicke's argument

Dicke (1978): „**No support** is found for the conventional view of the sunspot cycle, that there exists a large **random walk** in the phase of the cycle. Instead, both sunspots and the [D/H] solar/terrestrial weather indicator **seem to be paced by an accurate clock inside the sun.**“

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## Is there a chronometer hidden deep in the Sun?

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### R. H. Dicke

Joseph Henry Laboratories, Physics Department, Princeton University, Princeton, New Jersey 08540

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*No support is found for the conventional view of the sunspot cycle, that there exists a large random walk in the phase of the cycle. Instead, both sunspots and the [D/H] solar/terrestrial weather indicator seem to be paced by an accurate clock inside the Sun.*

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It has long been believed that “the sunspot disturbances, like the eruptions of a geyser, are inherently only roughly periodic”<sup>1</sup>. Observations show a large variation in the ~11 yr

cycle as follows: “It was previously believed that the sunspot cycle resulted from the superposition of different periodic cycles. . . . Since then it has become clear that the rise and fall in the number of spots is due to a number of practically independent individual processes. Thus the idea of a true periodic phenomenon was dropped in favour of the so-called ‘eruption hypothesis’. On this hypothesis, each cycle represents an independent eruption of the Sun which takes about 11 yr to die down”. This conception of an irregular sunspot cycle, implying a random walk in the phase of the cycle, seems to agree with the Babcock theory and with subsequent modifications of the

Dicke, R.H., Nature 276 (1978), 676

# Planetary motion and the solar cycle: Dicke's argument reconsidered

Checking Dicke's argument with Schove's and Hathaway's data:  
Discriminating between a random walk (RW) process and a clocked process (CP) for the years  $y_n$  of sunspot maxima (Dicke) or minima (here)

Residuals (phase errors):  $\delta y_n = y_n - y_0 - p(n-1)$ ,  $p$  ... average cycle length

A significant discriminating measure is the **RATIO** of the mean square of  $\delta y_n$  to the mean square of  $(\delta y_n - \delta y_{n-1})$

	<b>RATIO</b>	<b>Dicke (N=25)</b>	<b>Here (N=90)</b>
Random walk	$(N+1)(N^2-1)/3(5N^2+6N-3)$	1.72	6.12
Clocked process	$(N^2-1)/2(N^2+2N+3)$	0.46	0.49
Observation		<b>0.87</b>	<b>1.19</b>

Dicke, R.H., Nature 276 (1978), 676

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Observation		<b>0.87</b>	<b>1.19</b>

**Can this be sharpened (with better Be or tree ring data) ?**

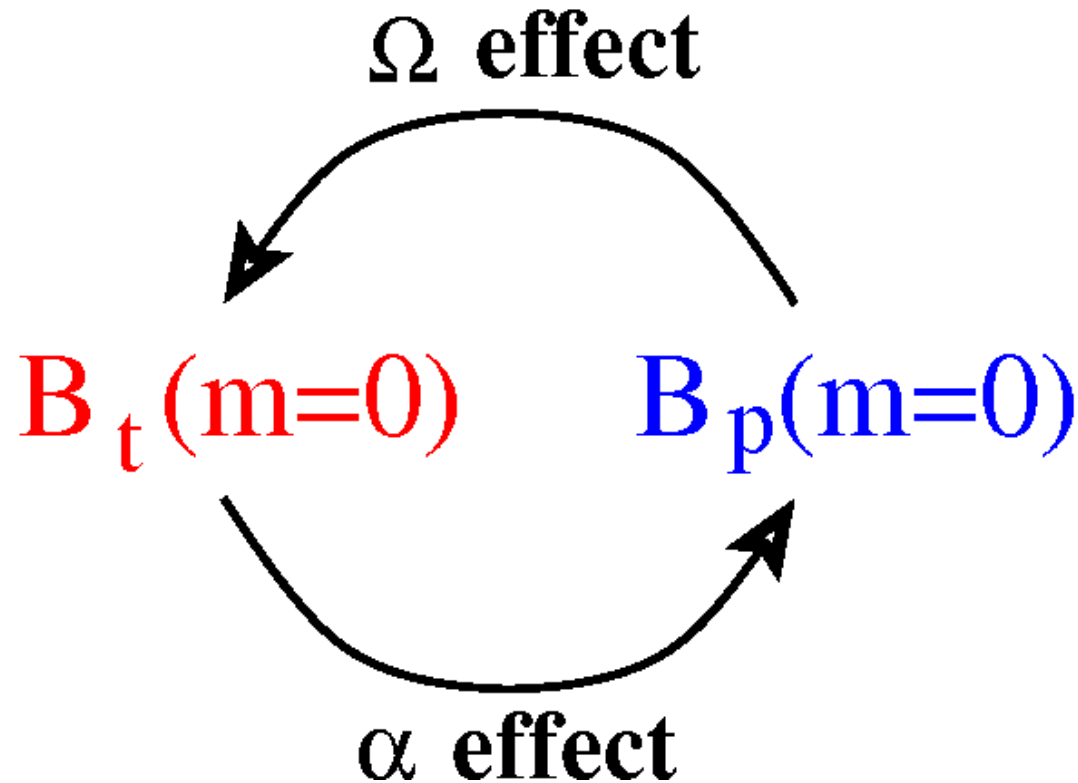
Dicke, R.H., Nature 276 (1978), 676

# Solar dynamo models (the main road)

# Solar dynamo models: Basics

Any solar dynamo needs:

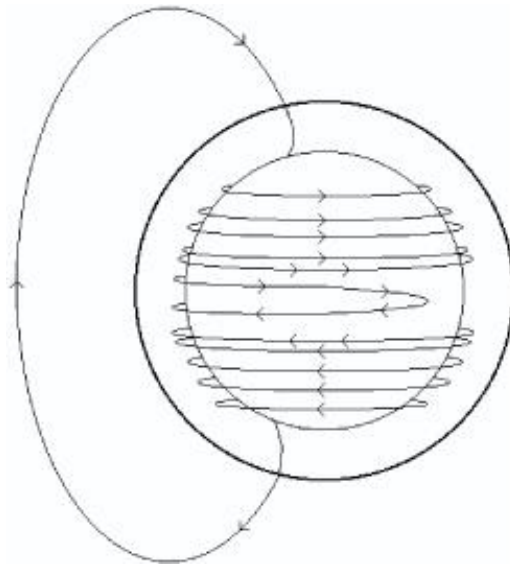
- some  $\Omega$  effect to generate toroidal field from poloidal field
- some  $\alpha$  effect to regenerate poloidal field from toroidal field



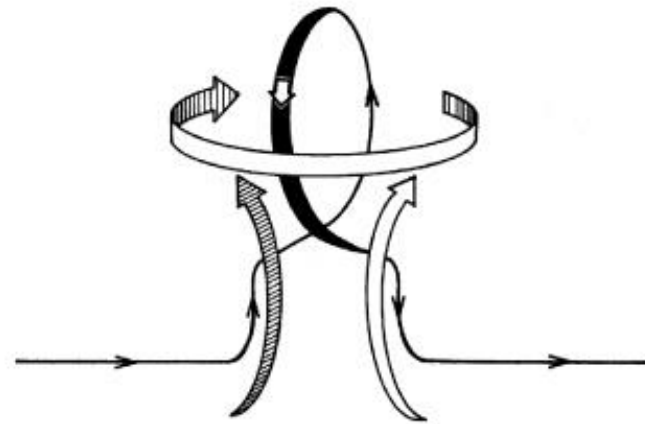
# Solar dynamo models: Basics

Any solar dynamo needs:

- some  $\Omega$  effect to regenerate toroidal field from poloidal field
- some  $\alpha$  effect to regenerate poloidal field from toroidal field



$\Omega$  effect



$\alpha$  effect

Solanki et al., Rep. Progr. Phys. 69 (2006), 563

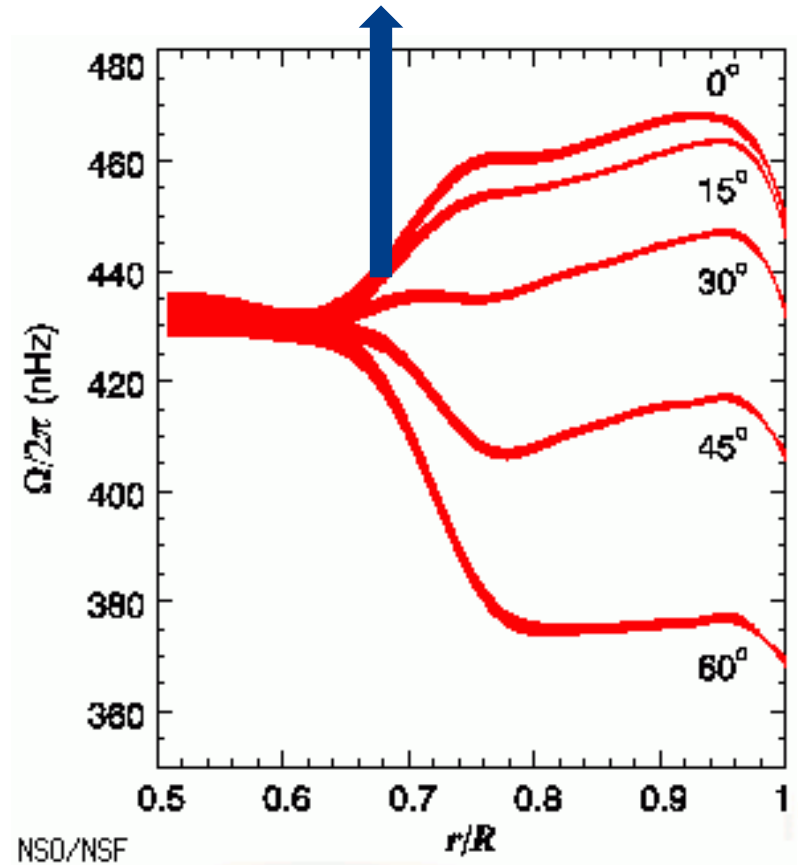
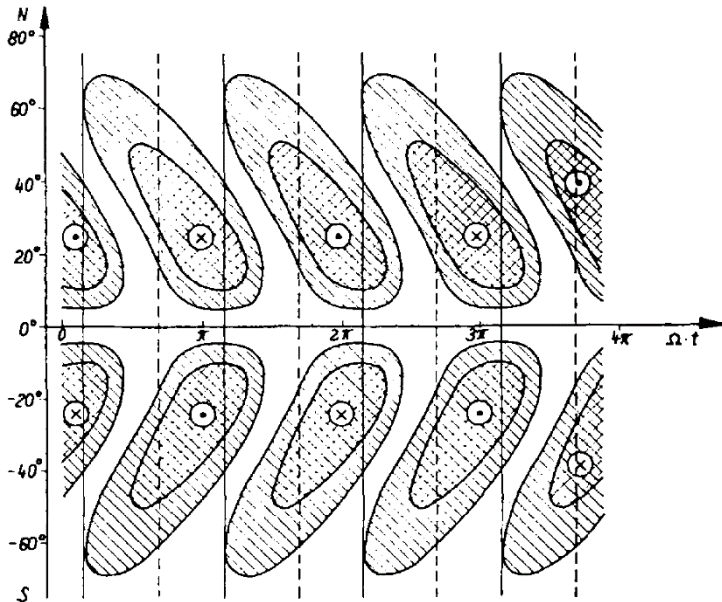


# Solar dynamo models: Butterfly diagram of sunspots

Parker-Yoshimura rule: **product of  $\alpha$  and  $d\Omega/dr$  must be negative** to provide the correct butterfly diagram of sunspots

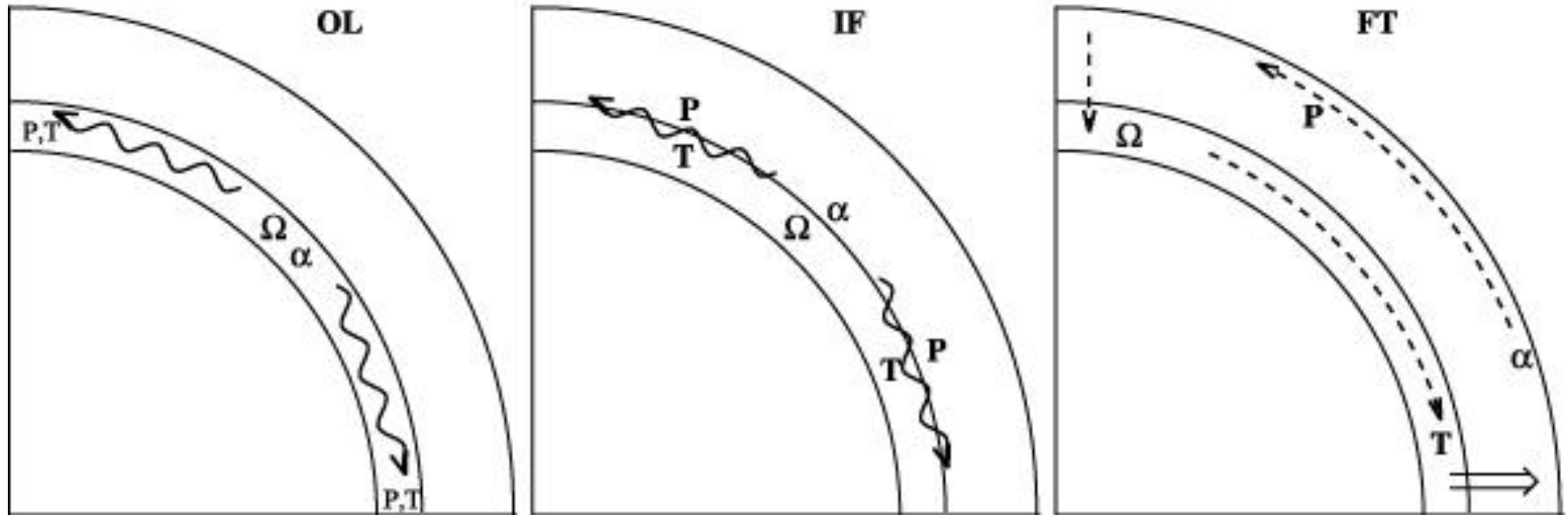
First numerical butterfly diagram for **positive  $\alpha$  and negative  $d\Omega/dr$**

But: helioseismology showed a **positive  $d\Omega/dr$**  in a equator-near strip



Steenbeck and Krause, Astron. Nachr. 291 (1969), 49

# Solar dynamo models: possible solutions to Parker-Yoshimura puzzle



Overshoot layer  
beneath the  
convection zone

Interface

Flux transport  
(Babcock-Leighton)

Solanki et al., Rep. Progr. Phys. 69 (2006), 563

# Synchronized Babcock-Leighton type dynamos?

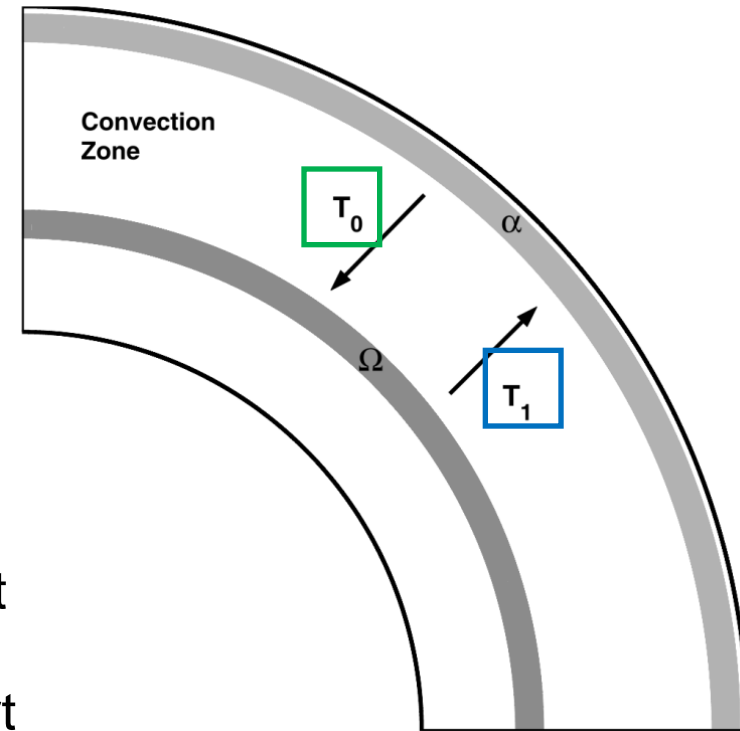
(Motivated by Antonio's idea!)

# Basis: A time-delay model emulating a Babcock-Leighton dynamo

Wilmot-Smith et al., *Astrophys. J.* 652 (2006), 696

$$\dot{A}(t) = \alpha(t - T_1)B(t - T_1) - \tau^{-1}A(t)$$

$$\dot{B}(t) = \Omega A(t - T_0) - \tau^{-1}B(t)$$



$T_1$  Time delay for toroidal field transport

$T_0$  Time delay for poloidal field transport

$$\alpha(t) = \frac{\alpha_0}{4} [1 + \operatorname{erf}(B^2(t) - B^2_{Min})][1 - \operatorname{erf}(B^2(t) - B^2_{Max})]$$

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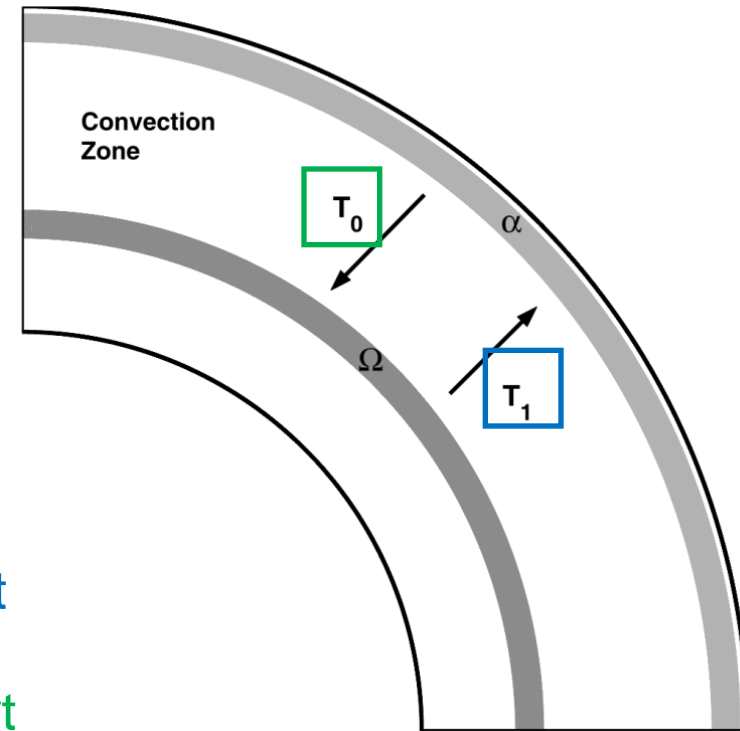
$$\dot{B}(t) = \Omega(t)A(t - T_0) - \tau^{-1}B(t)$$

$T_1$  Time delay for toroidal field transport

$T_0$  Time delay for poloidal field transport

$$\alpha(t) = \frac{\alpha_0}{4} [1 + \operatorname{erf}(B^2(t) - B_{Min}^2(t))] [1 - \operatorname{erf}(B^2(t) - B_{Max}^2)]$$

**NEW:** Considering  $\Omega(t)$  or  $B_{Min}(t)$  as time-dependent

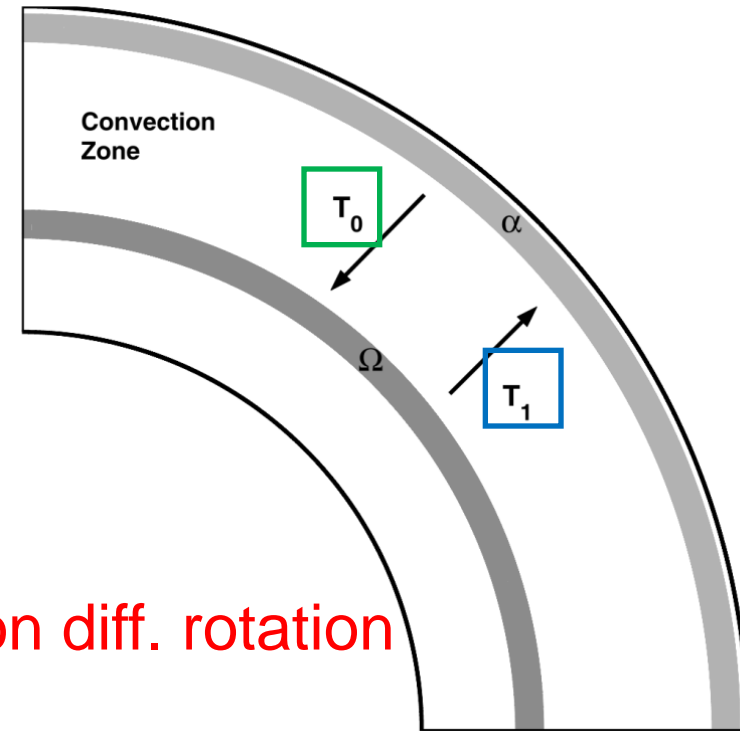


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$$\dot{B}(t) = \Omega(t)A(t - T_0) - \tau^{-1}B(t)$$



$\Omega(t)$  emulates putative tidal influence on diff. rotation

$$\alpha(t) = \frac{\alpha_0}{4} [1 + \operatorname{erf}(B^2(t) - B_{Min}^2(t))] [1 - \operatorname{erf}(B^2(t) - B_{Max}^2(t))]$$

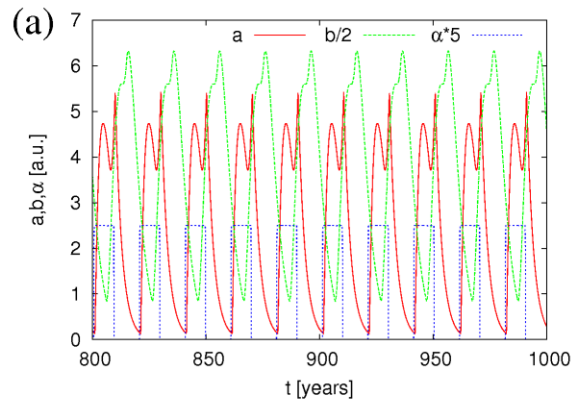
$B_{Min}(t)$  emulates a varying flux storing capacity of the tachocline due to **high sensitivity of sub-adiabaticity** on tidal forcing (idea of Antonio Ferriz Mas)

# Numerical parameter studies

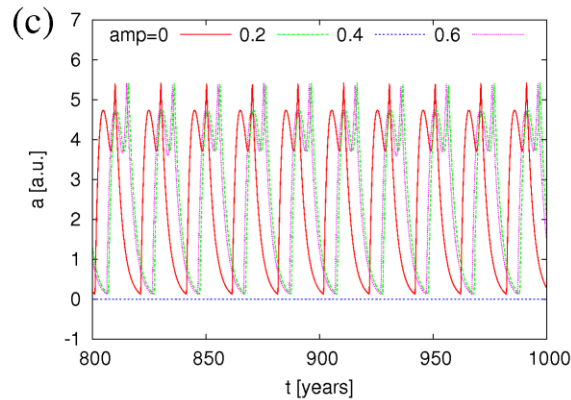
	Flux-transport dominated $\tau > T_0 + T_1$	Diffusion dominated $\tau < T_0 + T_1$	Intermediate regime $T_1 < \tau < T_0$
$\alpha\Omega > 0$	(sawtooth-shaped) pulsations	(sawtooth-shaped) pulsations	(sawtooth-shaped) pulsations
$B_{\text{Min}}(t) = B_0(1+a \sin(2\pi t/11.07))$	No synchronization	No synchronization	No synchronization
$\Omega(t) = B_0(1+a \sin(2\pi t/11.07))$	No synchronization	No synchronization	<b><u>Synchronization (only for large a)</u></b>
$\alpha\Omega < 0$	(sawtooth-shaped) oscillations	(complicated) oscillations	(sawtooth-shaped) oscillations
$B_{\text{Min}}(t) = B_0(1+a \sin(2\pi t/11.07))$	No synchronization	No synchronization	No synchronization
$\Omega(t) = B_0(1+a \sin(2\pi t/11.07))$	No synchronization	No synchronization	<b><u>Synchronization (only for large a)</u></b>

# Example: Intermediate regime $T_1 < t < T_0$ , $\alpha\Omega > 0$ , Pulsations

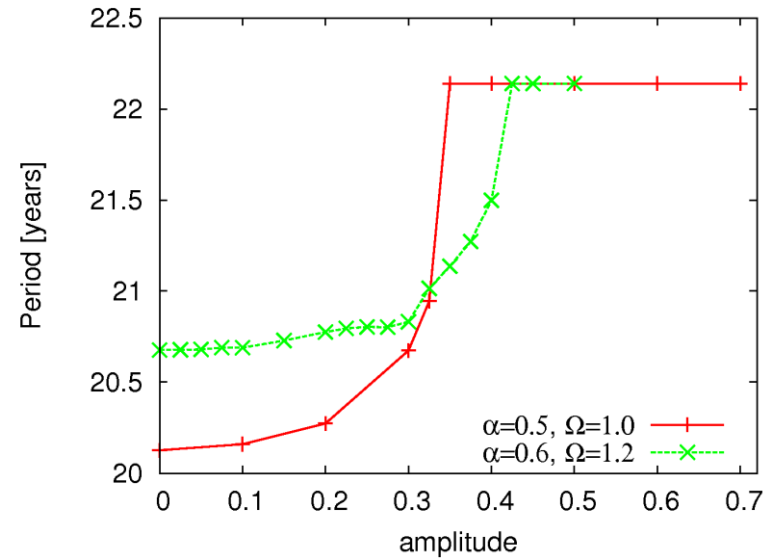
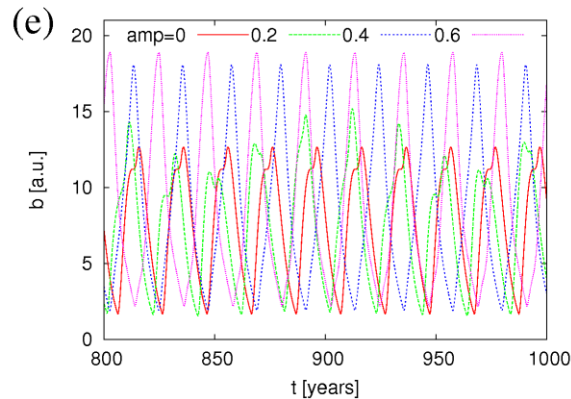
Unperturbed  
 $B_{\min}$  and  $\Omega$



Perturbed  
 $B_{\min}(t)$  with  
a  $\sin(2\pi t/11.07)$



Perturbed  
 $\Omega(t)$  with  
a  $\sin(2\pi t/11.07)$

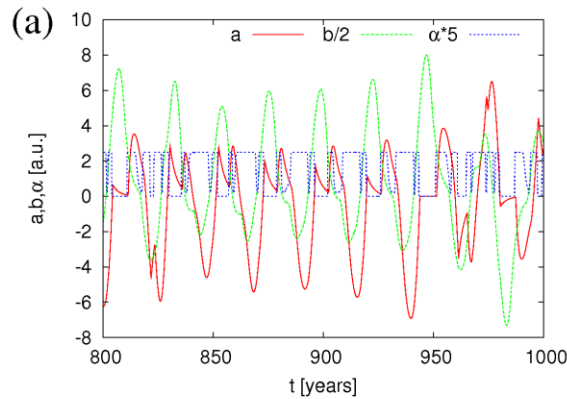


Synchronization, but only for unrealistically large a

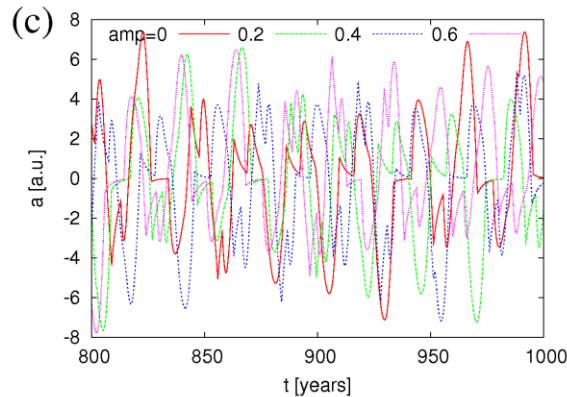


# Example: Intermediate regime $T_1 < t < T_0$ , $\alpha\Omega < 0$ , Oscillations

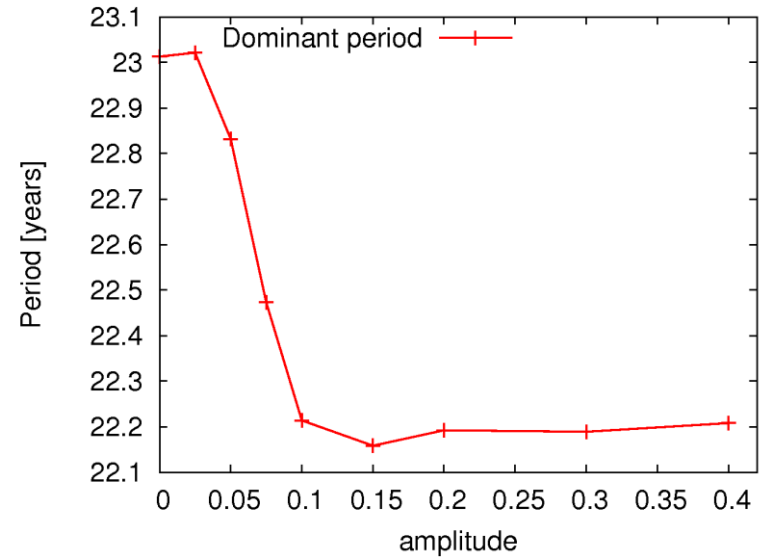
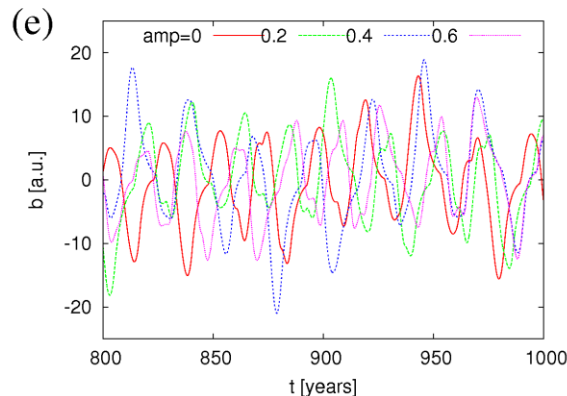
Unperturbed  
 $B_{\text{Min}}$  and  $\Omega$



Perturbed  
 $B_{\text{min}}(t)$  with  
 $a \sin(2\pi t/11.07)$



Perturbed  
 $\Omega(t)$  with  
 $a \sin(2\pi t/11.07)$



A sort of synchronization, but only for large a

# Taylor-Spruit dynamo and the helicity question

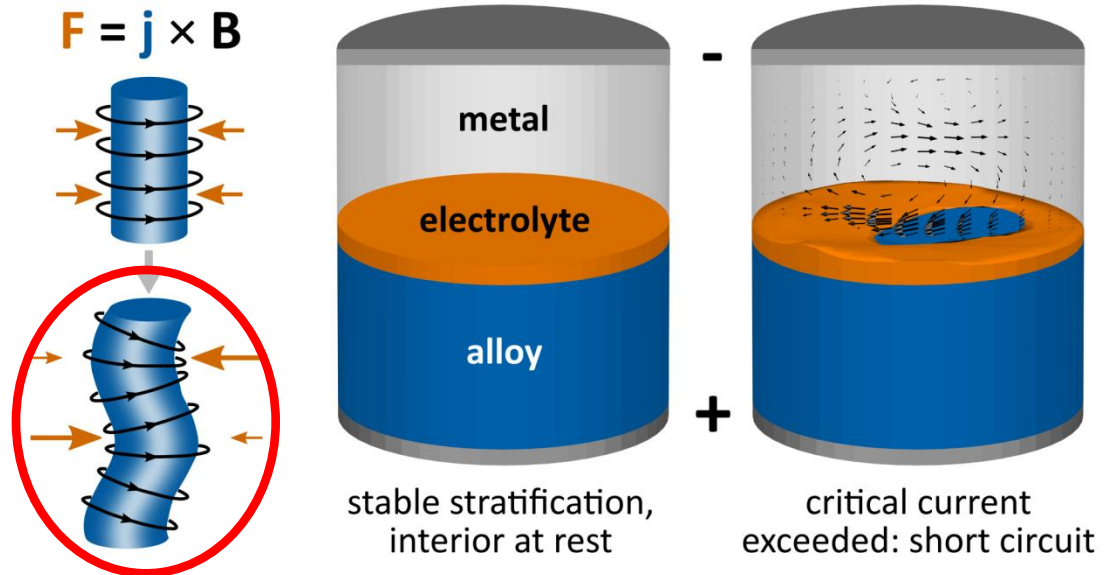
# Kink-type Tayler instability (TI) at low Pm

Astrophysical motivation:

- Alternative mechanism of solar dynamo (**Taylor-Spruit dynamo**)
- Structure formation in cosmic jets

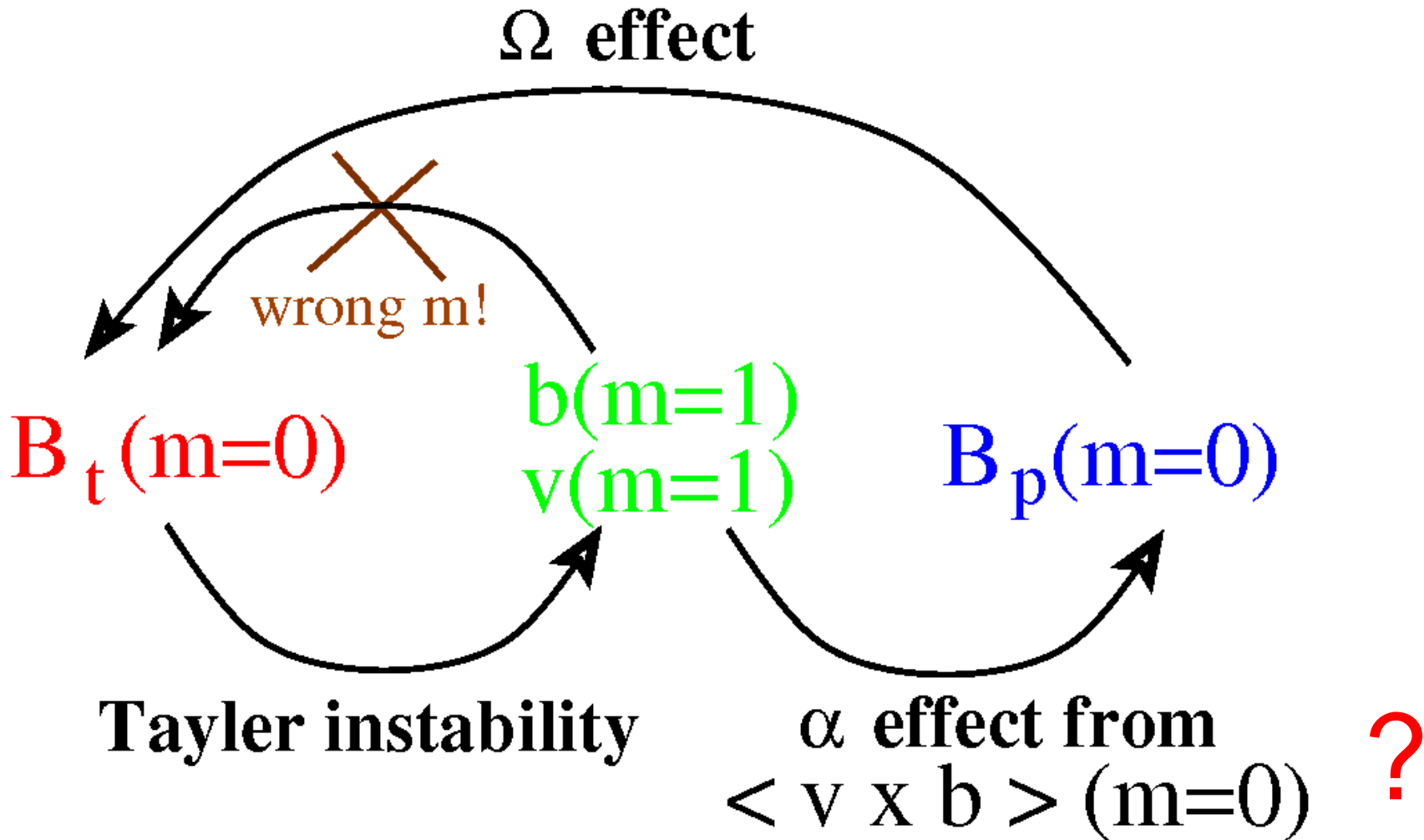
Technical motivation:

- Understanding and controlling the complex MHD of **liquid metal batteries**



Stefani et al., Energy Conv. Managem. 52 (2011), 2982;  
Weber et al., J. Power Sources 265 (2014), 166

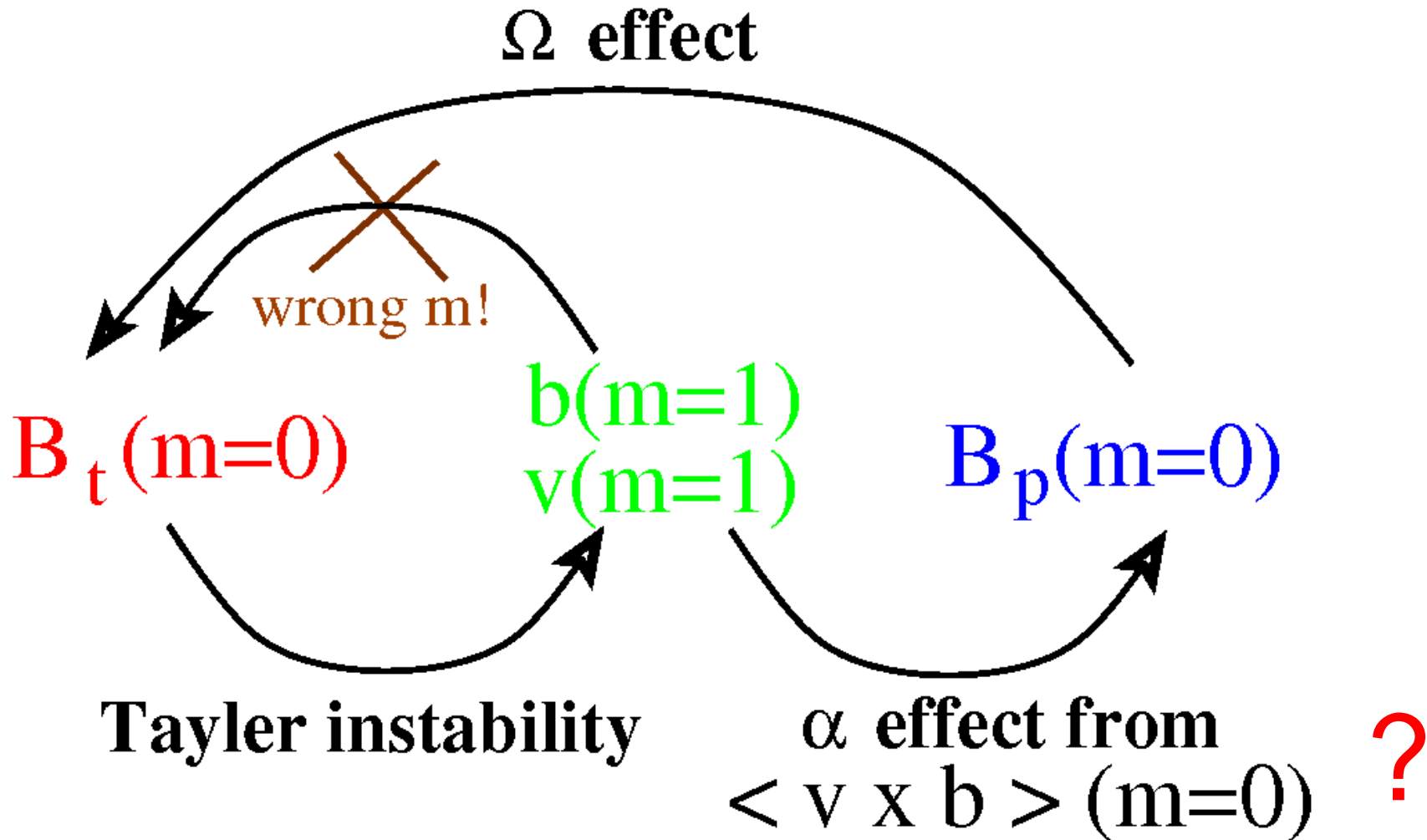
# Taylor-Spruit dynamo: the main problem



Spruit, Astron. Astrophys. 381 (2002) 923;

Zahn et al., Astron. Astrophys. 474 (2007) 147

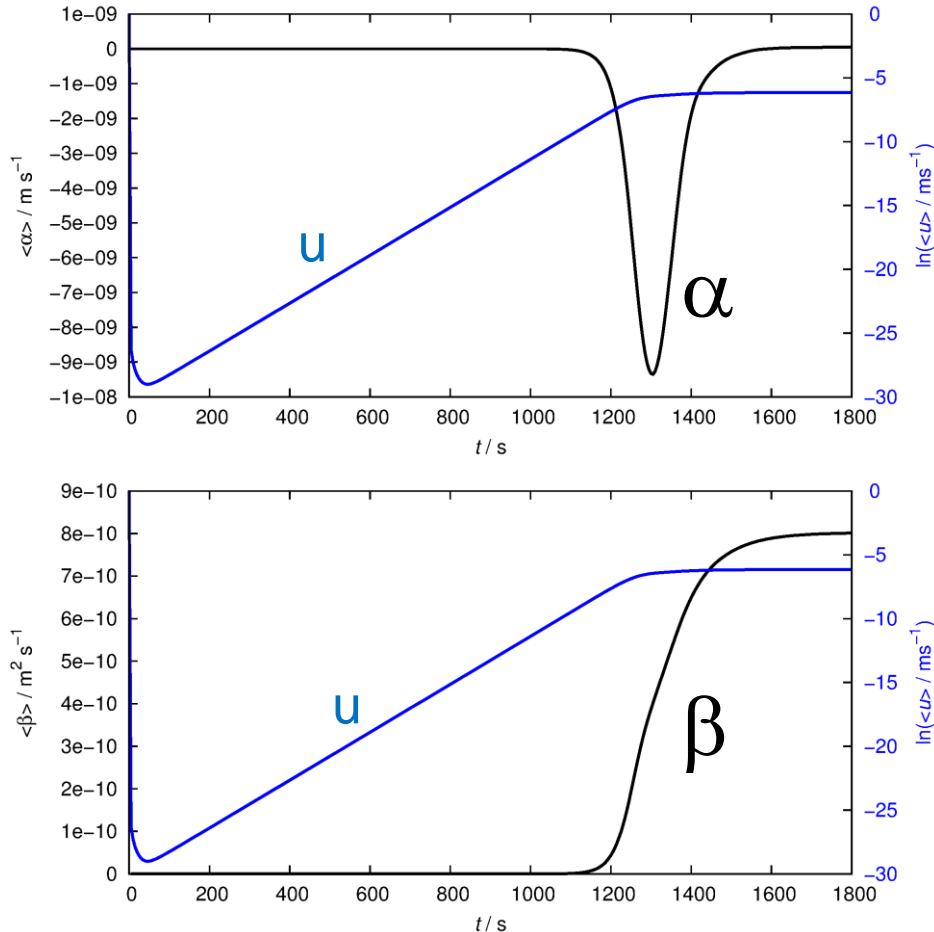
# Taylor-Spruit dynamo: the main problem



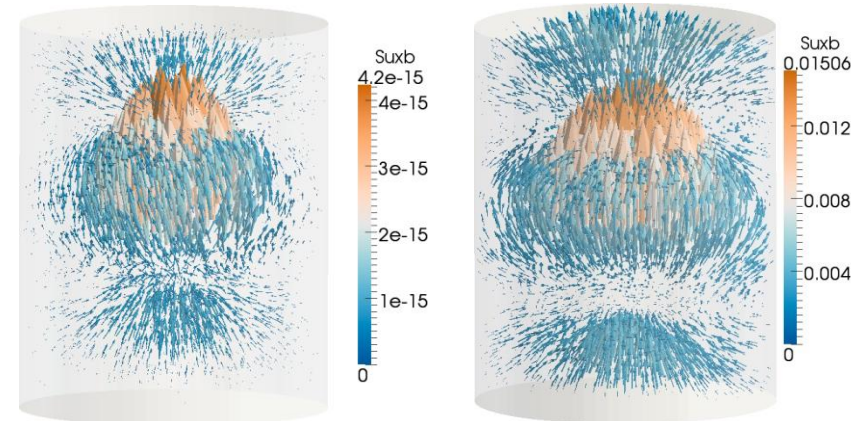
See Ferriz Mas, Schmitt, Schüssler, Astron. Astrophys. 289 (1994), 949 for fluxtube picture

# Taylor-Spruit dynamo: Any helical symmetry breaking at low Pm ?

At low Pm, neither the  $\beta$  effect nor the  $\alpha$  effect are strong enough to change the magnetic base configuration.  **$\alpha$  effect appears only in the exponential growth phase and disappears in the saturation regime.**



Induced current at...



500 s

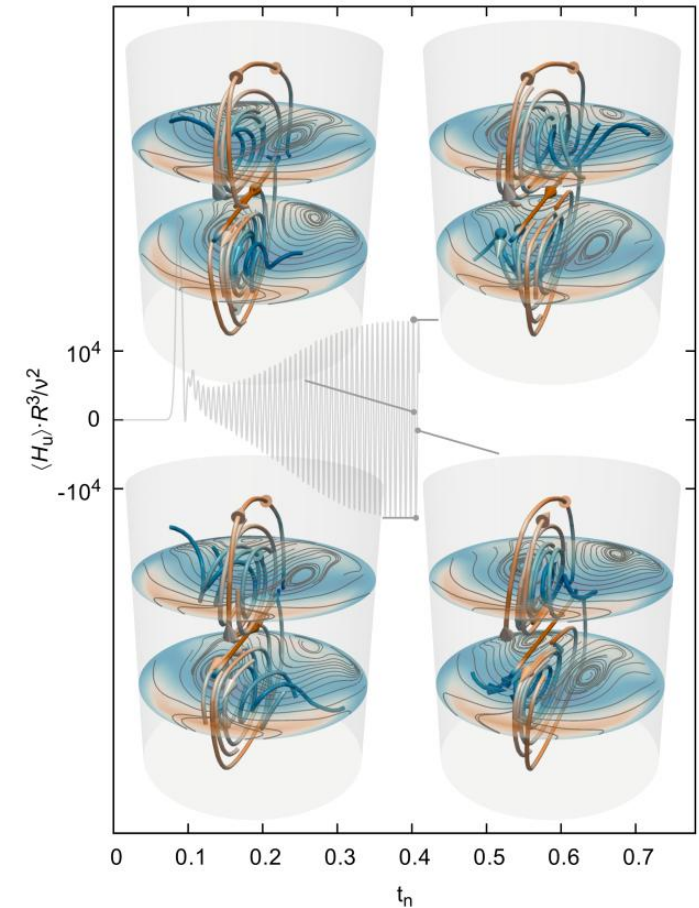
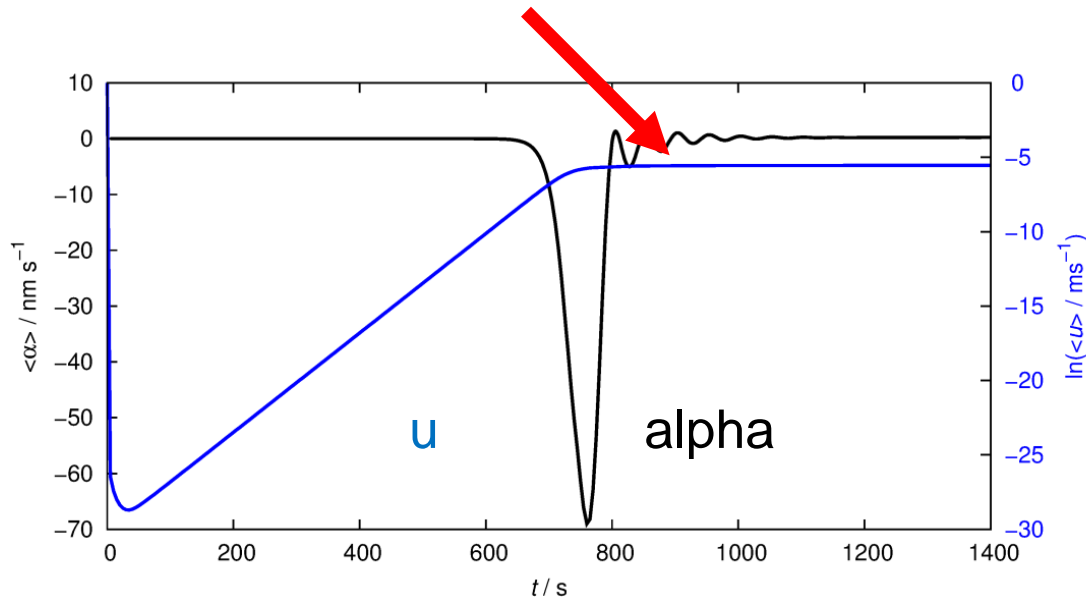
1800 s

Example:  $h/d=1.25$ ,  $Ha=55$

Weber et al., New J. Phys. 17 (2015), 113013

# Taylor-Spruit dynamo: Saturation and helicity oscillations at $Pm=10^{-6}$

(Damped) helicity oscillations  $Ha = 70$

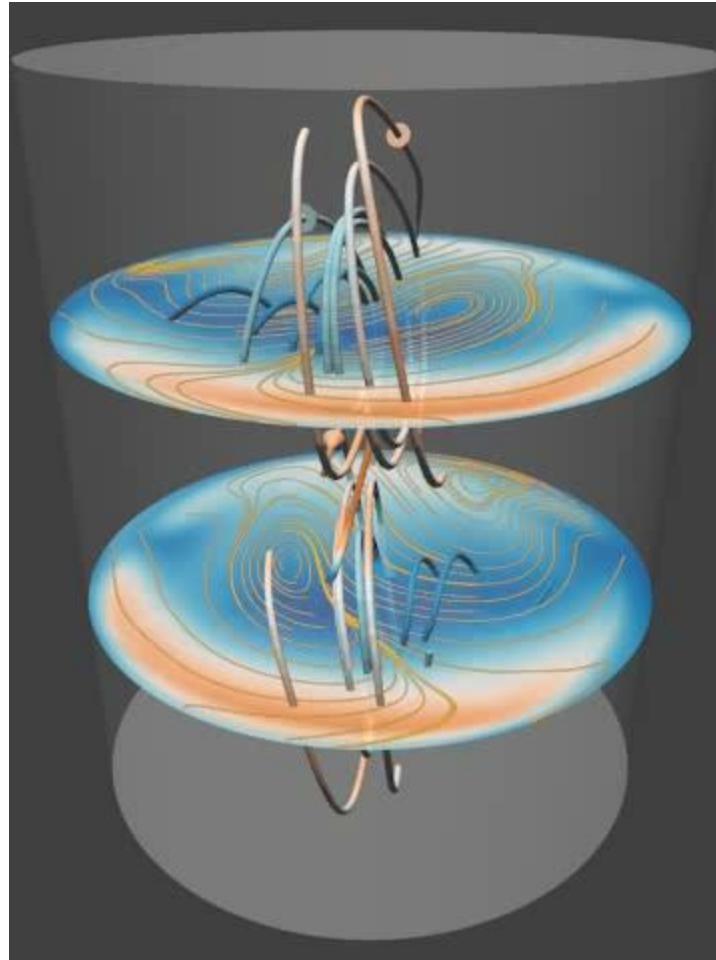


$Ha = 100$

Weber et al., New J. Phys. 17 (2015), 113013



# Taylor-Spruit dynamo: Character of the helicity oscillations



$Ha = 100$   
 $Pm = 10^{-6}$

Weber et al., New J. Phys. 17 (2015), 113013



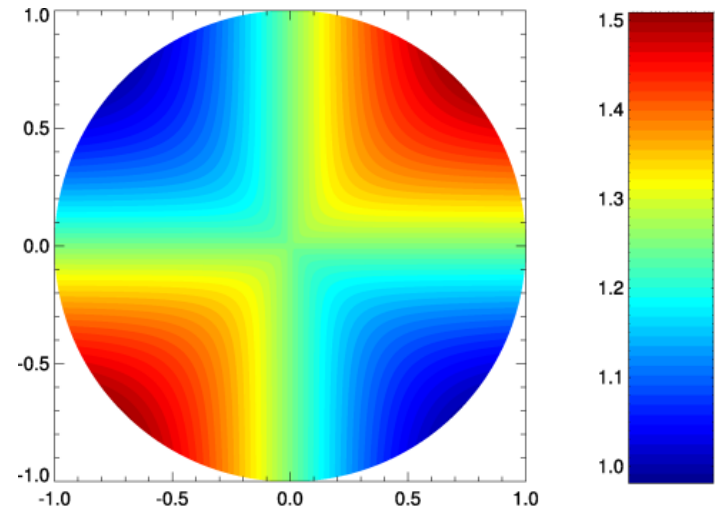
# Resonant excitation of helicity oscillations

# Resonant excitation of helicity oscillations

Taylor-Spruit-like dynamo:

- $\Omega$ -effect due to differential rotation
- $\alpha$ -effect relies on chiral symmetry breaking
- $\alpha$ -oscillation could be triggered and **synchronized** by planetary torques (emulated here by a  $m=2$  viscosity oscillation) **with very low energy input**

$m=2$  viscosity perturbation

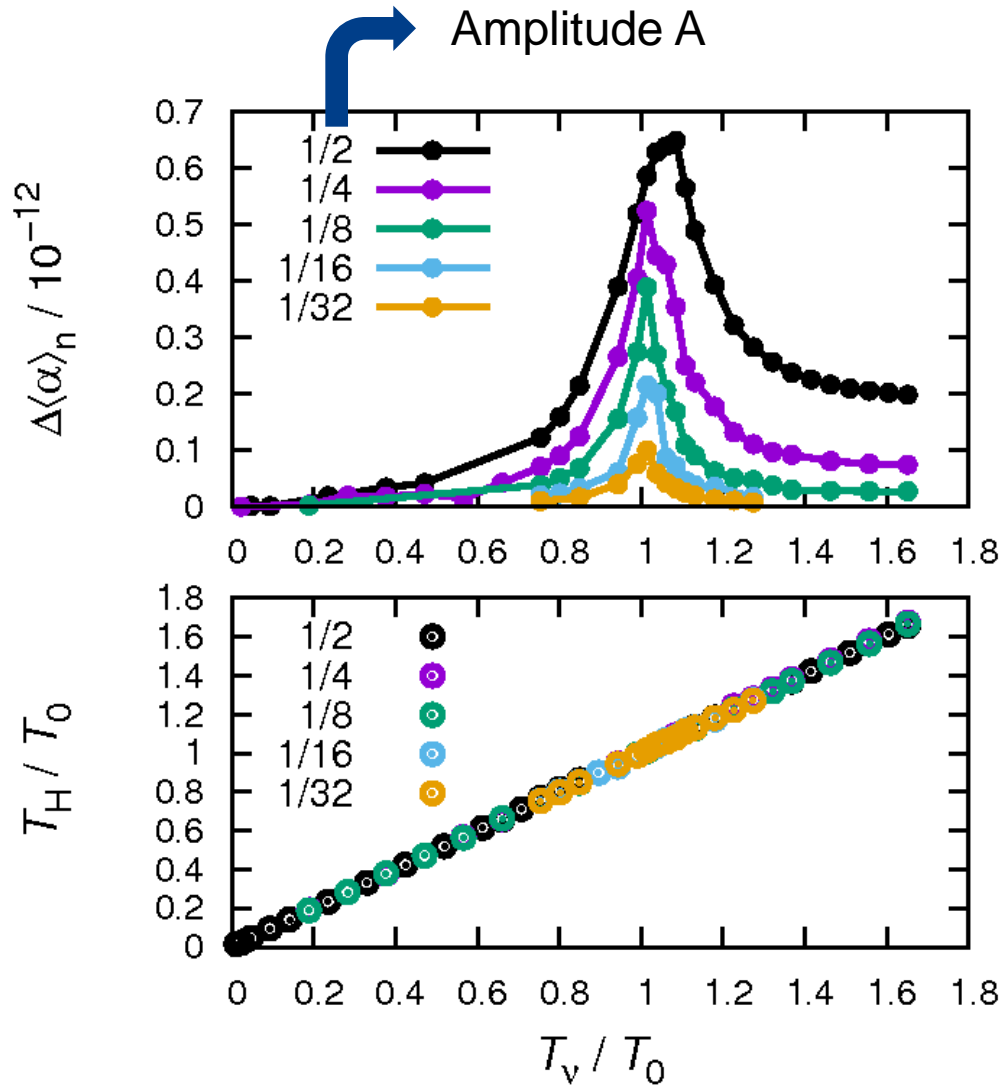


$$\nu(r, \phi, t) = \nu_0 \{1 + A[1 + 0.5r^2 / R^2 \sin(2\phi)(1 + \cos(2\pi t / T_V))]\}$$



Amplitude A

# Resonant excitation of helicity oscillations



Amplitude of  $\alpha$  oscillations in dependence on the ratio of the period of the  $m=2$  viscosity oscillation to the period of intrinsic helicity oscillations

Period of  $\alpha$  oscillations shows 1:1 resonance.

# A simple, and a slightly more complicated model of a synchronized dynamo

# A simple model of a synchronized dynamo

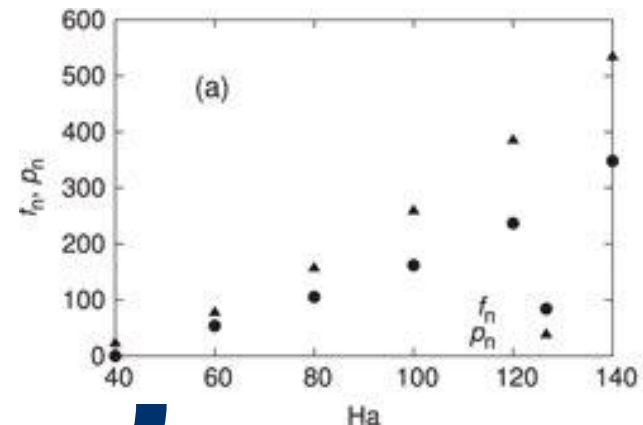
$$\dot{A}(t) = \alpha(t)B(t) - \tau^{-1}A(t)$$

$$\dot{B}(t) = \omega A(t) - \tau^{-1}B(t)$$

$$\alpha(t) = \frac{c}{1 + gB^2(t)} + \frac{pB^2(t)}{1 + hB^4(t)} \sin(2\pi t / T_V)$$

Constant  $\alpha$  term  
with quenching

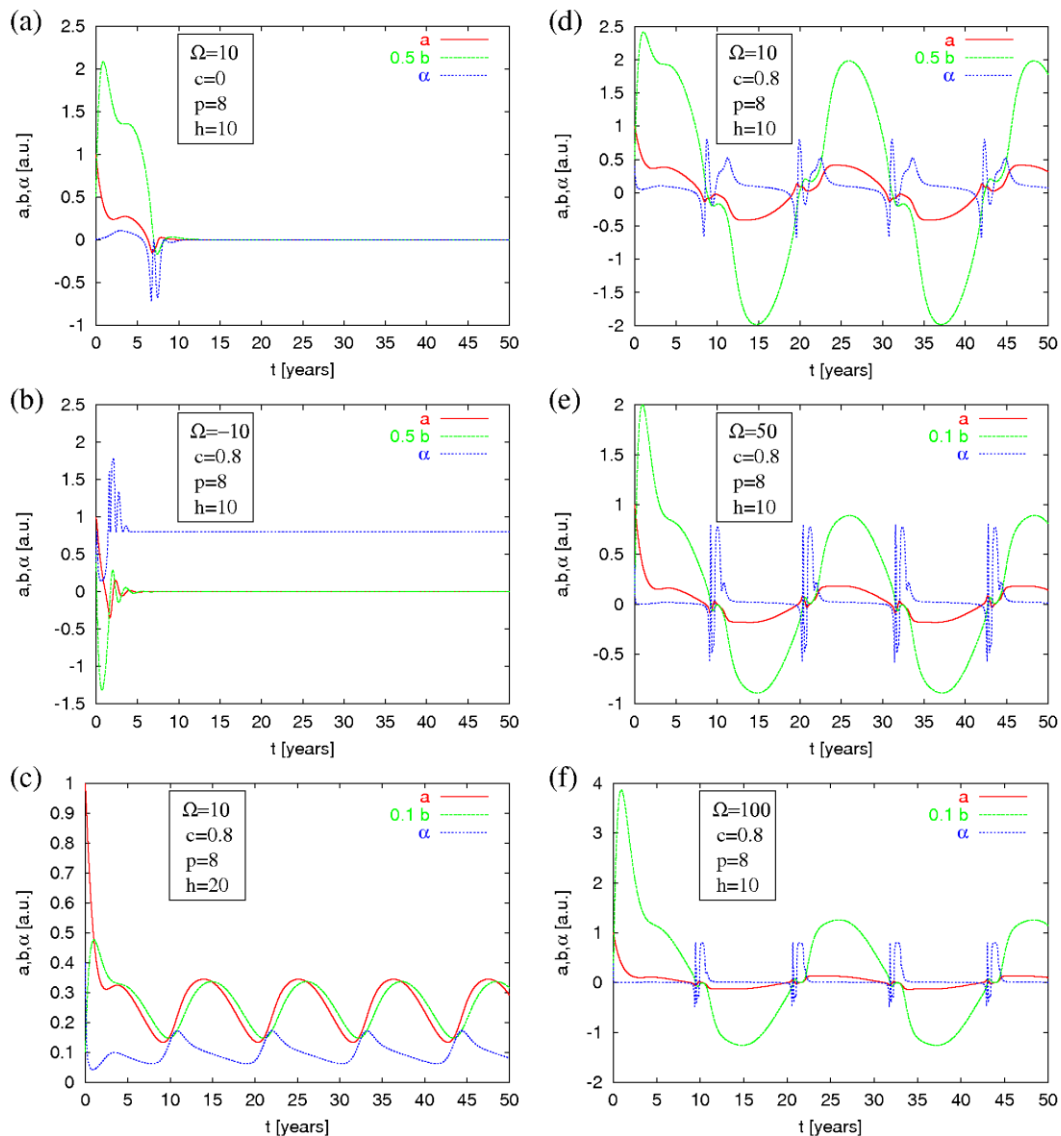
Oscillating  $\alpha$  term with  
resonant dependence  
on the field strength  
(i.e., on the frequency  
of helicity oscillations)



Stefani et al, Solar Phys. 291 (2016), 2197;  
arXiv:1511.09335

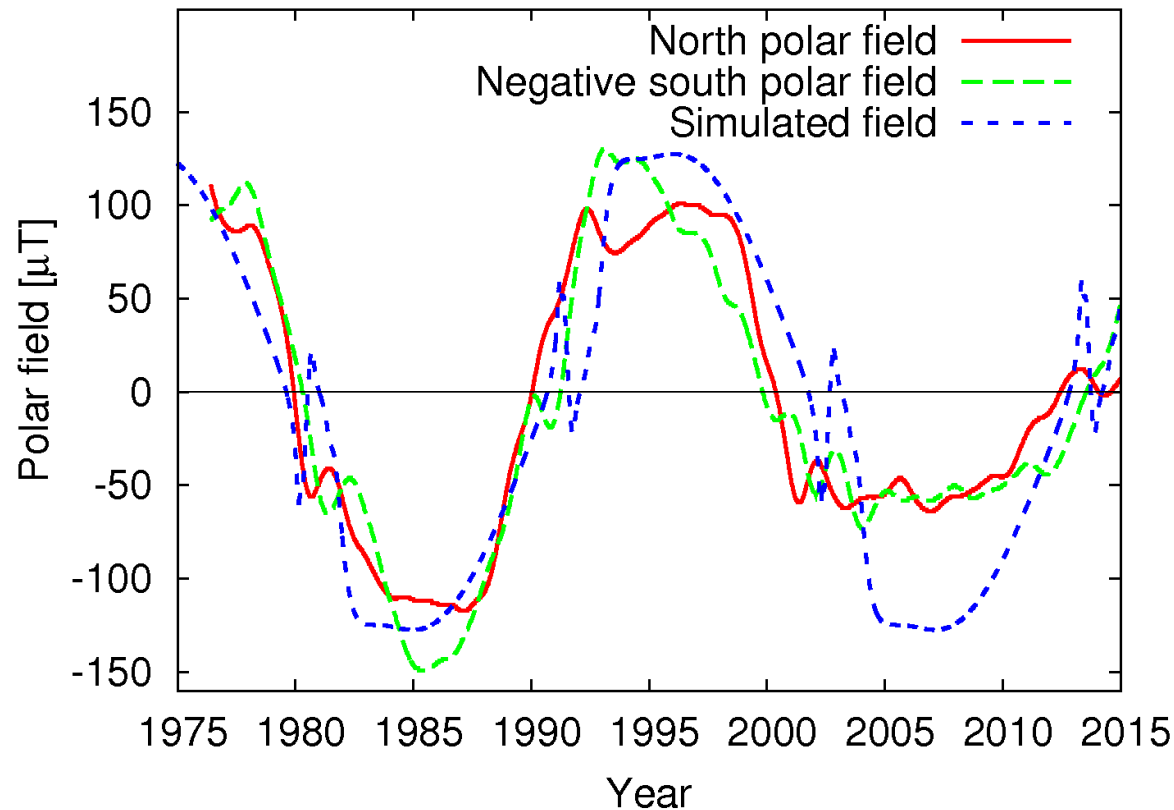
# A simple model of a synchronized dynamo

For a wide range of parameters, a robust 22.14 years Hale cycle appears...

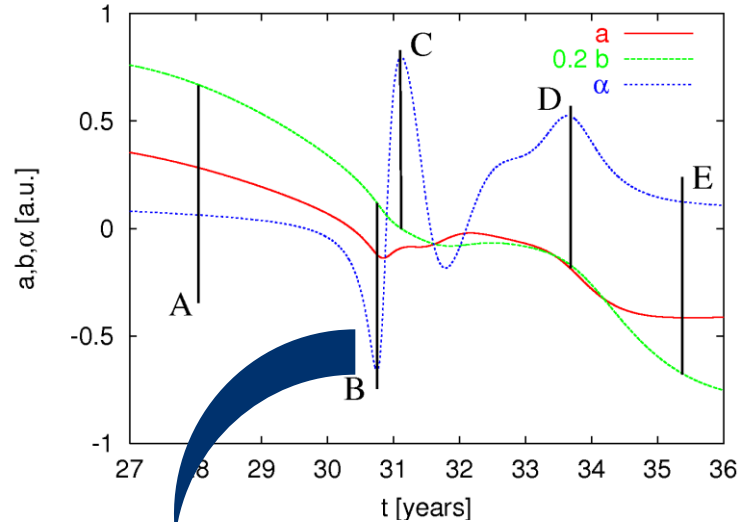


# A simple model of a synchronized dynamo

Comparison with measured polar field shows interesting „mid-term oscillations“

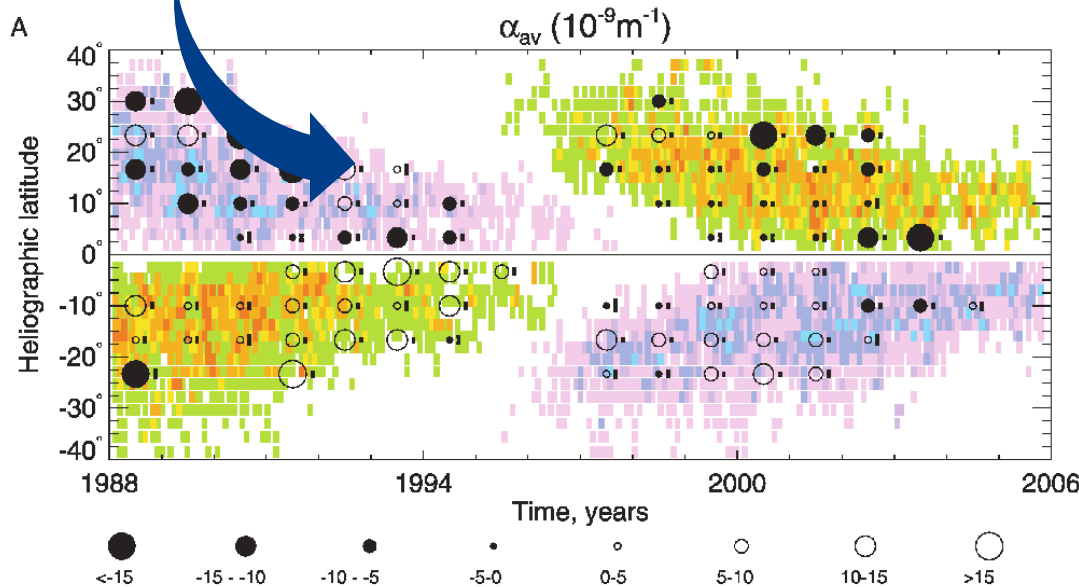


# A simple model of a synchronized dynamo: „wrong“ helicities?



$\alpha$  acquires the „wrong“ sign for a short period before the field reversal

Connection with observation?

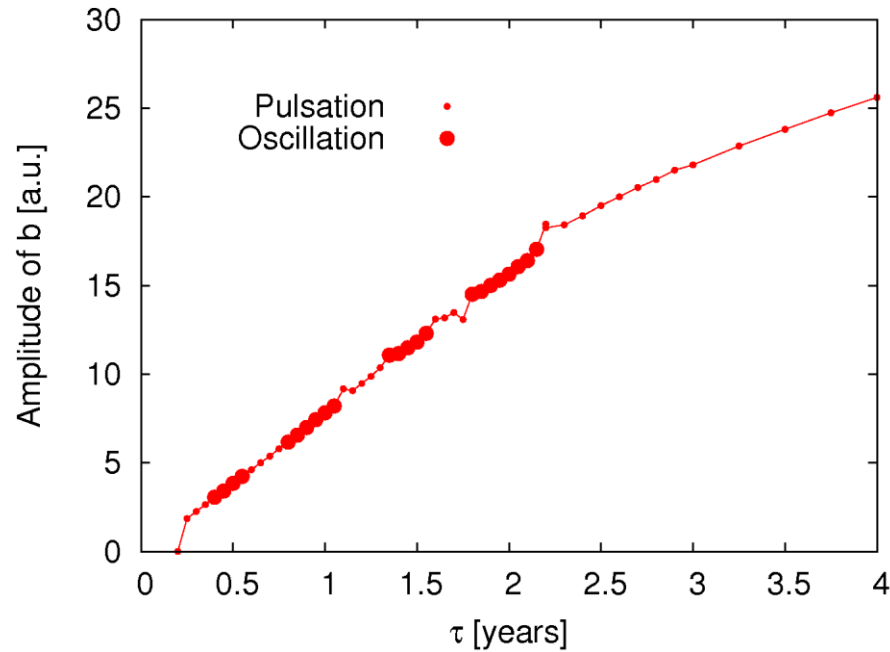


Zhang et al., MNRAS 402 (2010), L30



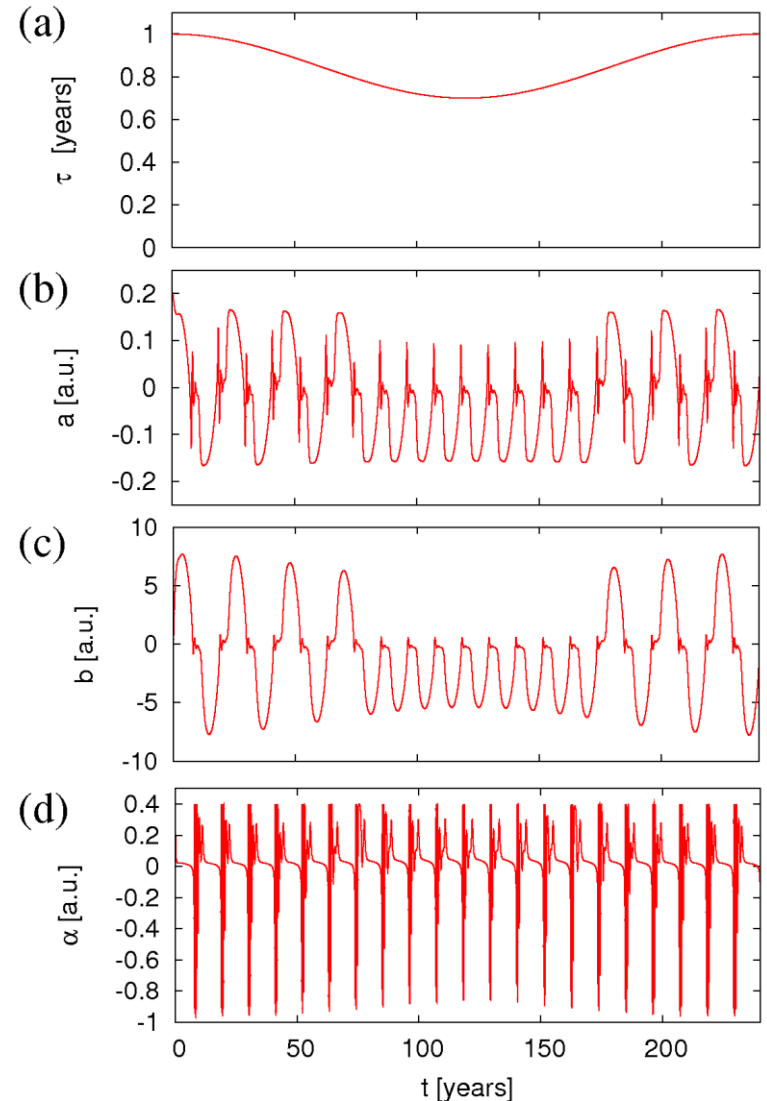
# Transitions between oscillations and pulsations → „Grand minima“ ?

## Bands of oscillations and pulsations



„Grand minima“ with phase coherence?

Stefani et al., arXiv:1706.07638

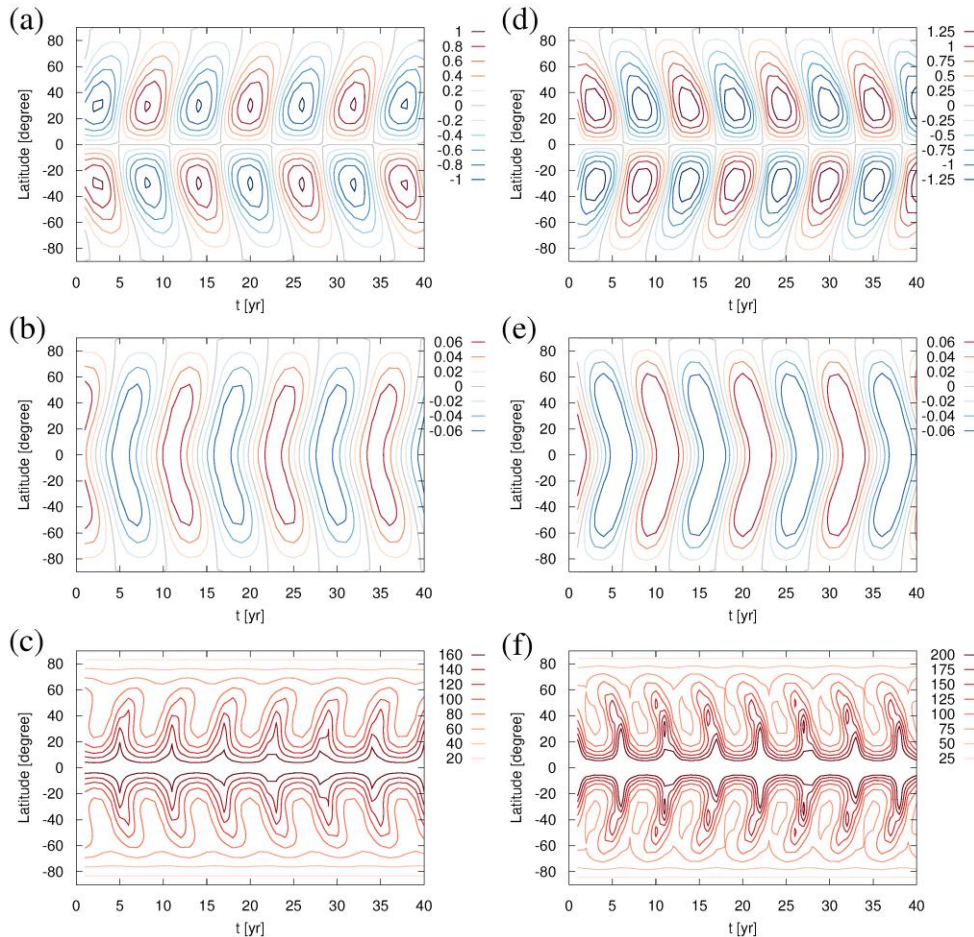


# First results with a 1D-model: Test case with $\Omega$ -quenching

$$\alpha = -1, \tau = 1$$

$$\omega_0 = 170$$

$$\omega_0 = 250$$



$$\frac{\partial B(\theta, t)}{\partial t} = \omega(\theta, t)A(\theta, t) - \frac{1}{\tau} \frac{\partial^2 B(\theta, t)}{\partial \theta^2}$$

$$\frac{\partial A(\theta, t)}{\partial t} = \alpha(\theta, t)B(\theta, t) - \frac{1}{\tau} \frac{\partial^2 A(\theta, t)}{\partial \theta^2},$$

$B(\theta, t)$

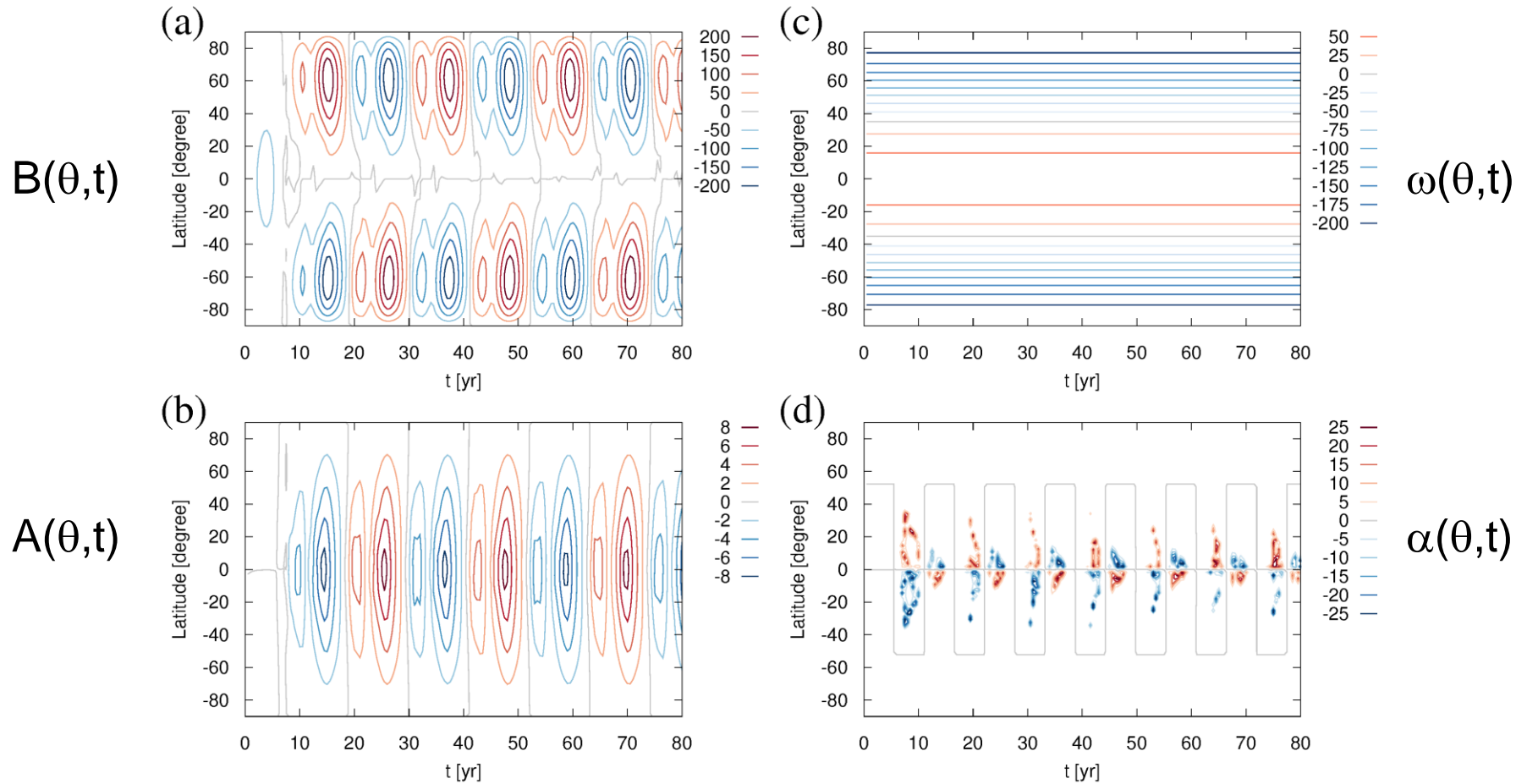
$A(\theta, t)$

$\omega(\theta, t)$

Stefani et al., arXiv:1803.08692

# First results with a 1D-model: Test case with resonant and periodic $\alpha$

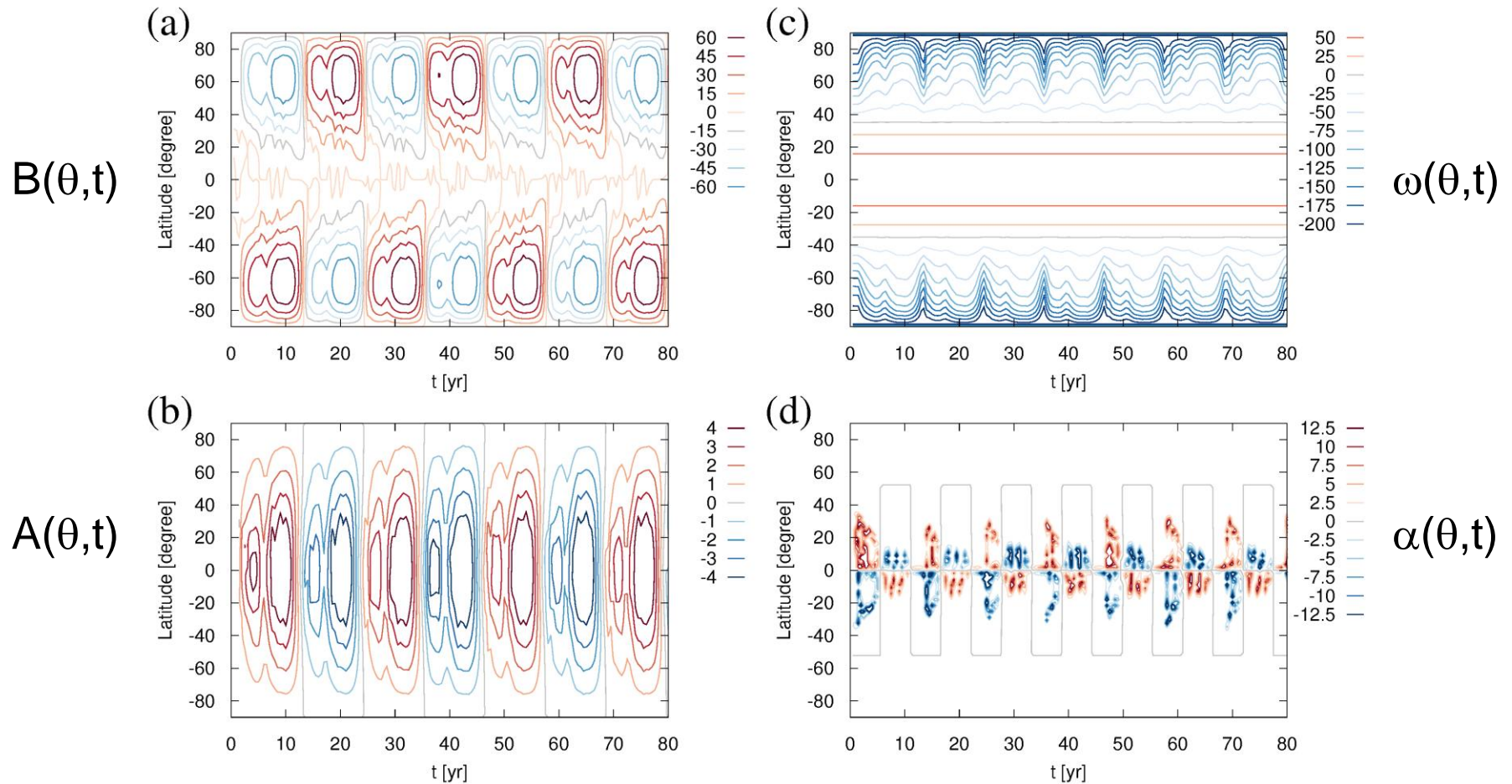
$$\omega(\theta, \tau) = \omega_0(1 - 0.939 - 0.136 \cos^2(\theta) - 0.1457 \cos^4(\theta)), \quad \omega_0 = 1000, \quad \tau = 2, \quad \alpha_{\text{periodic}} = 60$$



Stefani et al., arXiv:1803.08692

# First results with a 1D-model: Test case with resonant and periodic $\alpha$

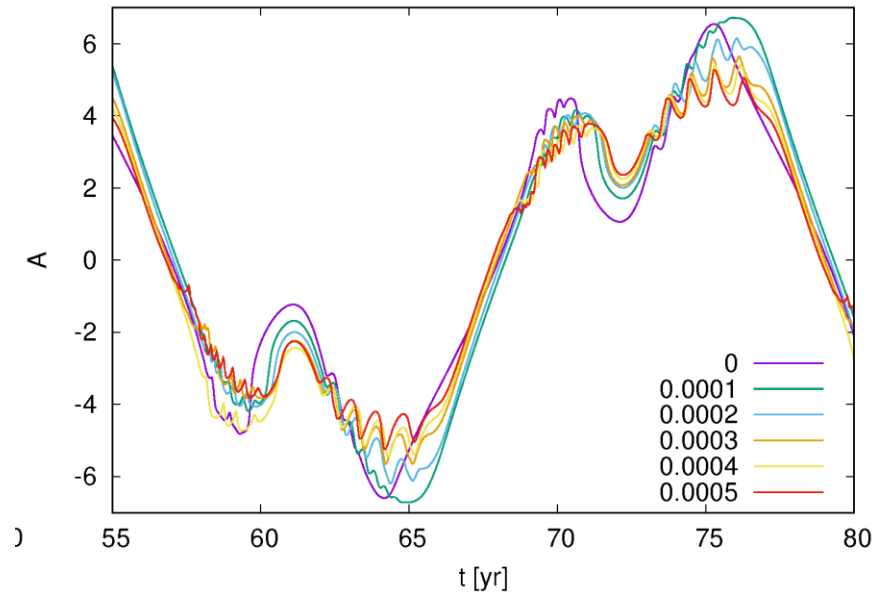
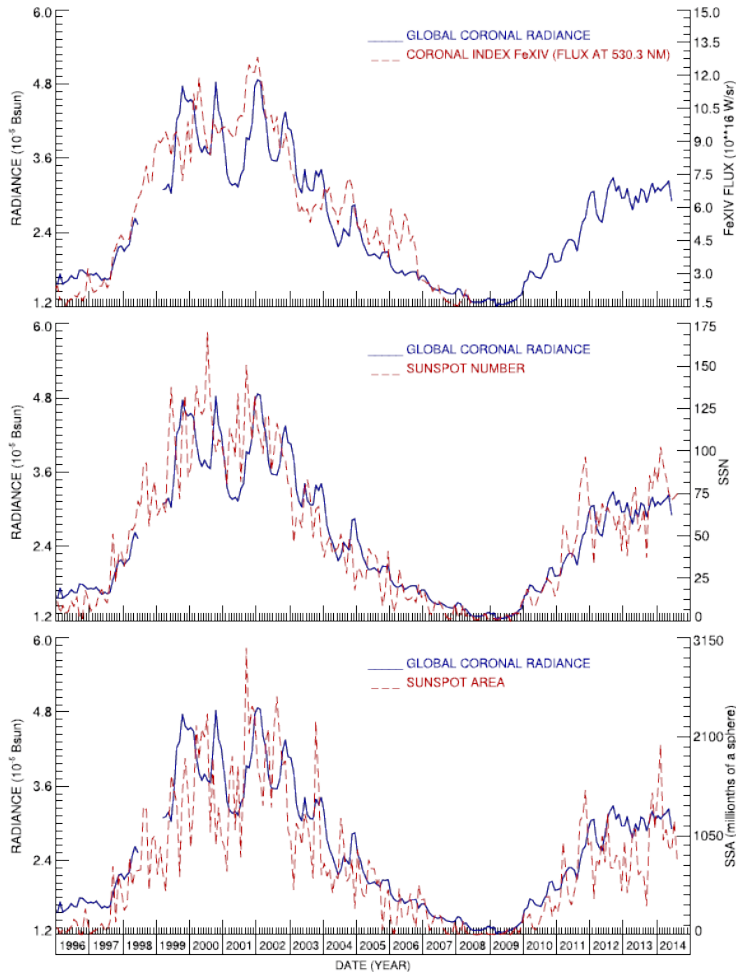
$\omega_0=1000$ ,  $\tau=2$ ,  $\alpha_{\text{periodic}}=40$  + quenching of  $\omega$  (or  $\sigma$ ) in the pole-near regions



Stefani et al., arXiv:1803.08692

# Some interesting features: Mid-term oscillations

Mid-term oscillations ( $\sim 1.7$  years), as visible in many solar data, appear naturally in the synchronization model...

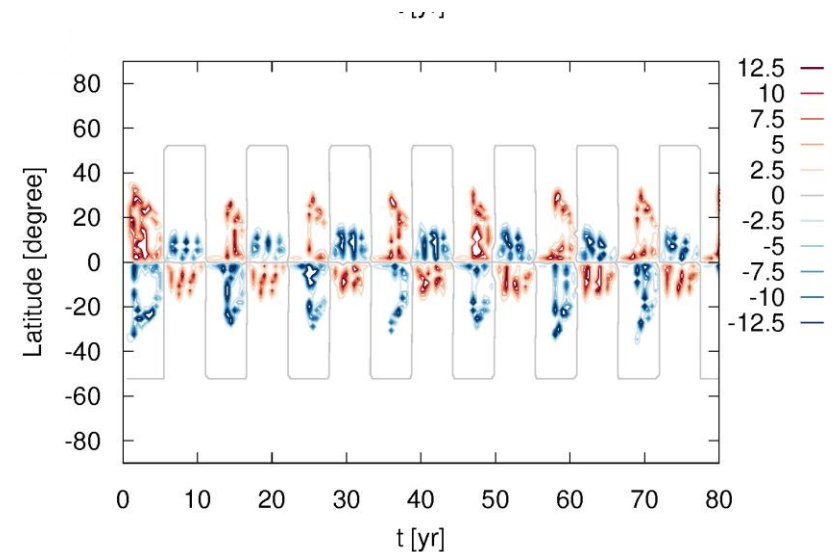
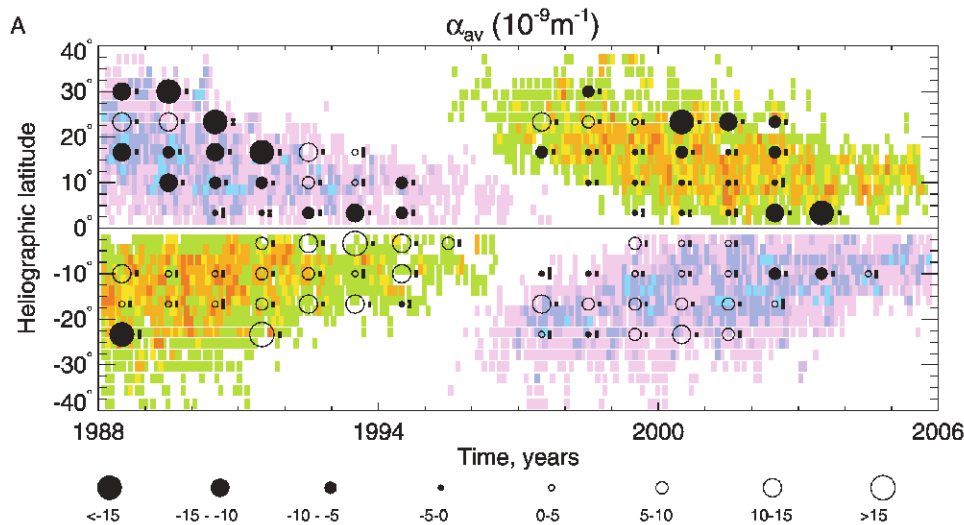


Stefani et al., arXiv:1803.08692



# Some interesting features: Appearance of „wrong“ helicities

„Wrong“ helicities are frequently observed in the two hemispheres. They come out as an essential feature of the synchronization model...

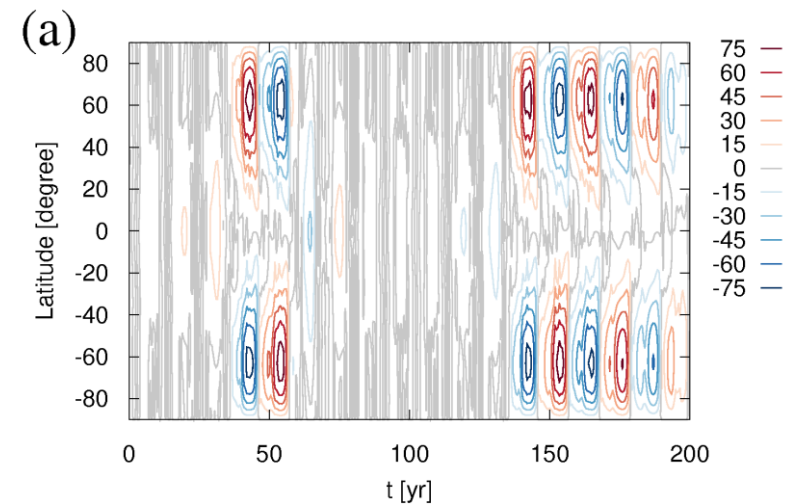
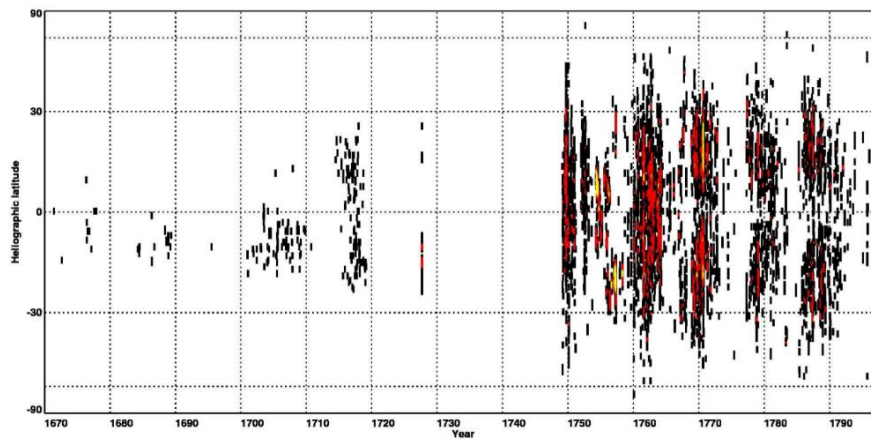


Zhang et al., MNRAS 402 (2010), L30

Stefani et al, arXiv:0803.08692

# Some interesting features: Quadrupole fields around grand minima

During (or shortly after) grand minima, dipole fields may be replaced by quadrupole fields. These **transitions** appear in the synchronization model **with maintained phase coherence**.



Arlt and Weiss, Space Sci. Rev. 186 (2014), 525

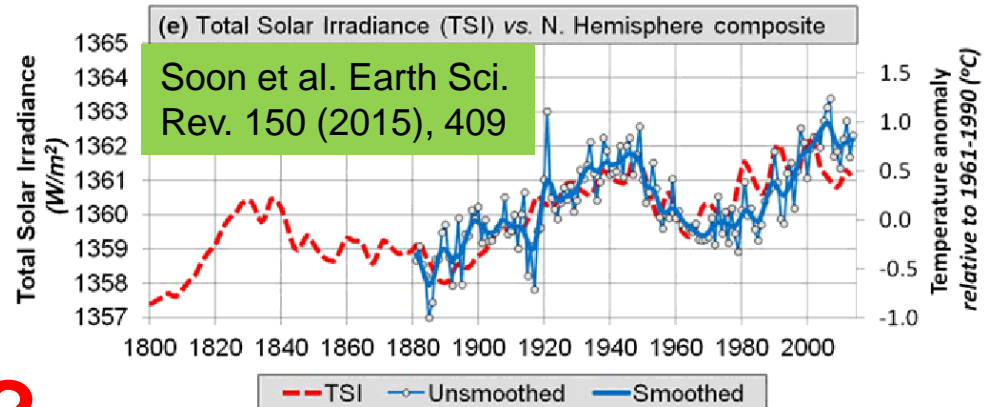
Stefani et al, arXiv:0803.08692

# Any planetary synchronization model for the 60+++ years cycle?

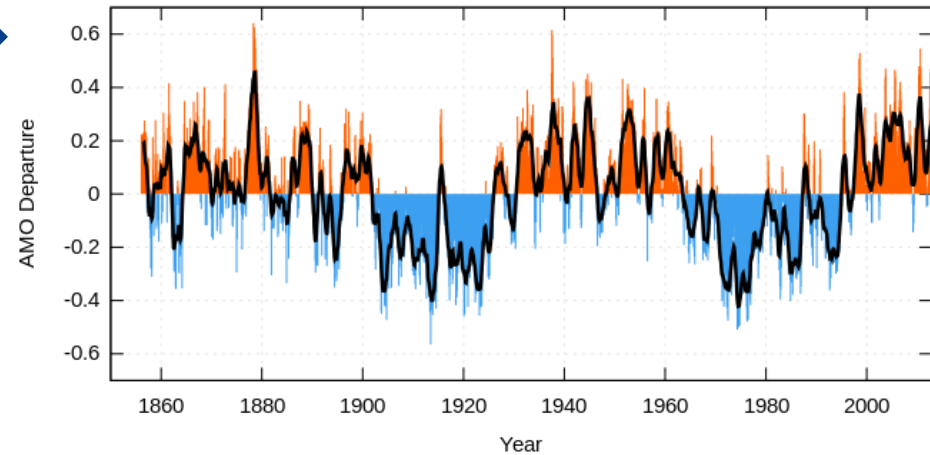
Perhaps...

Mercury has nearly as strong a tidal forcing as Earth.

Period of Mercury, Venus, Earth, Jupiter alignments:  
**66.4 years**



Monthly values for the AMO index, 1856 -2013



Atlantic Multidecadal Oscillation:  
A simple sinus fit gives **66.3 years**

Verma, S.D., 1986 Influence of planetary motion and radial alignment of planets on sun. In: Bhatnagar, K.B. Space dynamics and celestial mechanics, Astrophys. Space Sci. Libr. 127, Springer, 143

Soon, Connoly, Connoly, Earth-Science Review 150 (2015), 409

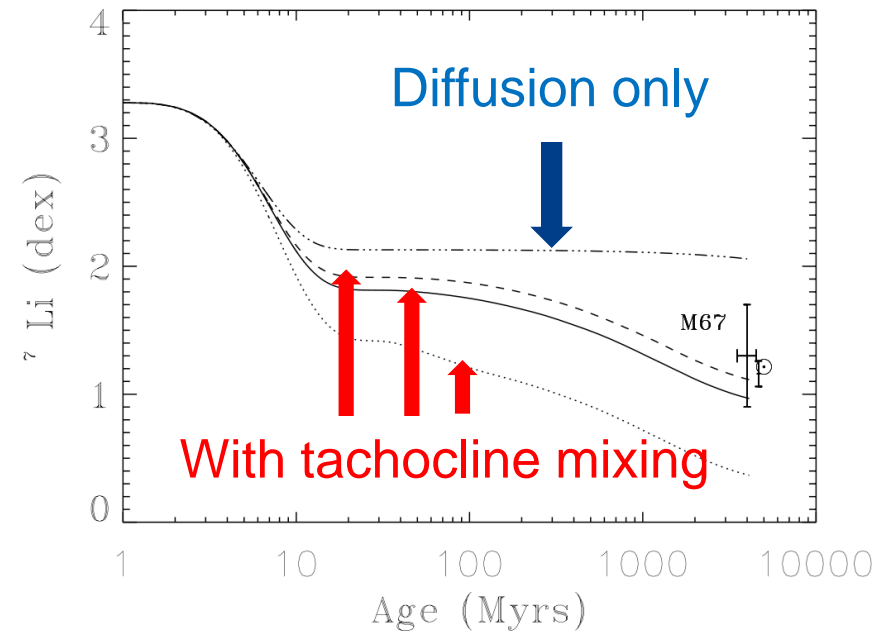


## Summary and prospects

- **Tiny tidal heights of 1 mm** may translate (by naive virial theorem) to  $v \sim (2 g h)^{1/2} \sim 1 \text{ m/s}$  because of  $g = 500 \text{ m/s}^2$
- **Babcock-Leighton** models are rather stubborn to synchronization (but it's not excluded)
- **Taylor instability** at small Pm tends to produce **intrinsic oscillations of the helicity** and the corresponding  $\alpha$  effect
- These helicity oscillations can be **resonantly excited by  $m=2$  perturbations** (with minor energy input)
- Simple 0D and 1D  $\alpha$ - $\Omega$  dynamo models with an 11.07 years  $\alpha$  oscillation produce a **22.14 years solar cycle**
- High sensitivity of oscillations/pulsations on dynamo parameters (e.g.  $\tau$ )  $\rightarrow$  **grand minima** ?
- Higher dimensional models with realistic tachocline geometry and rotation are urgently needed!

## Further prospects: Lithium depletion problem in the sun and stars

- Lithium and Beryllium are destroyed by proton captures at very high temperatures (Li  $2.5 \times 10^6$  K, Be at even higher).
- These elements survive in outer (colder) convection zone of sun-like stars. Their observed continuous decrease suggests a transport process from the convection zone into the radiation zone, where they are destroyed.
- The transport mechanism needs to be rather slow (compared to turbulent convective motion).



${}^7\text{Li}$  surface abundances in stars with solar mass and composition. Tachocline mixing with disk coupling time 10, 3, and 0.5 Myr, respectively. 5777 K at 4.6 Gyr

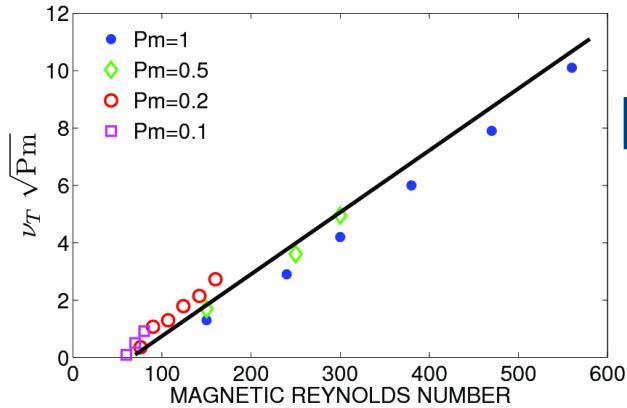
Piau and Turck-Chieze, ApJ  
566, 419 (2002)

# Lithium depletion problem in the sun and stars

- Mainly 2 candidates: **convective overshooting**, or a **magnetic instability**
- Problem: treating of mixing is not enough, the outward transport of angular momentum is essential.
- Turbulent Schmidt number (ratio of turbulent viscosity and mixing  **$Sc = \nu^*/D^*$** ). A value  $Sc \approx 100$  is needed to explain the sun's Li depletion
- **Convective turbulence: strong mixing AND effective transport** of angular momentum, thus a resulting Schmidt number  $Sc \approx 1$ .
- **MHD instability: mild mixing and effective angular momentum transport**,  $Sc \approx 100$  easily reachable (Gellert et al. 2014).
- Explanation: angular momentum transport is due to Reynolds AND Maxwell stress, but mixing is ONLY a hydrodynamic process.  
**With a magnetically dominated state, the problem can be solved.**

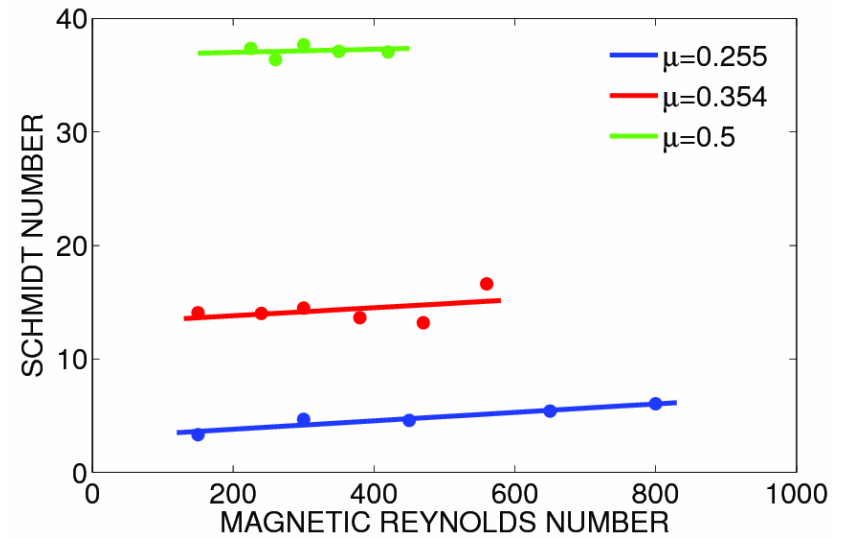
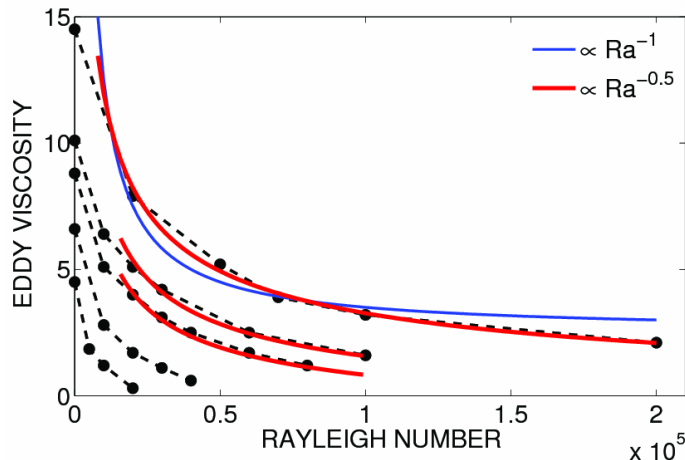
# Astrophysical application of AMRI: Turbulent Schmidt number in stars

How to reconcile the slow-down of stars and the mild mixing of chemicals? Turbulent viscosity  $\nu_T$  should be a few hundreds, the Schmidt number  $Sc = \nu_T/D_T$  around 100.



Final scaling:

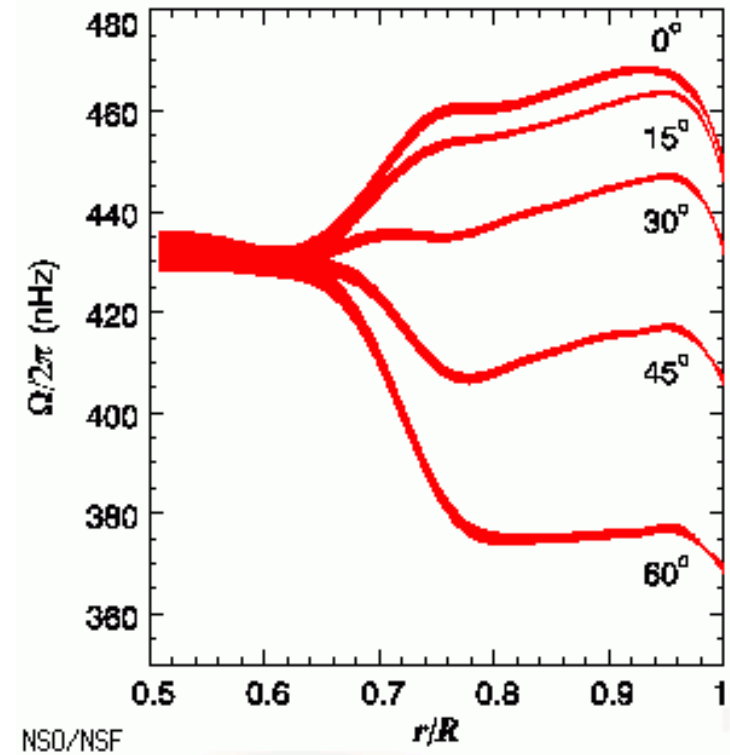
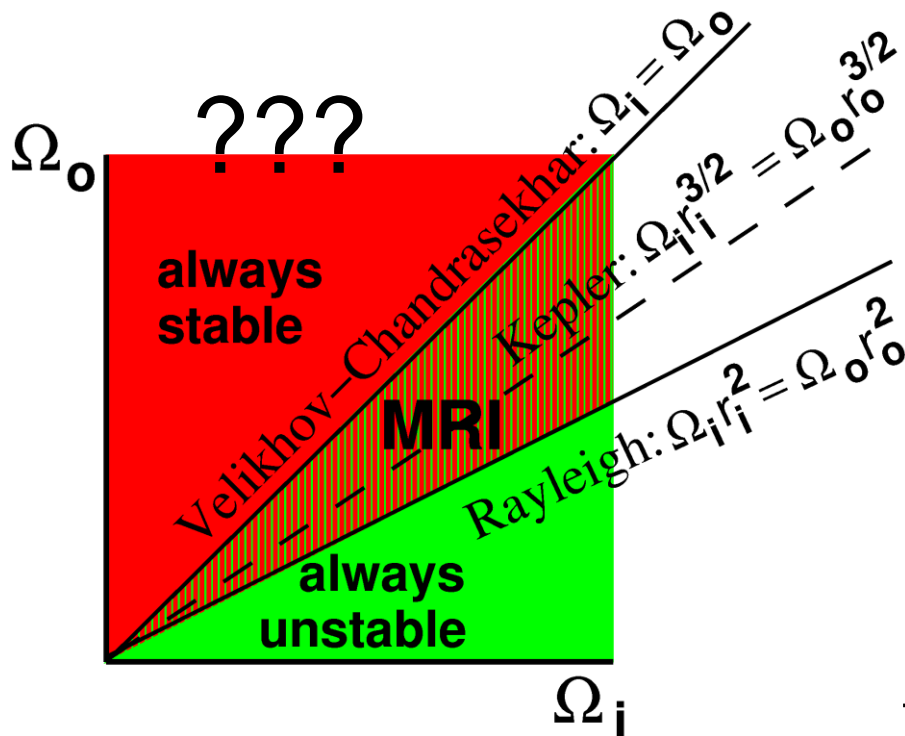
$$\frac{\nu_T}{\nu} \sim \sqrt{\frac{Pm}{Ra}} \frac{Re}{\mu_\Omega^2}$$



Gellert et al., IAUS 307 (2014)

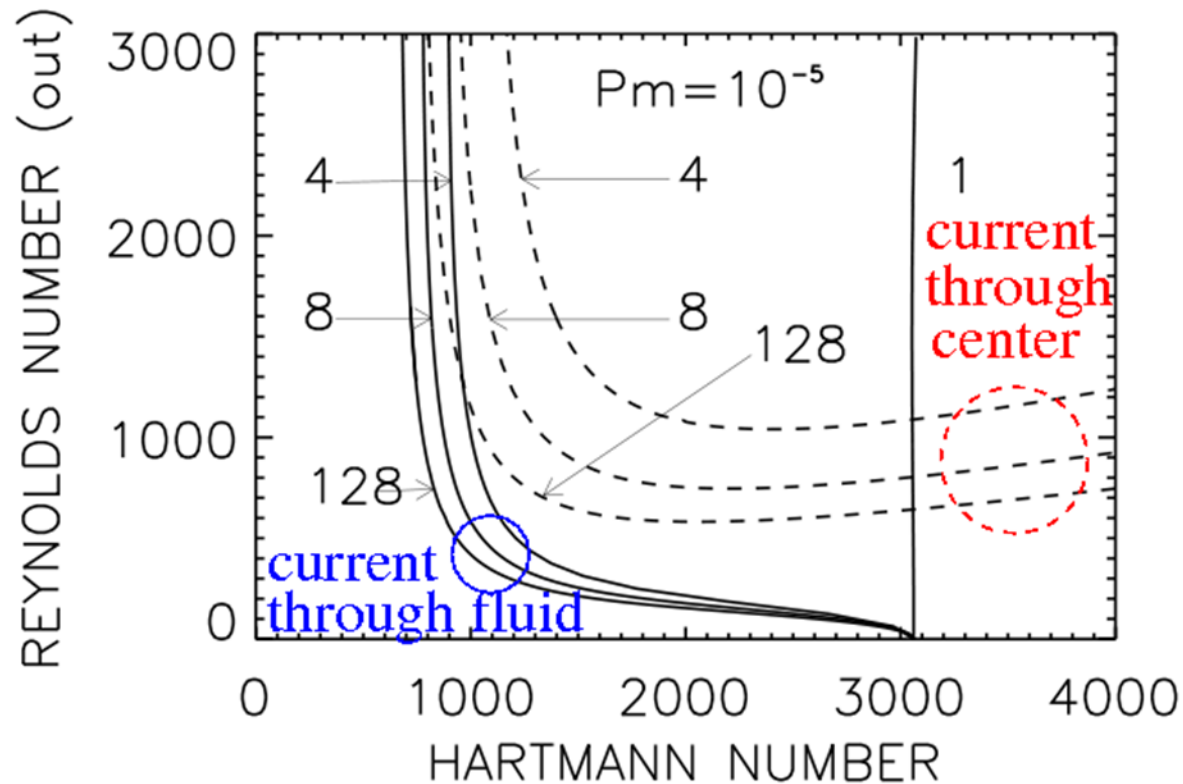
# Related questions: Any chance for a „Super-MRI“?

...i.e., can magnetic fields destabilize rotational flows with positive shear?



Possible relevance, e.g., for the solar tachocline in the equator-near strip...

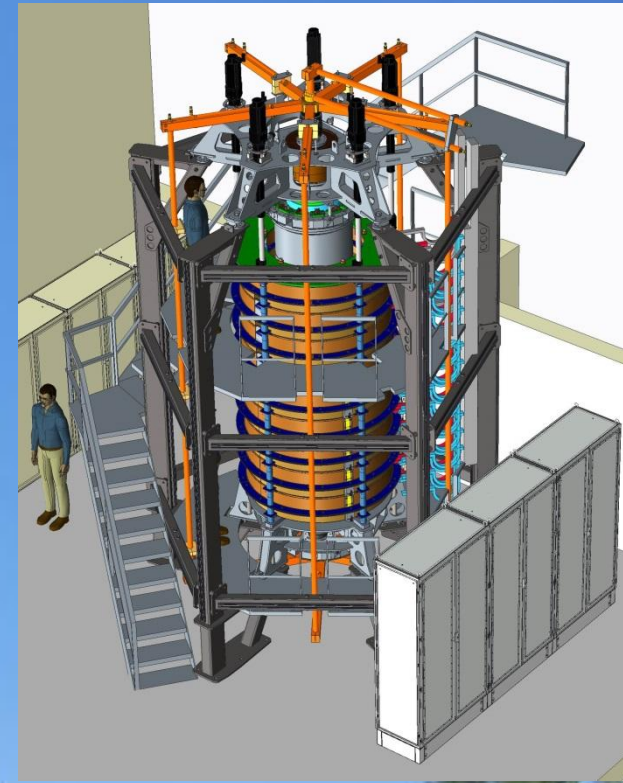
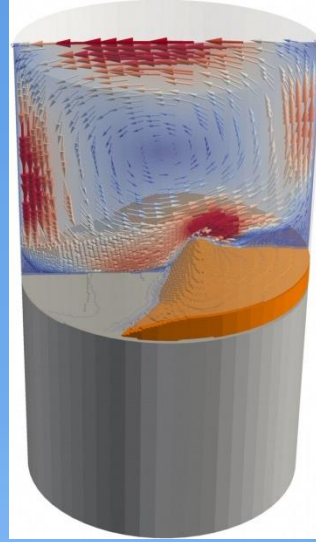
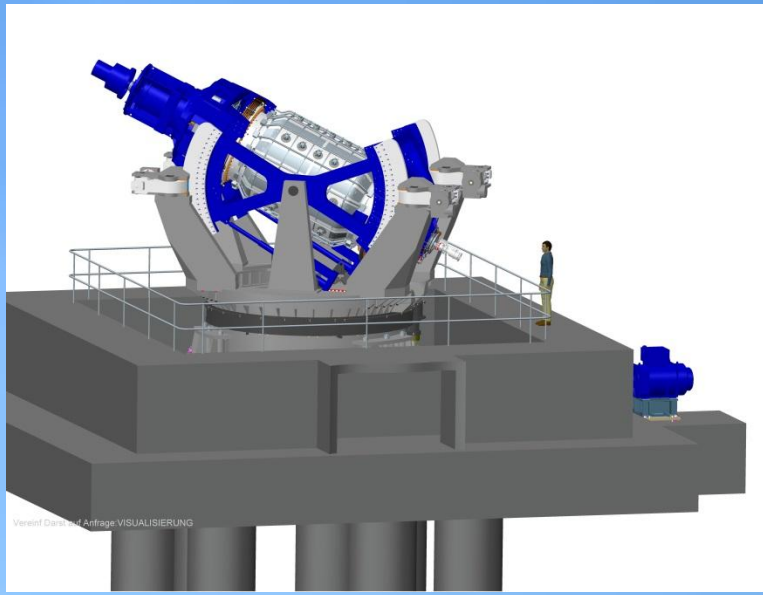
# Evidence for „Super-AMRI“ in WKB and in 1D-stability code



Stefani and Kirillov, Phys. Rev E 92 (2015) 051001(R)

Rüdiger et al, Phys. Fluids 28, 014105 (2016)





**Thank you for your attention**