Is there a planetary influence on solar activity?

Some insights from nonlinear dynamics and Bayesian statistics

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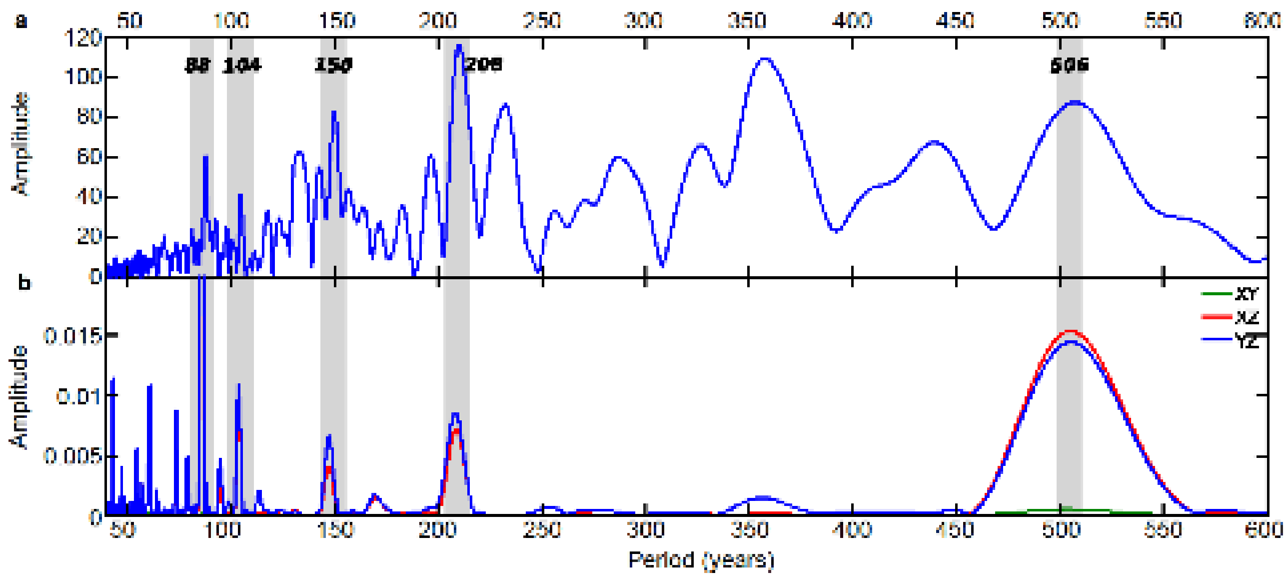
Fourier Spectra



Abreu et al. 2012

- Problem: Peaks have a **finite width**, due to the finite observation window
- This resolution has nothing to do with the **accuracy** lines can be determined with!
- How to measure accuracy -> **Bayesian statistics**!

The Bayesian method

- Bayesian statistics conceptualises **model-based learning from data** and allows us to **quantify knowledge** of model parameters by means of **probability distributions**.
- We need **prior knowledge** in the form of
  - a **parameterised stochastic model,** for the data generating process, informed by our physical understanding of the system and/or analysis of the features of the data. Mathematically cast in the form of a probability distribution, for model outputs given model parameters -> **likelihood function** $L(\mathbf{y}|\boldsymbol{\theta})$.
  - a **probability distribution** $f_{prior}(\boldsymbol{\theta})$, expressing our prior knowledge about the parameters of the model.
- **Bayes' equation** tells us how to update knowledge about parameters when new data becomes available, which is believed to be a realisation of the model:

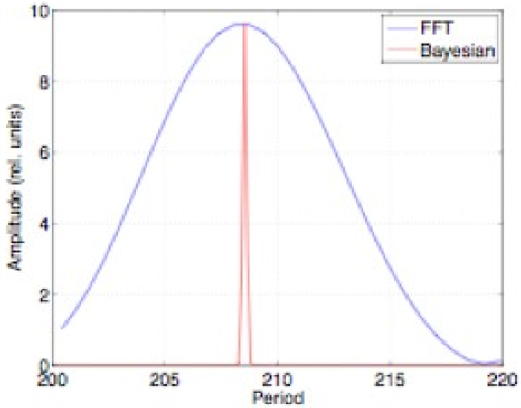
$$f_{post}(\boldsymbol{\theta}|\mathbf{y}) = \frac{L(\mathbf{y}|\boldsymbol{\theta}) f_{prior}(\boldsymbol{\theta})}{\int L(\mathbf{y}|\boldsymbol{\theta}') f_{prior}(\boldsymbol{\theta}') d\boldsymbol{\theta}'}$$

The Bretthorst model

- **Data generating model**: Harmonic(s) plus red noise (Palonen 2004),

  ⋙ = period, phase and amplitude of harmonic(s) and amplitude of red background noise.

- Allows us to determine lines (periods of harmonics) with **phenomenal accuracy**!



unpublished!

The Bretthorst model

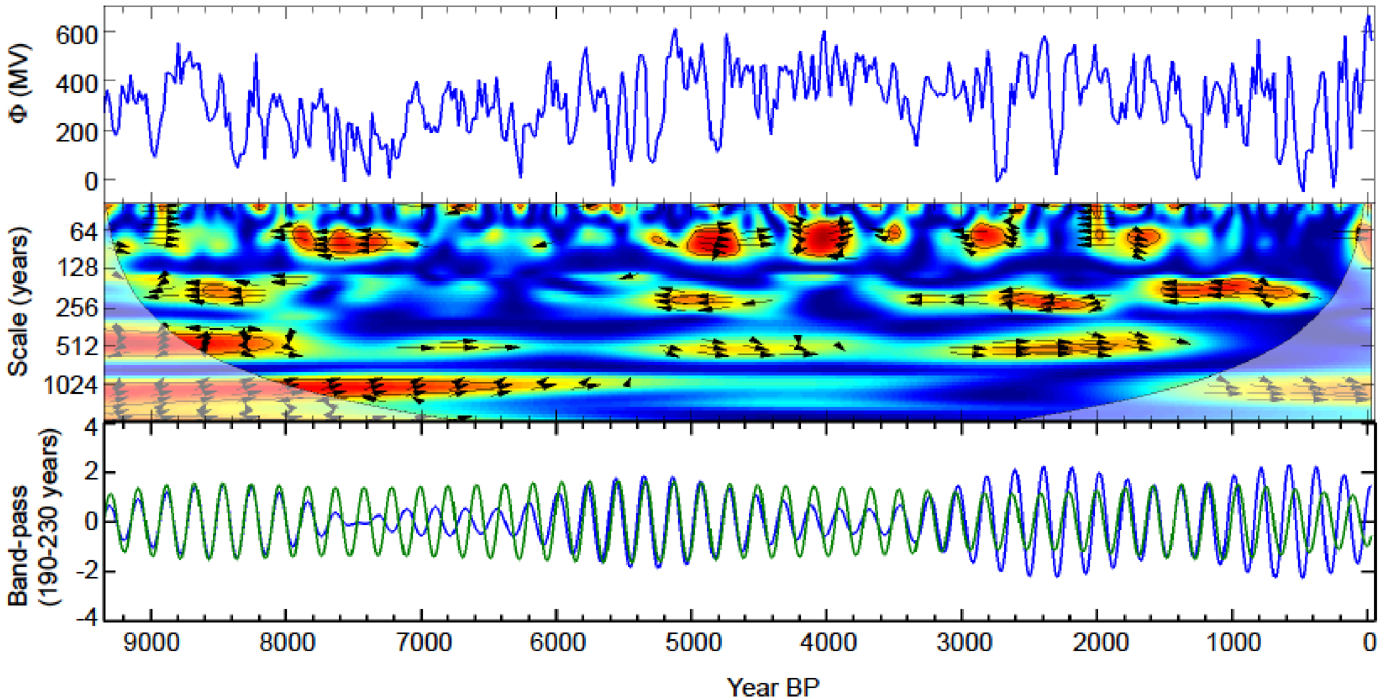
Extremely high significance!

But:

- Strongest amplitudes only match if we average over exactly 1 year!
- Are these cycles random or is there a structure?
- Shouldn’t we rather look at the phases?

Phase synchronisation

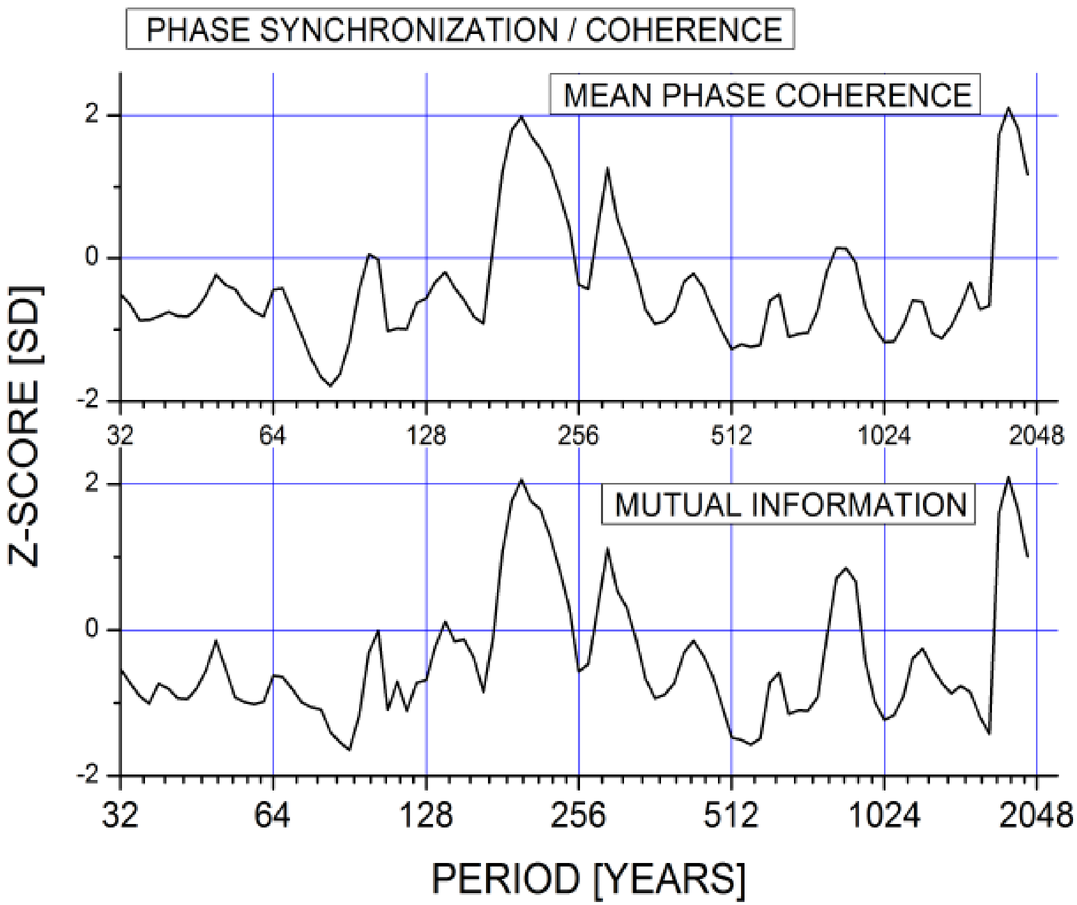
Phase-locking while cycle is strong!



Abreu et al. 2012

Phase synchronisation

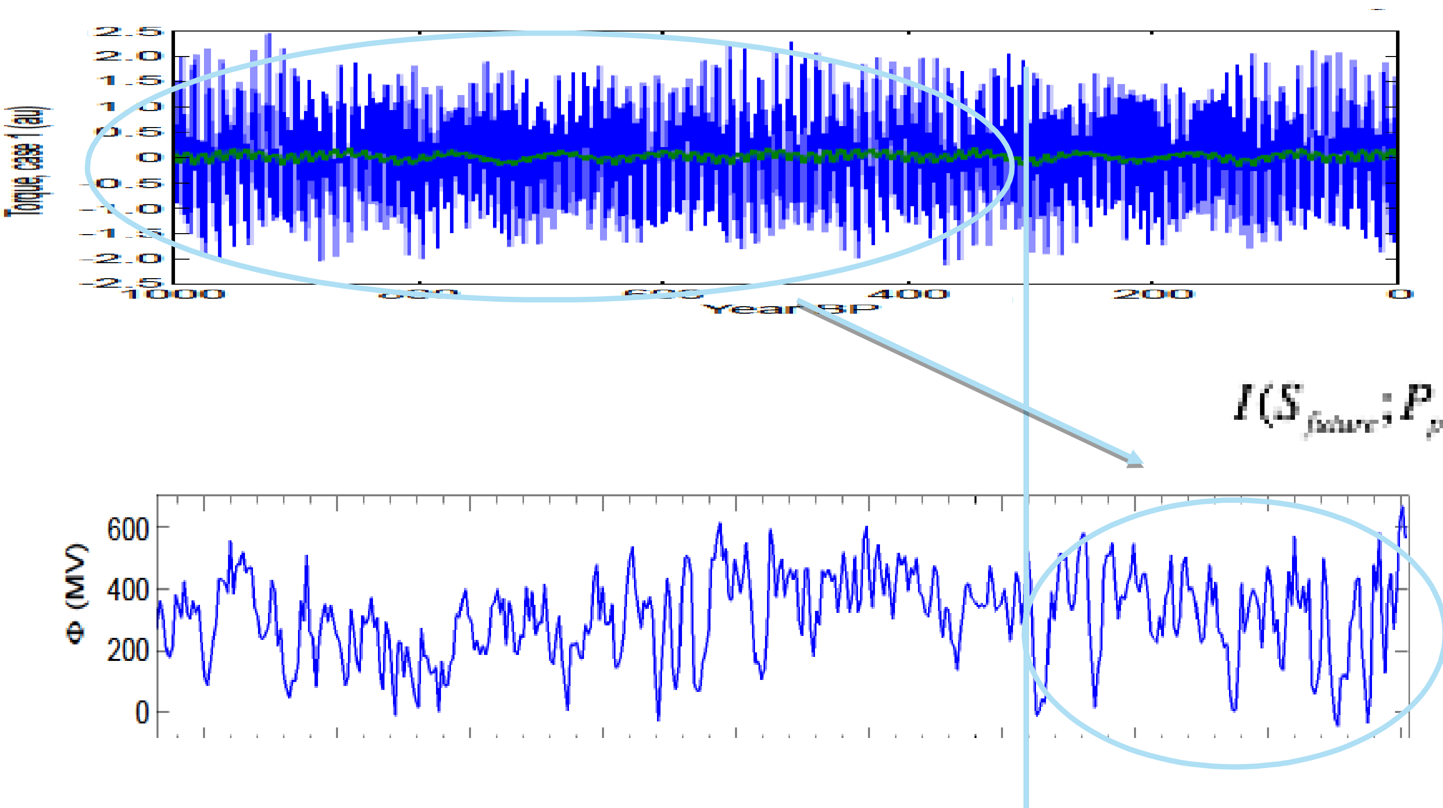
Coherence of **instantaneous phases** of planetary torques (P) and solar activity (S) time series, for each period:



Palus 2016 - unpublished!

Phase causality

Transfer entropy *P* -> *S* (Schreiber 2000) measures the information content in the past of time series P about the future of time series *S* that is not already contained in the past of time series *S*.

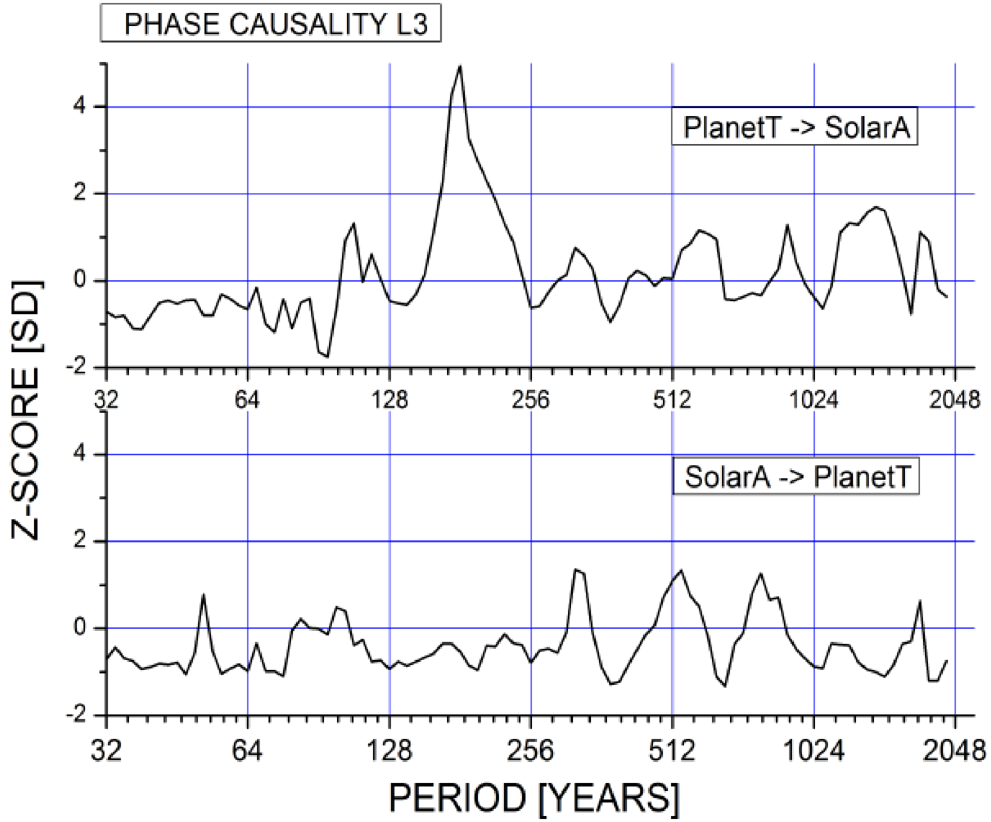


P

$$I(S_{future}; P_{past} \mid S_{past})$$

S

Phase causality



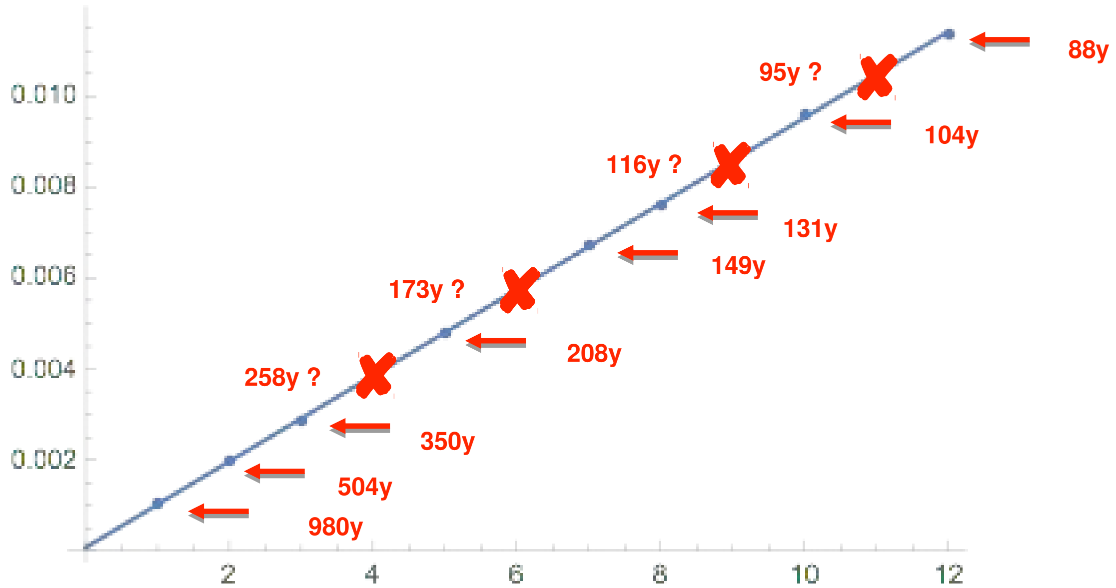
Palus 2016 - unpublished!

- How important is the **averaging**? (For the above analysis we need synchronous time-series -> 5y averaging)
- What are the **mechanisms** causing these cycles/couplings?

Mechanisms behind the cycles

Cycles follow a pattern!

$$\omega_n = \beta n$$



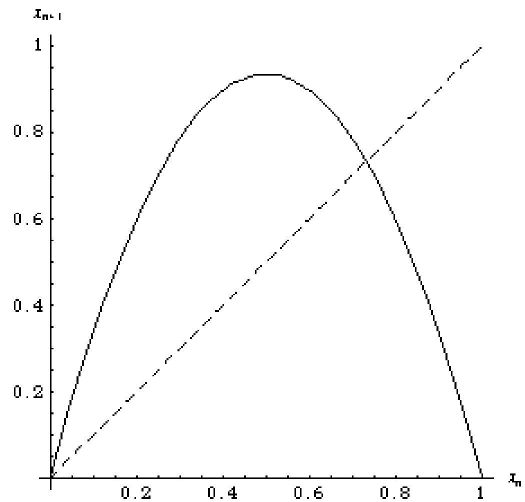
Abreu 2014 - unpublished!

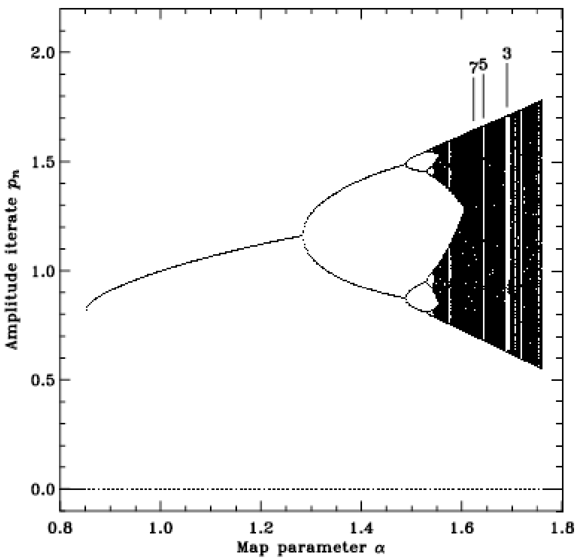
Mechanisms behind the cycles

This is the pattern we expect to see in a system on a **period-doubling route to chaos**!

The simplest example is an iterative map with a single-hump shaped function:

$$p_{n+1} = \alpha f(p_n)$$





After N bifurcations, the spectrum consists of peaks at:

$$\omega_n = n\frac{\omega_0}{2^N}, \quad n = 1, \ldots, 2^N .$$

Mechanisms behind the cycles

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FLUCTUATIONS IN BABCOCK-LEIGHTON DYNAMOS. I. PERIOD DOUBLING AND TRANSITION TO CHAOS

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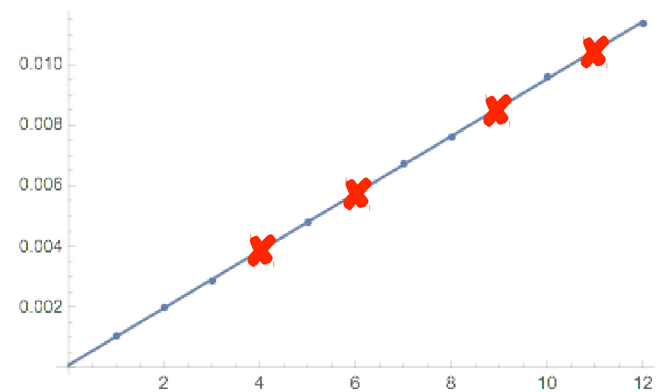
$$p_{n+1} = \alpha f(p_n)$$

$$\omega_n = n\frac{\omega_0}{2^N}, \quad n \le 2^N .$$



Possible solution, compatible with data: $N = 7, \quad \omega_0^{-1} = 8.27\,y.$ “Edge of chaos”

Do we still need the planets to explain all the cycles?

Bayesian inference and model selection

Infer model parameters (Bayesian inference) and compare different models (Bayesian model selection) based on records of solar activity.

Observational features:

- Long periods
- Gnevyshev-Ohl rule
- Intermittency (Grand Minima)
- …

A more realistic, stochastic, version of the iterative map model (Charbonneau 2001):

$$p_{n+1} = \alpha_n f(p_n) + \epsilon_n$$

Fluctuations in emergence rates of sunspots

Fluctuations from turbulent convection

Bayesian inference and model selection

Bayesian inference with nonlinear stochastic models is **expensive**, because the likelihood function, $L(\boldsymbol{y}|\theta)$, is typically expensive to evaluate, for a given set of model parameters θ, and observed data $\boldsymbol{y}$ (time-series of solar activity, or certain features thereof like periods, etc.).

Possible solution: **Approximate Bayes Computation (ABC)**:

Don't evaluate the likelihood function, but simulate model outputs, ***ysim,*** for given parameters θ and compare them with the observations ***yobs***. Depending on how good the match is, accept or reject θ and iterate the procedure.

- **BISTOM** project, funded by the Swiss Data Science Center.

CSCS
Centro Svizzero di Calcolo Scientifico
Swiss National Supercomputing Centre

SDSC

BISTOM project

- Infer parameters of stochastic iterative map dynamo model based on:
  - Certain features of the data (ABC)
  - Full time-series (HMC)

- Do the same, but with the 1+1D dynamo model by Schmitt et al. 1996

→ Does predictive power of models increase if we add planetary forcing?