Can stochastic resonance explain the amplification of planetary tidal forcing?

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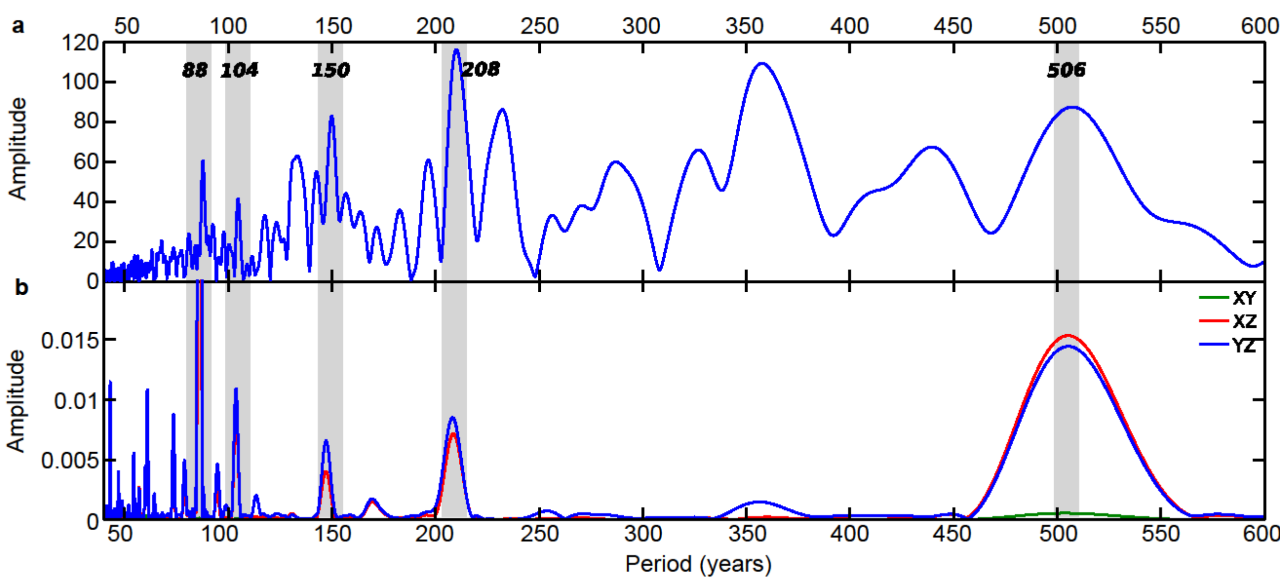
Long-period cycles in solar activity

Gleissberg de Vries



Solar activity from C^{14} and Be^{10} proxies.

Planetary torque

➡ Hypothesis of planetary influence

Abreu et al. 2012

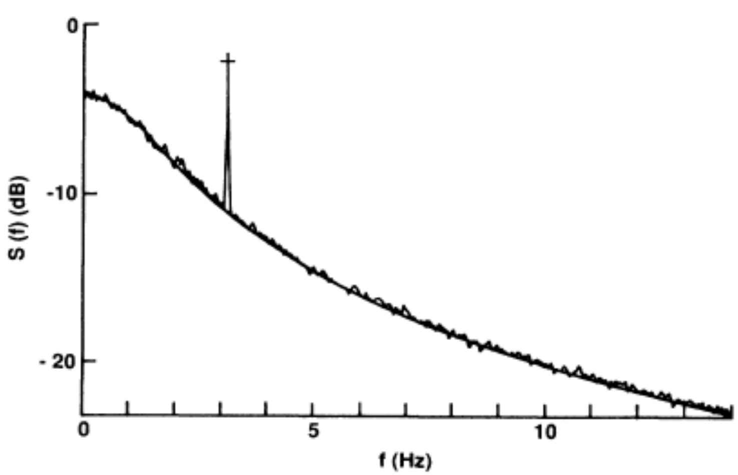
Stochastic Resonance

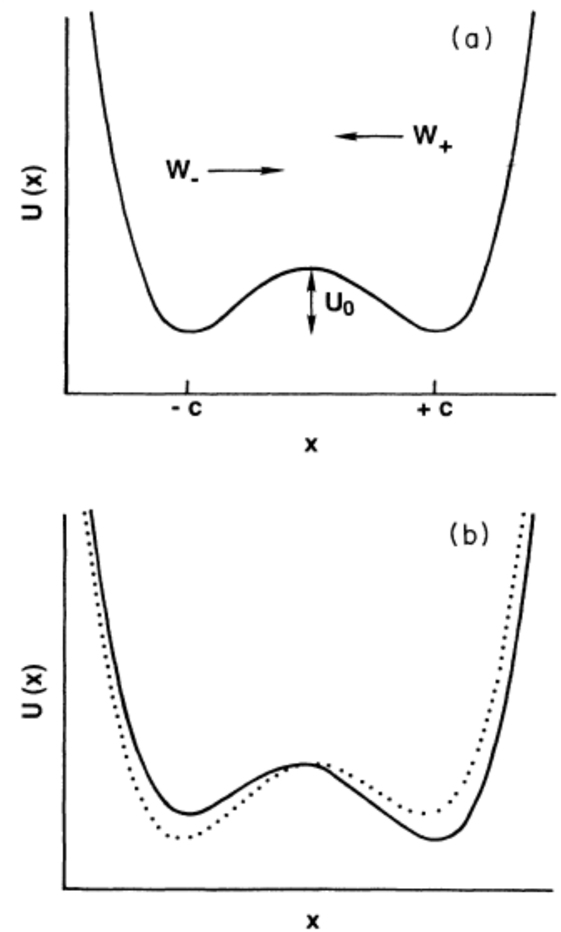
Transition probabilities:

$$W_{\pm}(t) = \frac{1}{2}(\alpha_0 \mp \alpha_1 \cos \omega_S t)$$

Output power spectrum:

$$S(\Omega) = \left(1 - \frac{\alpha_1^2}{2(\alpha_0^2 + \omega_S^2)}\right)\left(\frac{4c^2\alpha_0}{\alpha_0^2 + \Omega^2}\right) + \frac{\pi c^2 \alpha_1^2}{\alpha_0^2 + \omega_S^2}\delta(\Omega - \omega_S)$$





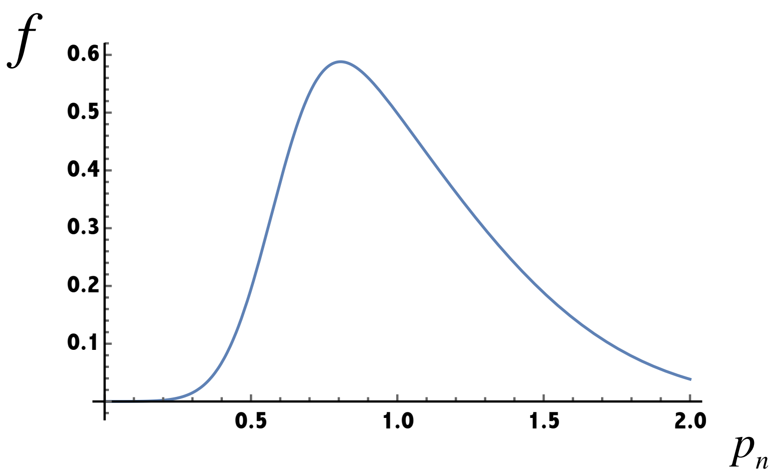
McNamara Wiesenfeld 1989

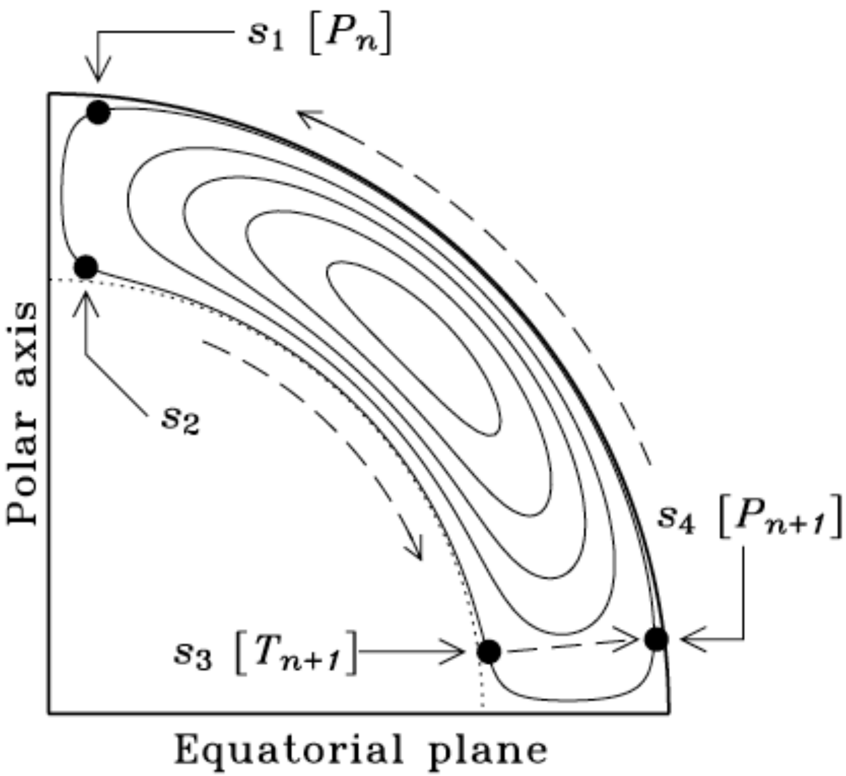
Babcock-Leighton-type dynamo models

$$T_{n+1} = aP_n\,,$$
$$P_{n+1} = f(T_{n+1})T_{n+1}\,,$$

$$\Rightarrow \quad p_{n+1} = \alpha f(p_n)p_n$$

$$f(T) = \frac{1}{4}\left(1+\mathrm{erf}\left(\frac{T-B_{min}}{w_1}\right)\right)\left(1-\mathrm{erf}\left(\frac{T-B_{\max}}{w_2}\right)\right).$$

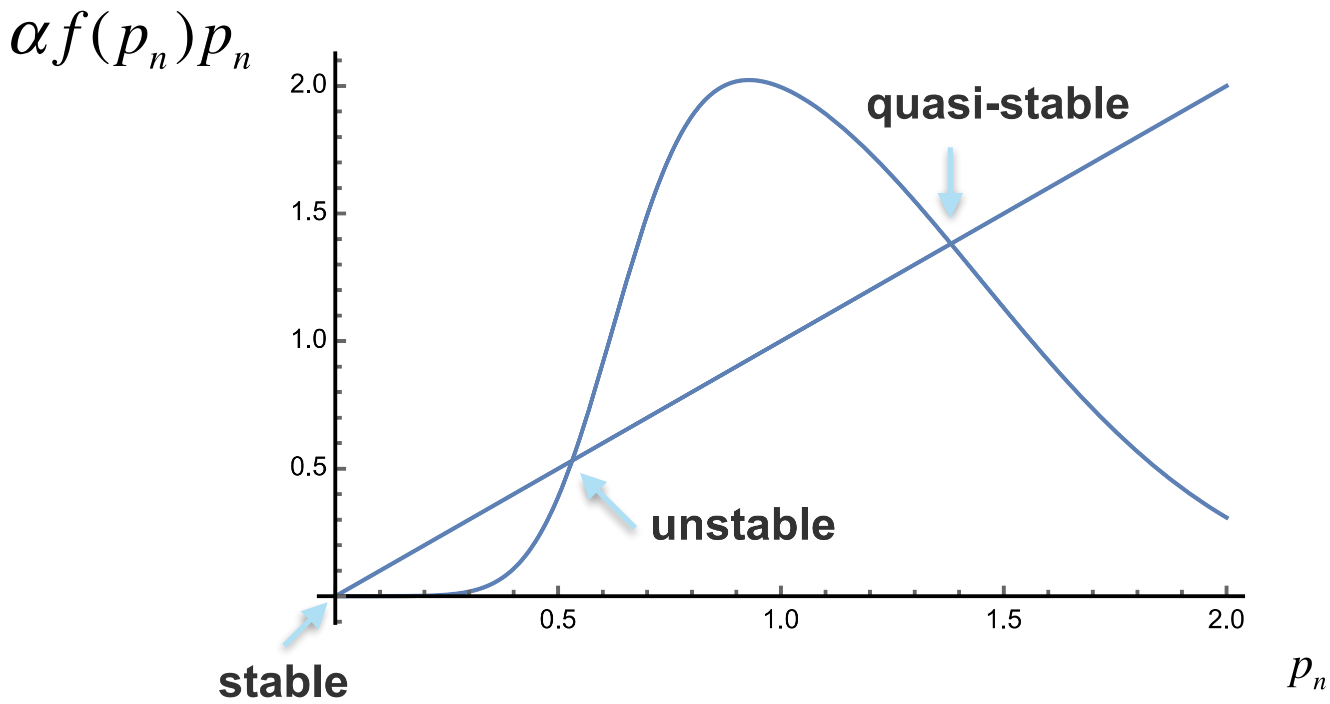




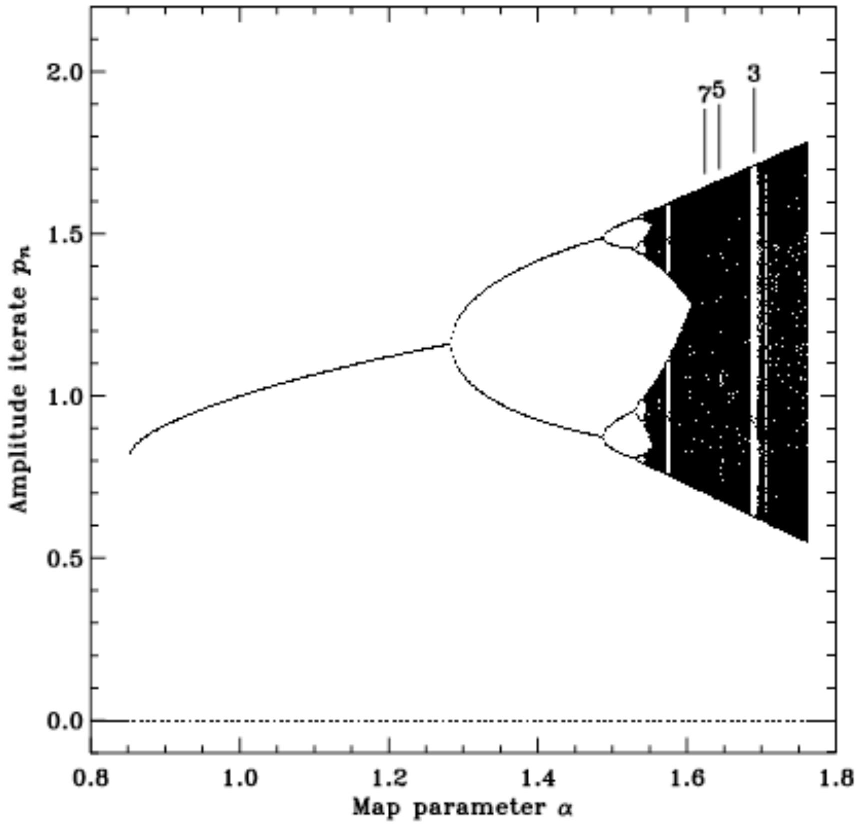
Charbonneau et al 2005

Babcock-Leighton-type dynamo models

$$p_{n+1} = \alpha f(p_n) p_n$$



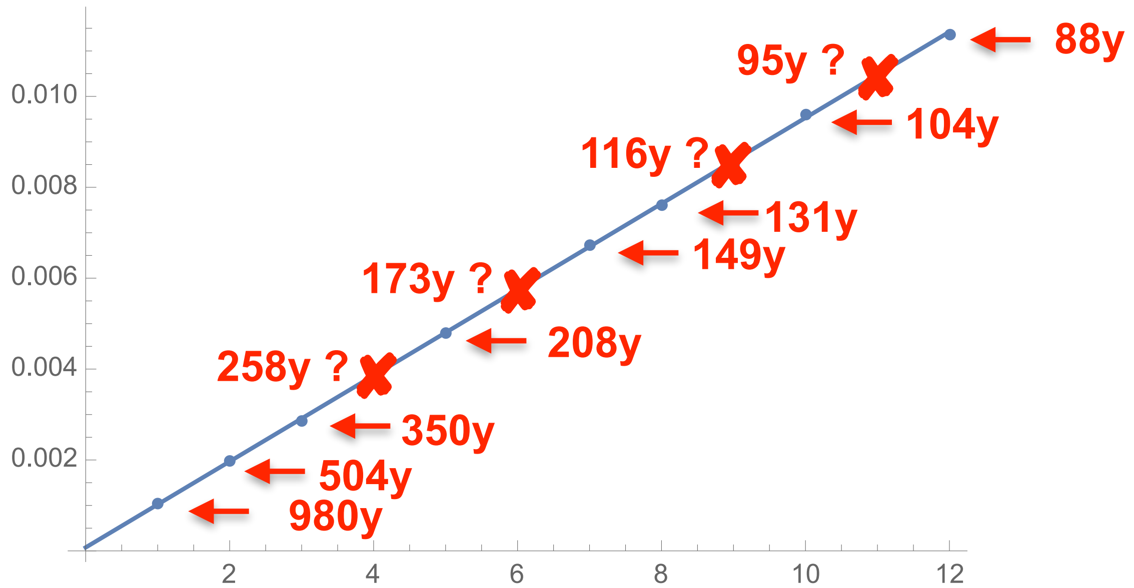
Babcock-Leighton-type dynamo models



Mechanisms behind the cycles

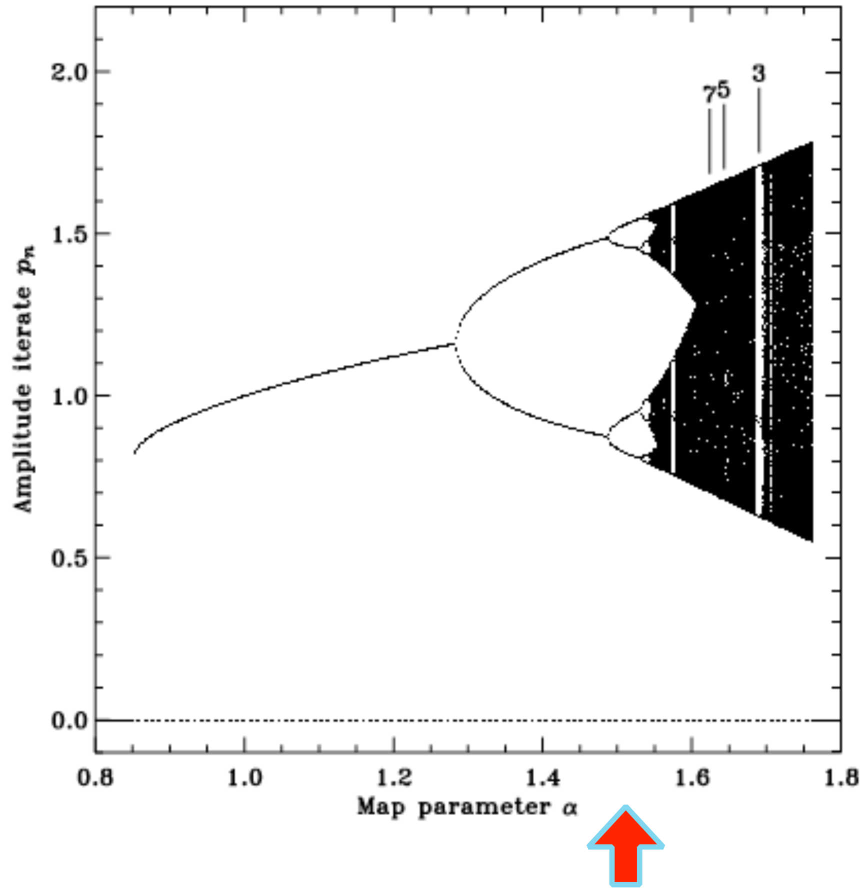
Cycles follow a pattern!

$$\omega_n \approx \beta n$$



Abreu 2014 - unpublished!

Babcock-Leighton-type dynamo models



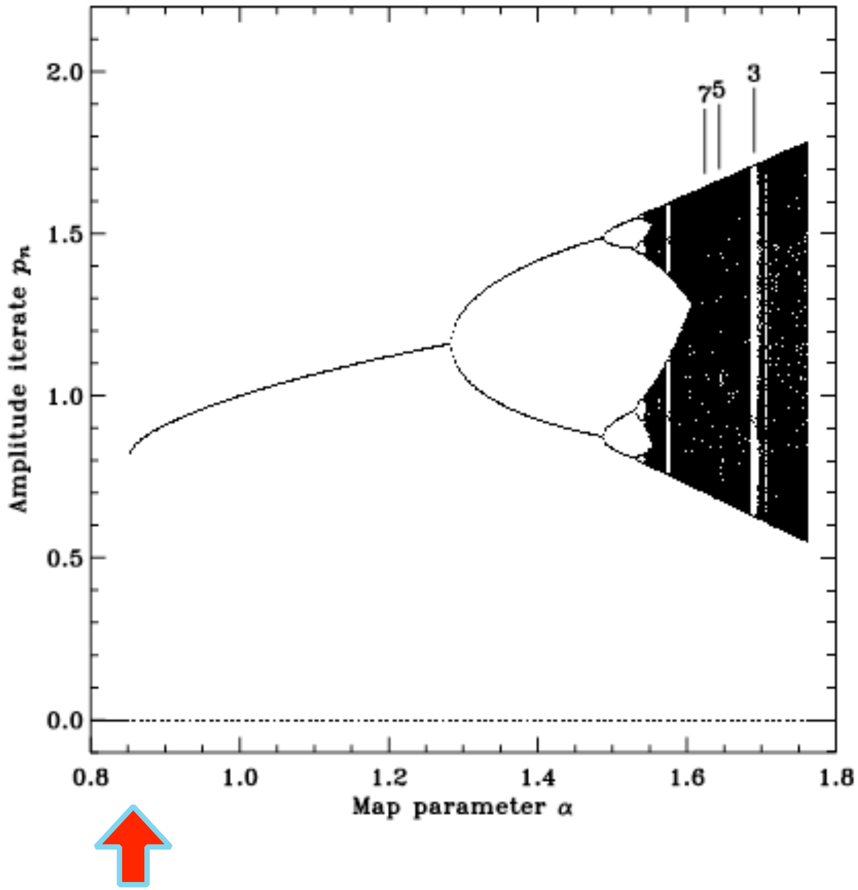
Hypothesis 1:
Cycles generated **intrinsically**

$$\omega_n = n\frac{\omega_0}{2^N}, \quad n \le 2^N .$$

$$N = 7, \quad {\omega_0}^{-1} = 8.27\,\mathrm{y} .$$

Very fine tuned to chaos transition!

Babcock-Leighton-type dynamo models

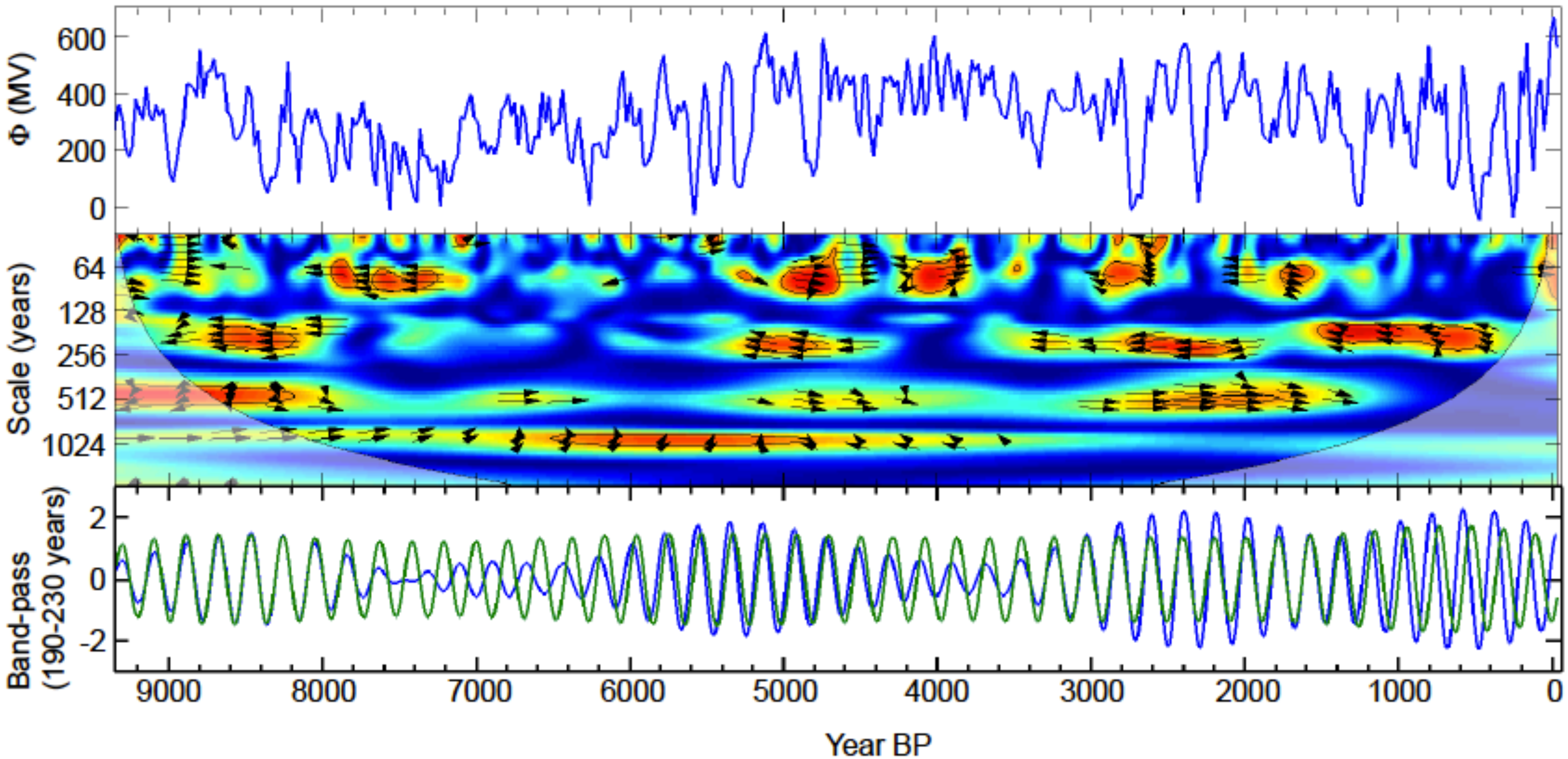


Hypothesis 2:
Cycles generated **extrinsically**

First transition from quiescent to oscillatory state.

eaawag aquatic research

Babcock-Leighton-type dynamo models

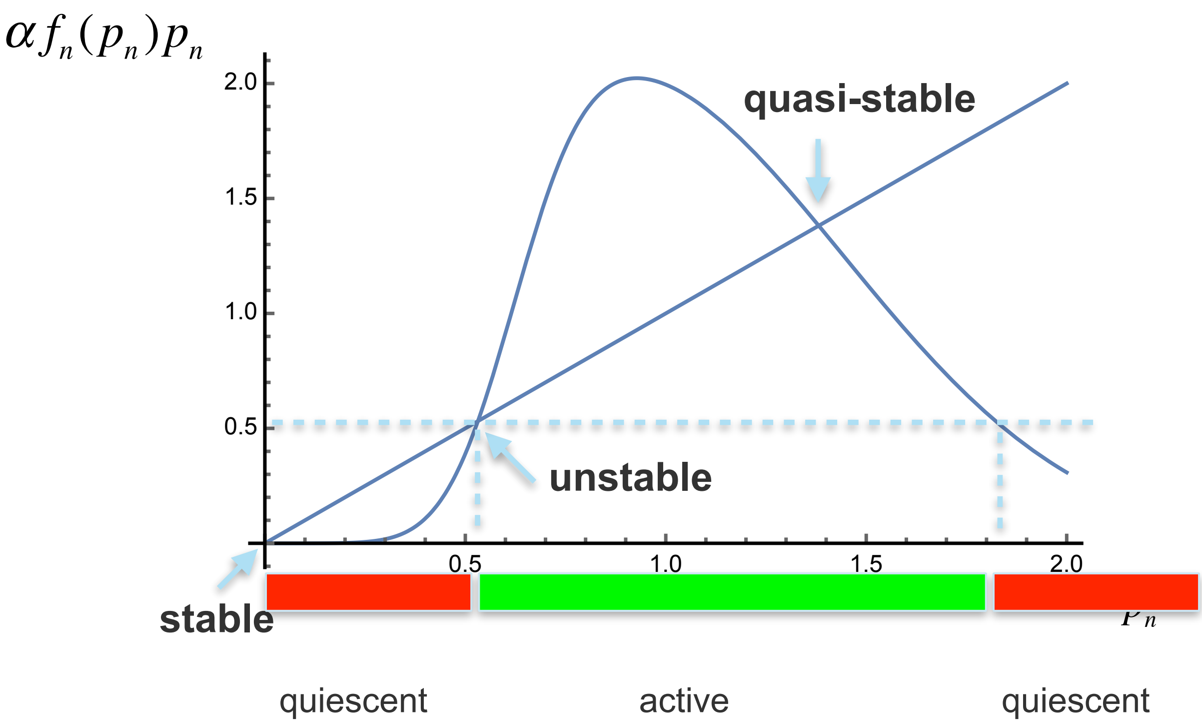


Abreu et al. 2012

Stochastic Resonance in Babcock-Leighton dynamos?

$$p_{n+1} = \alpha f_n(p_n) p_n + \epsilon_n \,, \quad \epsilon_n \sim \mathcal{U}[0,\epsilon],$$

$$f_n(p_n) = \frac{1}{4}\left(1 + \mathrm{erf}\left(\frac{p_n - B_{min}(1 + A\cos(22\pi n / P))}{w_1}\right)\right)\left(1 - \mathrm{erf}\left(\frac{p_n - B_{\max}}{w_2}\right)\right).$$



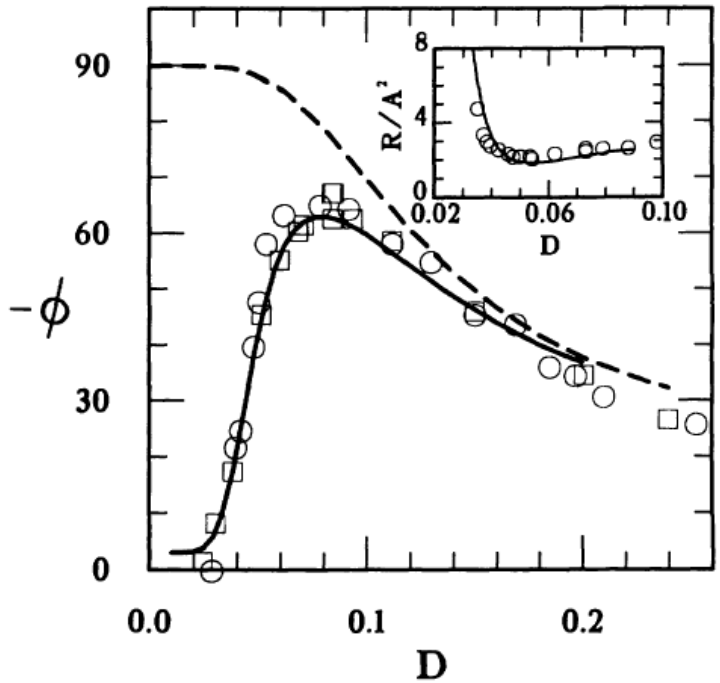
Summary

Evidence for stochastic resonance:

- Bi-stability of solar dynamo: Grand Minima
- Strong cycles in solar activity that coincide with cycles of weak external driver (planets)
- BL-type dynamos exhibit critical transition points of high susceptibility
- Some evidence that longer-period cycles are amplified stronger

To do

- Test “criticality hypotheses” with **data** (sunspots, proxies from radionuclides) using Bayesian inference (see Simone’s talk).
- Further evidence for SR from **phases**?



Dykman et al. 1992