Role of helicity in Solar and Stellar Dynamos

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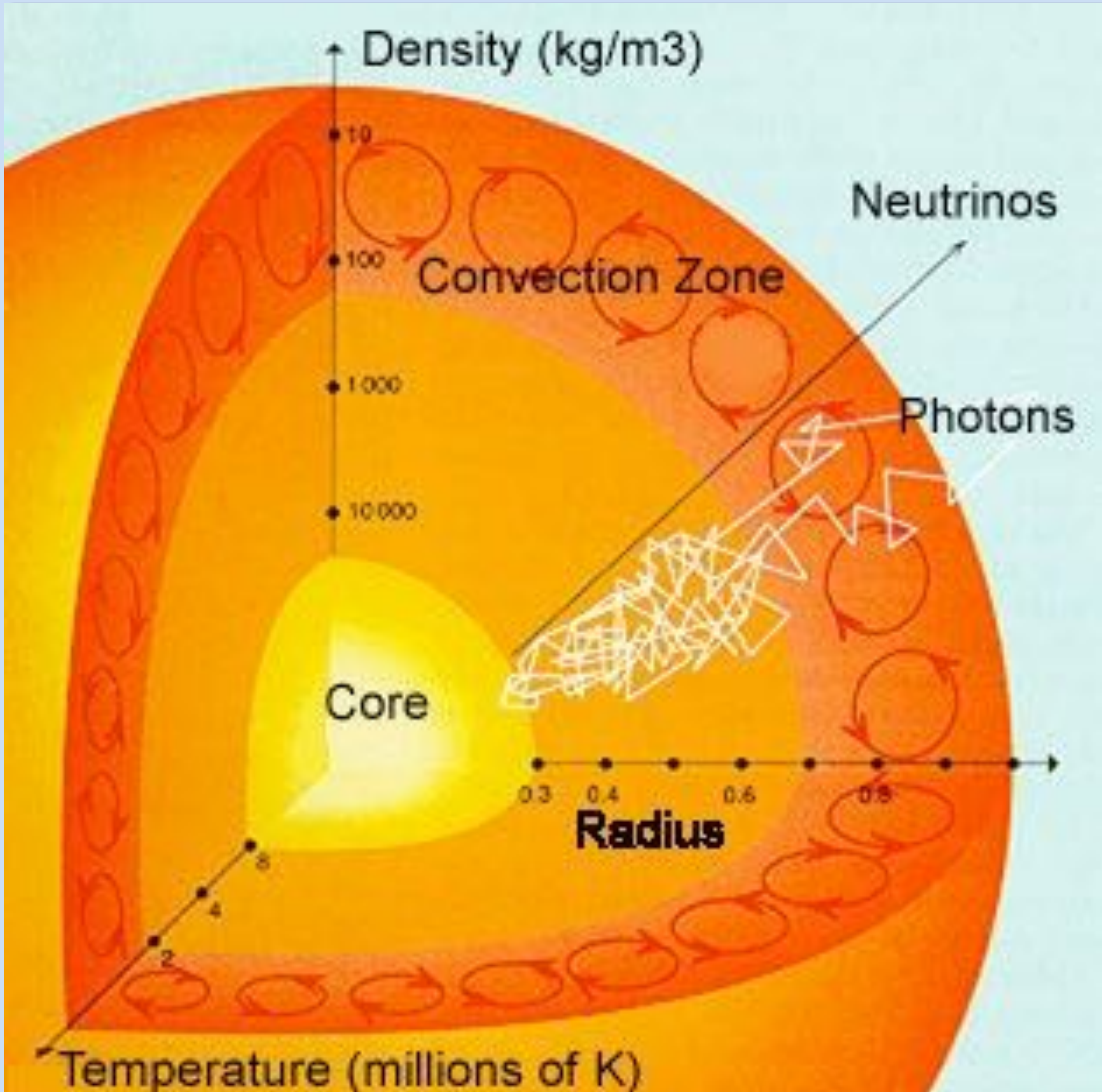
MPS

Helicity

The glue to connect them all.

Helicity in the Sun



Nonalignment of rotation and gravity

↓

Kinetic helicity

↓

Alpha-effect

↓

Magnetic helicity + catastr. quenching

↓

Space weather coronal heating

Dynamos

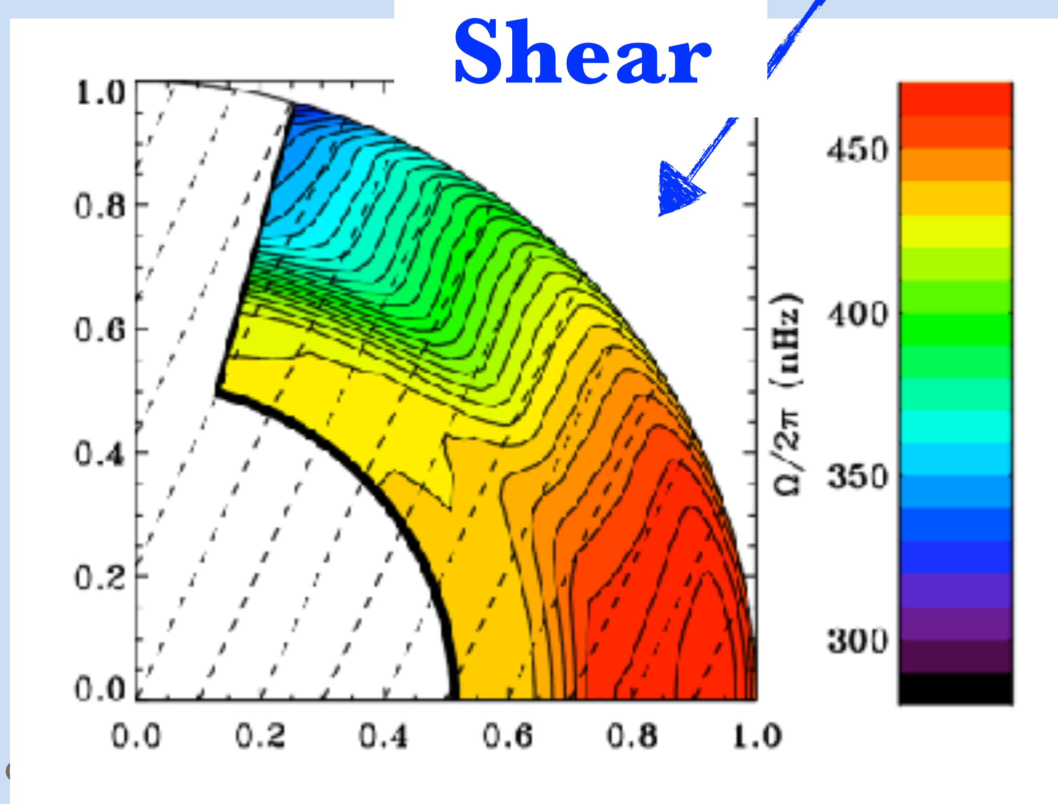
$$\frac{\partial B}{\partial t} = \nabla \times (u \times B) - \nabla \times \eta J$$

$$B = \overline{B} + b' \qquad u = \overline{U} + u'$$

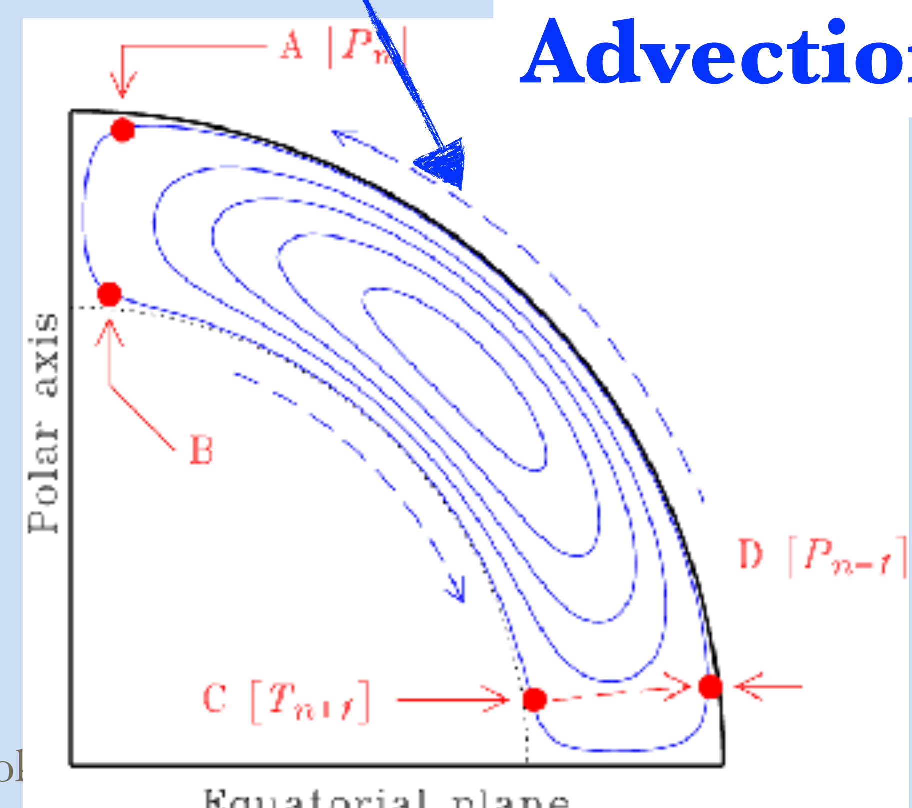
$$\frac{\partial \overline{B}}{\partial t} = \nabla \times (\overline{U} \times \overline{B} + \overline{u' \times b'}) - \nabla \times \eta \overline{J}$$

$$\nabla \times (\overline{U} \times \overline{B}) = (\overline{B} \cdot \nabla)\overline{U} - \overline{B}(\nabla \cdot \overline{U}) - (\overline{U} \cdot \nabla)\overline{B}$$

Shear



Advection



Alpha Omega Dynamo

$$\overline{\mathcal{E}} = \overline{u' \times b'}$$

Simplifications:

$$= \alpha \overline{B} + \eta_t \nabla \times \overline{B}$$

$$\alpha = \frac{\tau_c}{3}\left(-\overline{\boldsymbol{\omega}\cdot\boldsymbol{u}} + \frac{\overline{\boldsymbol{j}\cdot\boldsymbol{b}}}{\overline{\rho}}\right)$$

$$\frac{\partial \overline{B}_{\rm pol}}{\partial t} = \alpha \nabla \times \overline{B}_{\rm tor} + \eta_T \Delta \overline{B}_{\rm pol}$$

$$\overline{B} = \overline{B}_{\rm pol} + \overline{B}_{\rm tor}$$

$$\frac{\partial \overline{B}_{\rm tor}}{\partial t} = (\overline{B}_{\rm pol}\cdot\nabla)\overline{u}_{\rm tor} + \alpha \nabla \times \overline{B}_{\rm pol} + \eta_T \Delta \overline{B}_{\rm tor}$$

$\alpha\Omega$ dynamo

α^2 dynamo

$$\Omega = \overline{u}_{\rm tor}/r\sin\theta$$

Electromotive force

$$\boldsymbol{\mathcal{E}} = \boldsymbol{a} \cdot \overline{\boldsymbol{B}} + \boldsymbol{b} \cdot \nabla \overline{\boldsymbol{B}} + \ldots$$

$$\mathcal{E}_i = a_{ij} \overline{B}_j + b_{ijk} \partial_j \overline{B}_k + \ldots$$

$$\boldsymbol{\mathcal{E}} = \boldsymbol{\alpha} \cdot \overline{\boldsymbol{B}} + \boldsymbol{\gamma} \times \overline{\boldsymbol{B}} - \boldsymbol{\beta} \cdot (\nabla \times \overline{\boldsymbol{B}}) - \boldsymbol{\delta} \times (\nabla \times \overline{\boldsymbol{B}}) - \boldsymbol{\kappa} \cdot (\nabla \overline{\boldsymbol{B}})^{(S)}$$

Test-field method

Schrinner et al. 2005, 2007, 2012

$$\frac{\partial \overline{\boldsymbol{B}}}{\partial t} = \boldsymbol{\nabla} \times (\overline{\boldsymbol{u}} \times \overline{\boldsymbol{B}} + \overline{\boldsymbol{u}' \times \boldsymbol{B}'}) - \boldsymbol{\nabla} \times \eta \boldsymbol{\nabla} \times \overline{\boldsymbol{B}},$$

$$\boldsymbol{\mathcal{E}} = \boldsymbol{\alpha} \cdot \overline{\boldsymbol{B}} + \boldsymbol{\gamma} \times \overline{\boldsymbol{B}} - \boldsymbol{\beta} \cdot (\boldsymbol{\nabla} \times \overline{\boldsymbol{B}}) - \boldsymbol{\delta} \times (\boldsymbol{\nabla} \times \overline{\boldsymbol{B}}) - \boldsymbol{\kappa} \cdot (\boldsymbol{\nabla} \overline{\boldsymbol{B}})^{(S)}$$

$$\frac{\partial \boldsymbol{b}'_{\mathrm{T}}}{\partial t} = \boldsymbol{\nabla} \times \left(\boldsymbol{u}' \times \overline{\boldsymbol{B}}_{\mathrm{T}} + \overline{\boldsymbol{U}} \times \boldsymbol{b}'_{\mathrm{T}} + \boldsymbol{u}' \times \boldsymbol{b}'_{\mathrm{T}} - \overline{\boldsymbol{u}' \times \boldsymbol{b}'_{\mathrm{T}}} \right)$$
$$- \boldsymbol{\nabla} \times \eta \boldsymbol{\nabla} \times \boldsymbol{b}'_{\mathrm{T}}$$

The Simulation

Global convective dynamo simulations

$$\frac{\partial A}{\partial t} = u \times B + \eta \nabla^2 A$$

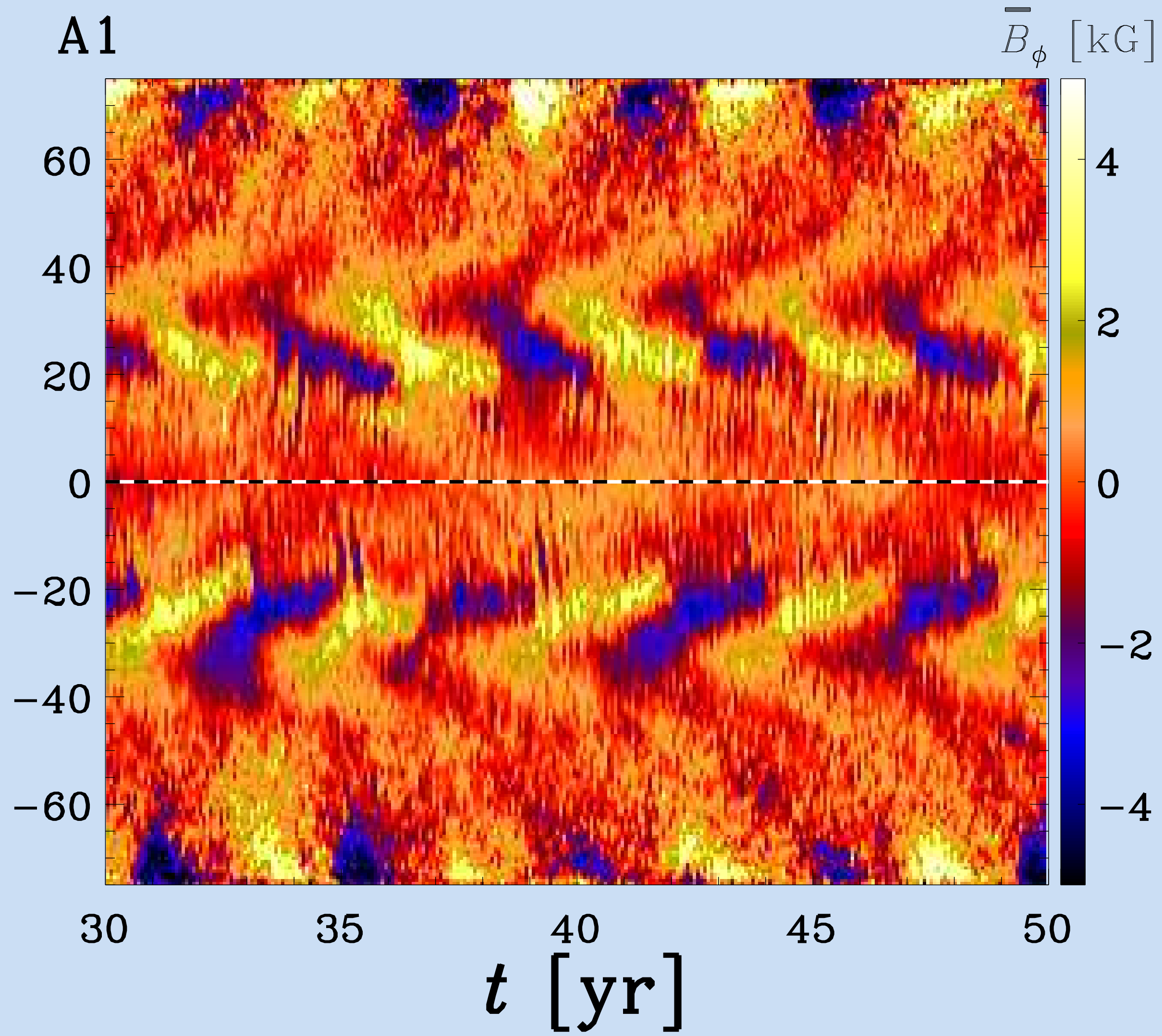
$$\frac{D \ln \rho}{Dt} = -\nabla \cdot u$$

$$\frac{Du}{Dt} = g - 2\Omega_0 \times u + \frac{1}{\rho}\left(J \times B - \nabla p + \nabla \cdot 2\nu\rho S\right)$$

$$T\frac{Ds}{Dt} = \frac{1}{\rho}\nabla \cdot \left(K\nabla T + \chi_t \rho T \nabla s\right) + 2\nu S^2 + \frac{\mu_0 \eta}{\rho} J^2 - \Gamma_{\rm cool}(r),$$

- high-order finite-difference code
- scales up efficiently to over 60.000 cores
- compressible MHD

https://github.com/pencil-code/pencil-code/

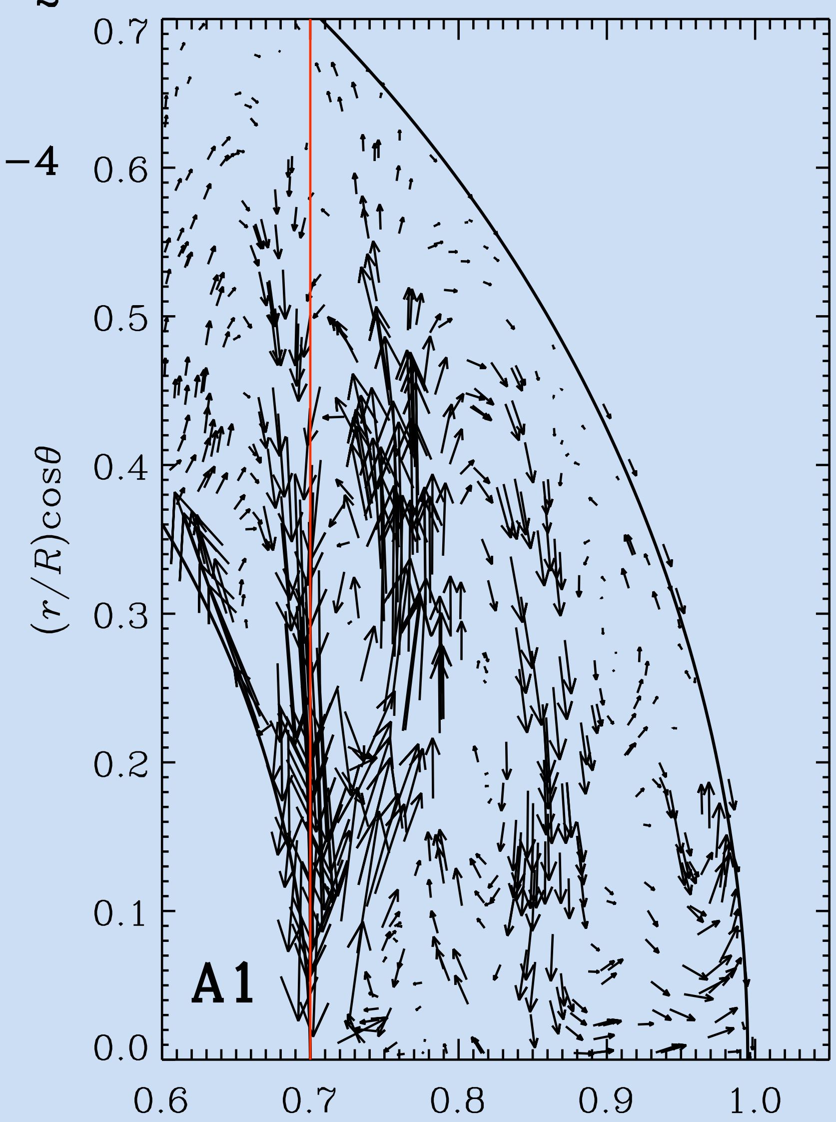


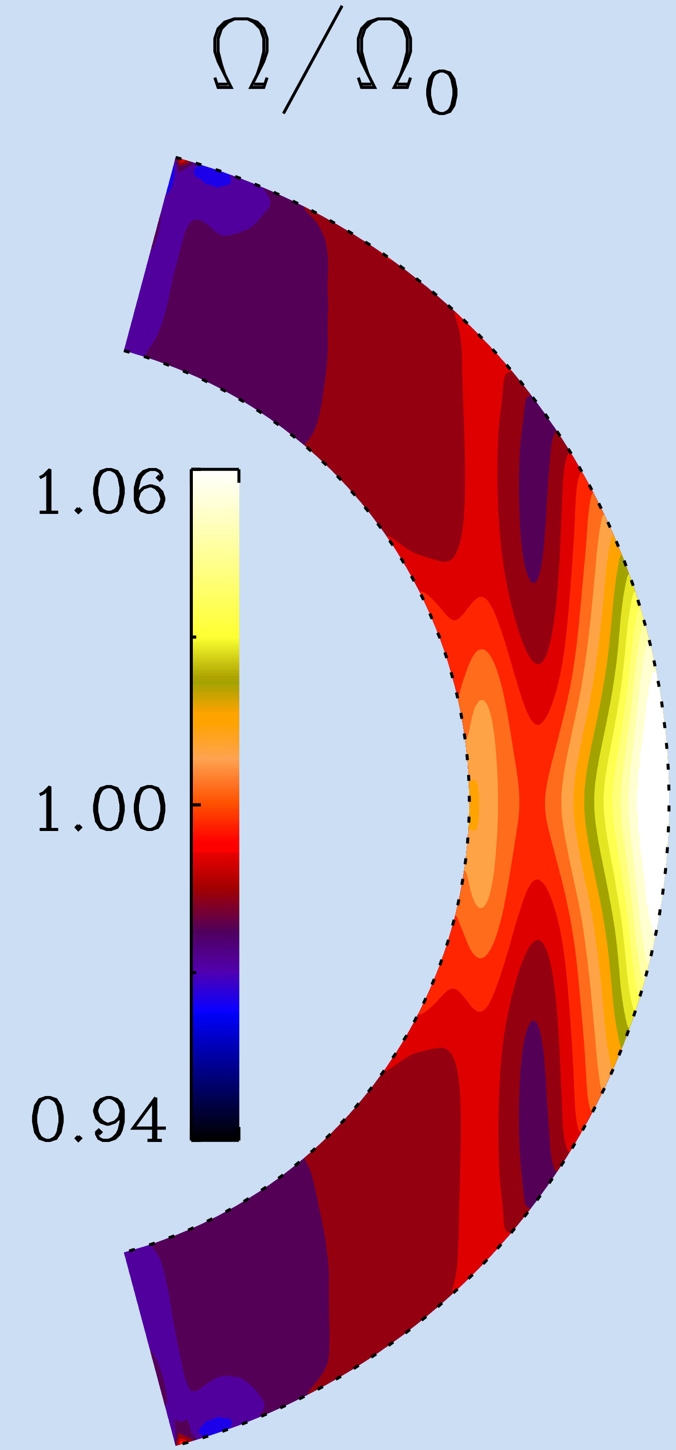
Pr=2 Pm=1

Re=Rm=30

Co=8.3

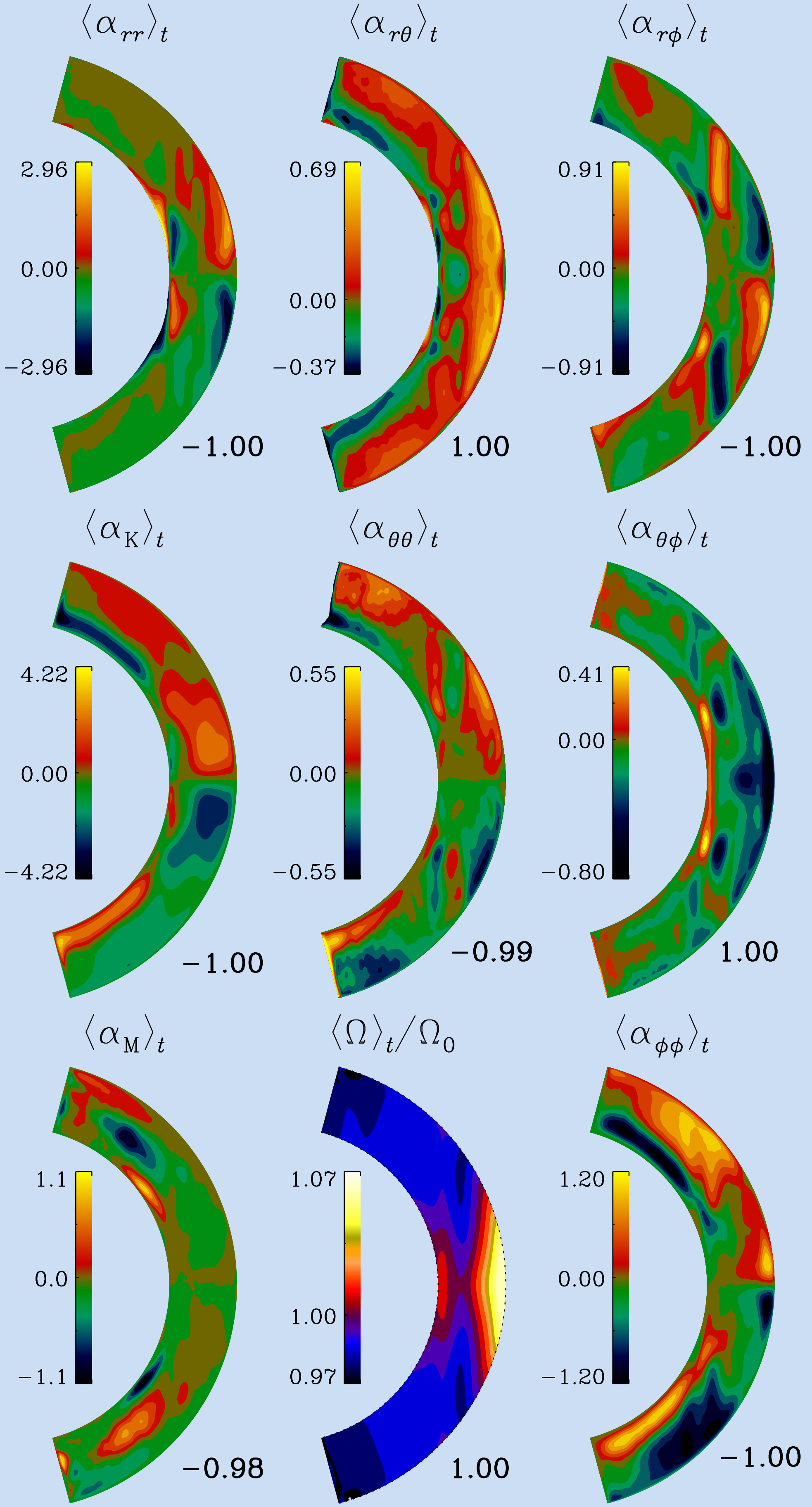
E=1.8x10^{-4}

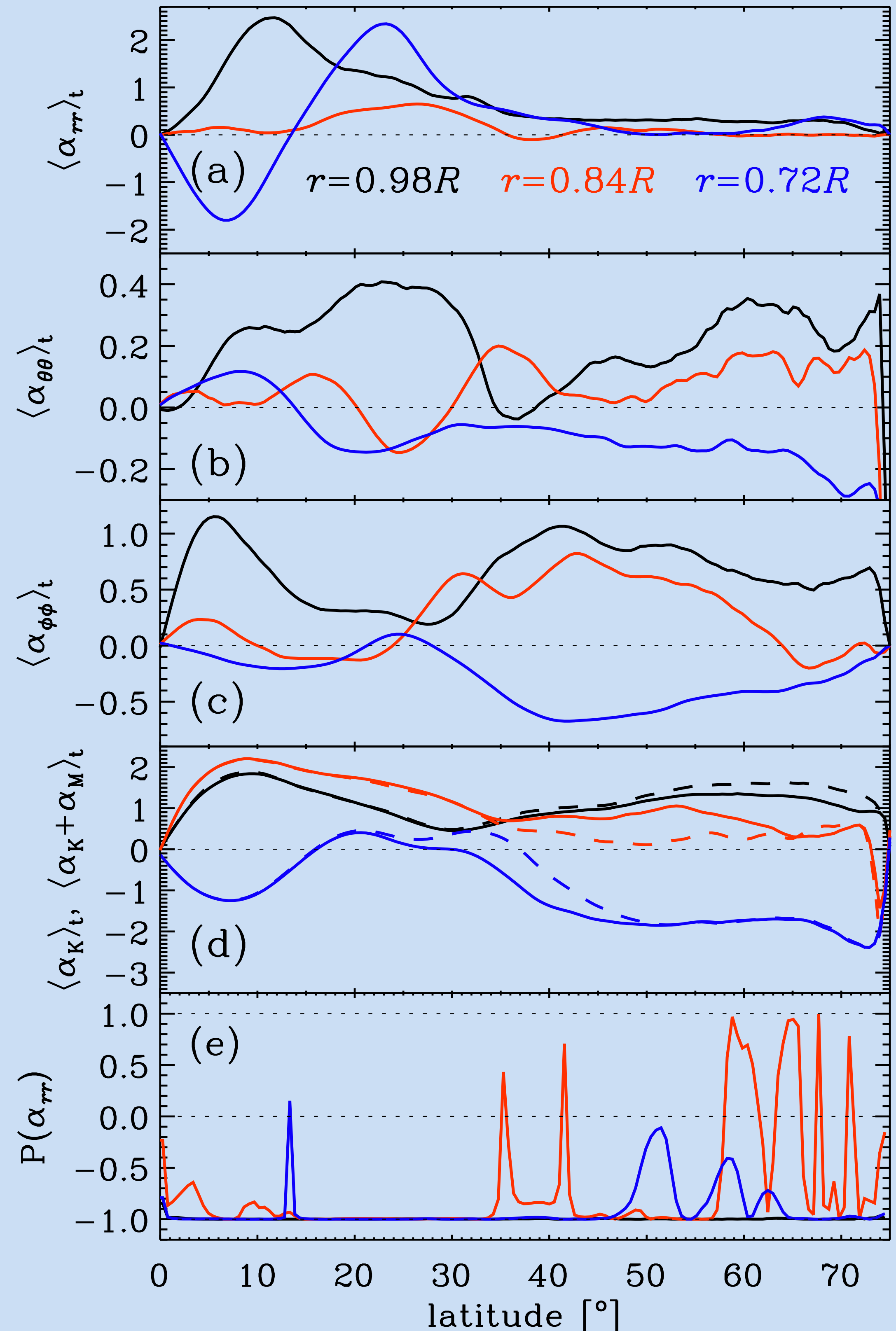




Käpylä et al.
2012, 2013, 2016, 2017

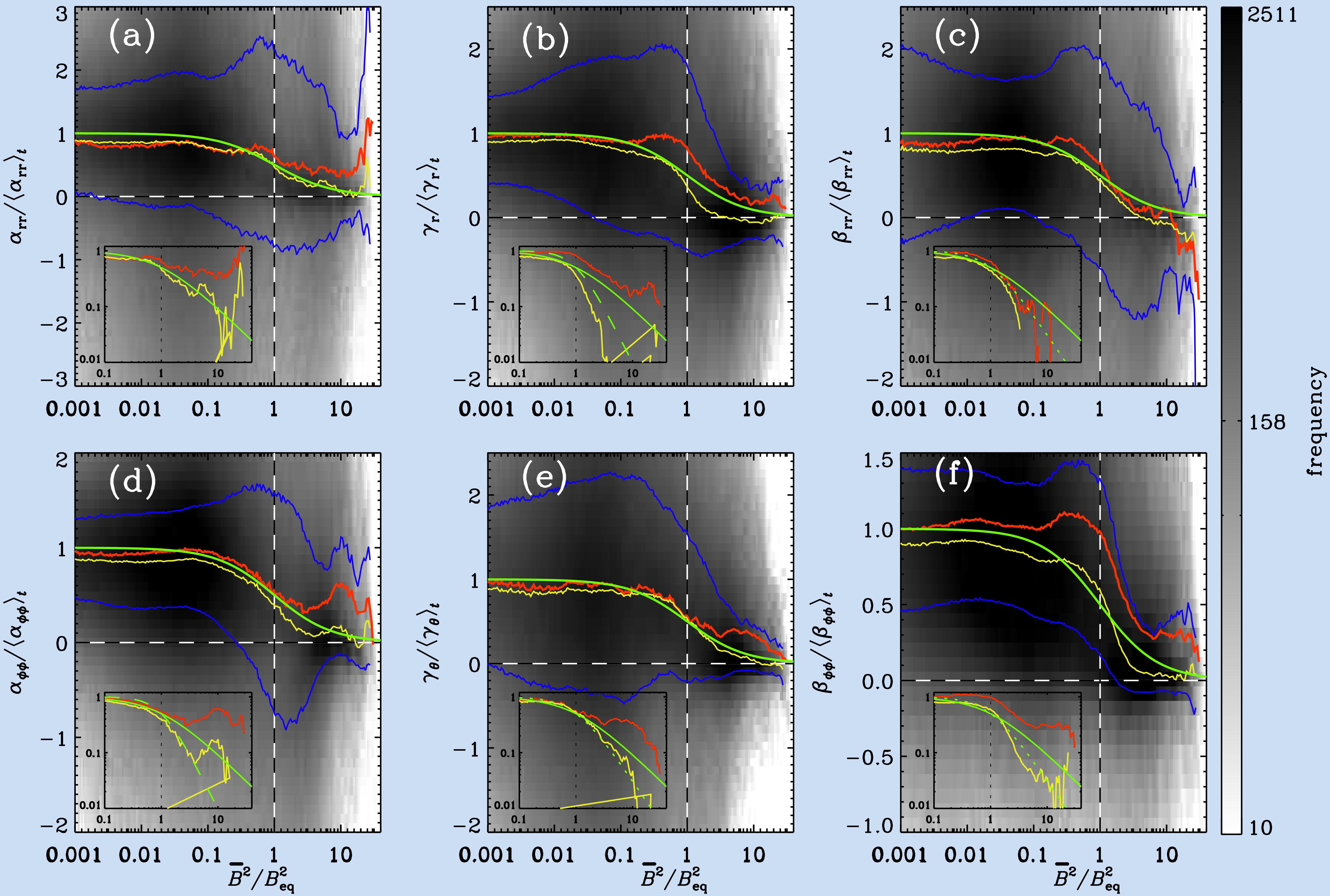
Warnecke et al.
2014, 2016

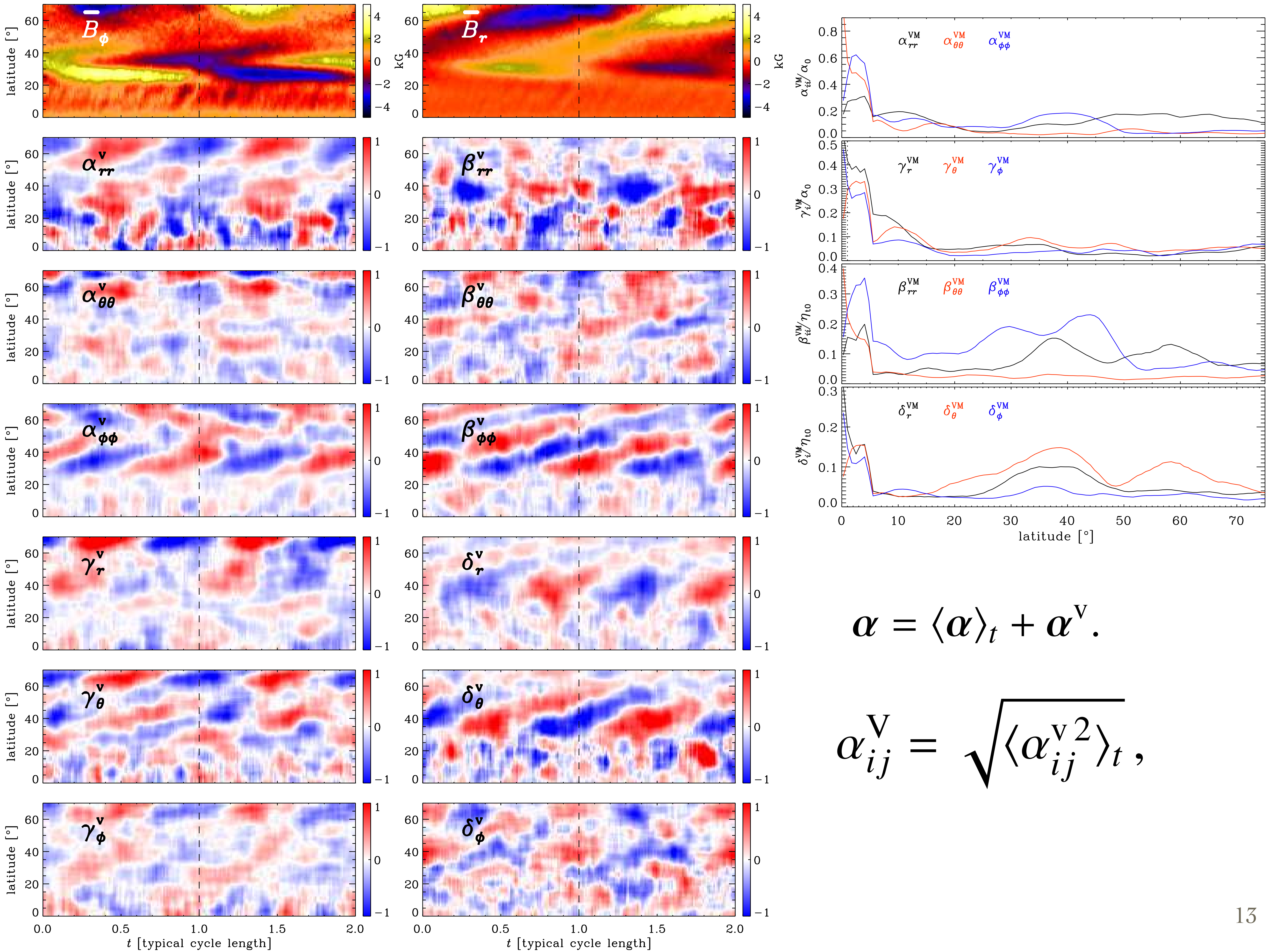




$$P(\alpha_{ij}) = \frac{\left(\alpha_{ij}^{\mathrm{es}}\right)^2 - \left(\alpha_{ij}^{\mathrm{ea}}\right)^2}{\left(\alpha_{ij}^{\mathrm{es}}\right)^2 + \left(\alpha_{ij}^{\mathrm{ea}}\right)^2},$$

Magnetic quenching

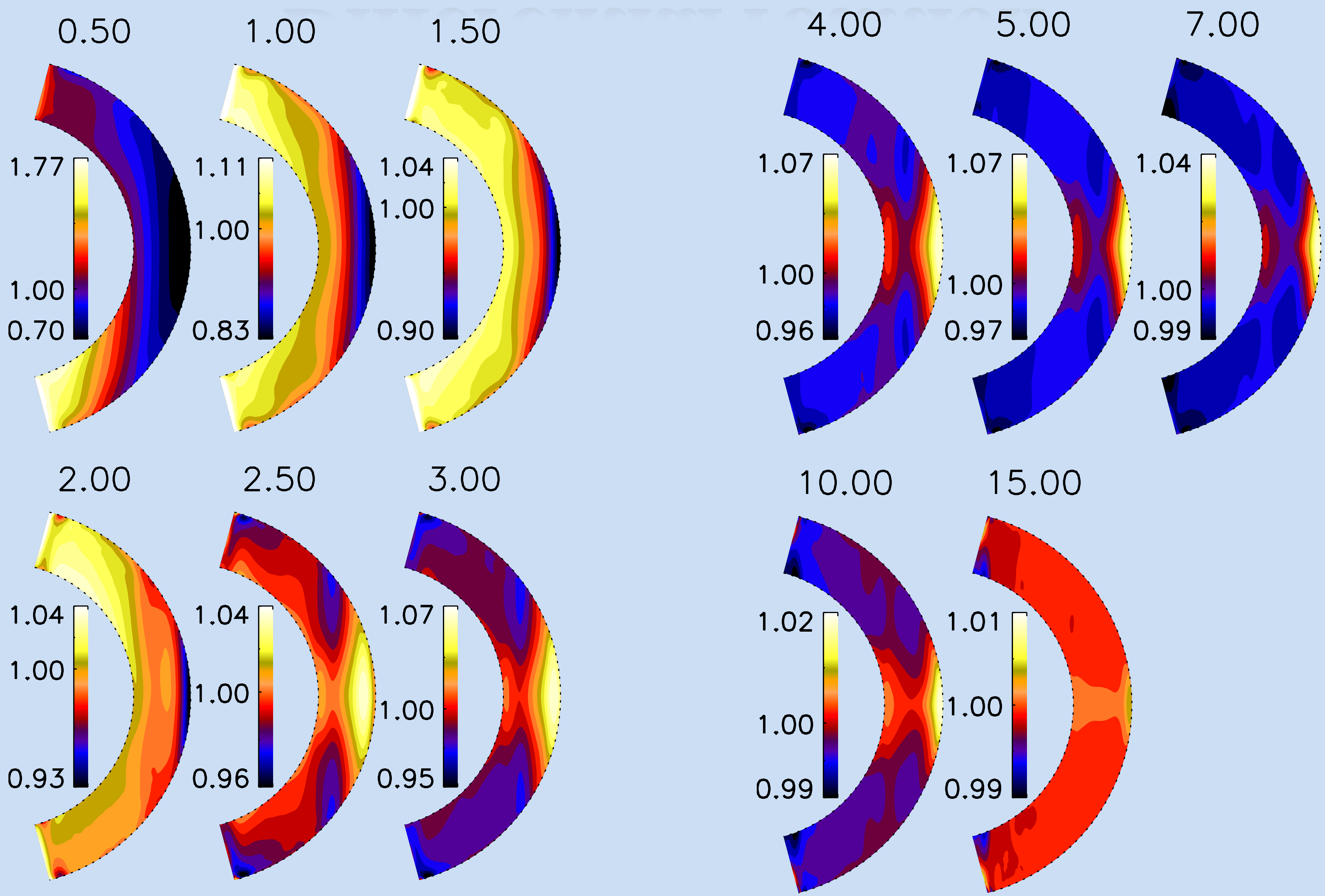




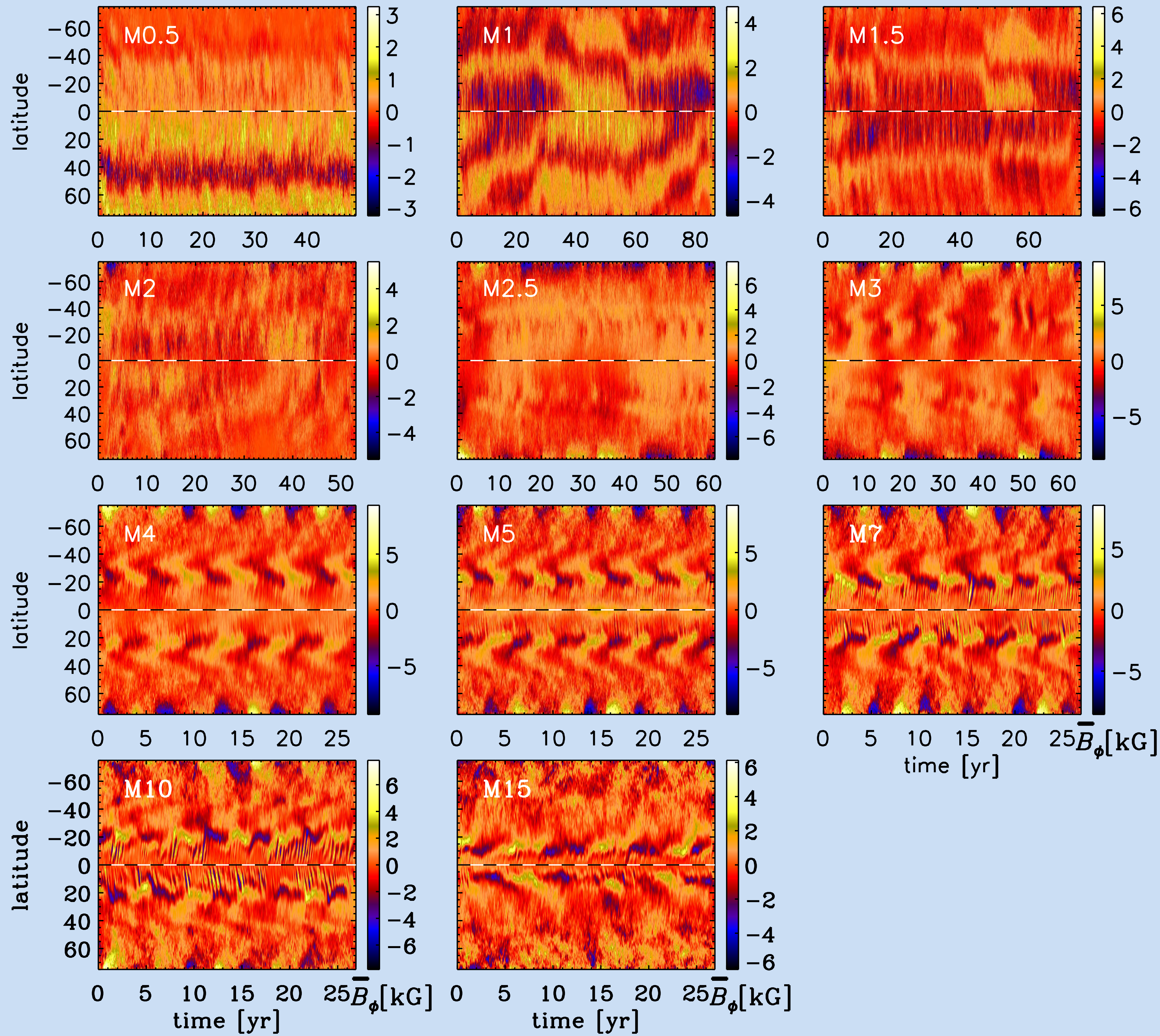
$$\alpha = \langle \alpha \rangle_t + \alpha^{\mathrm{v}}.$$

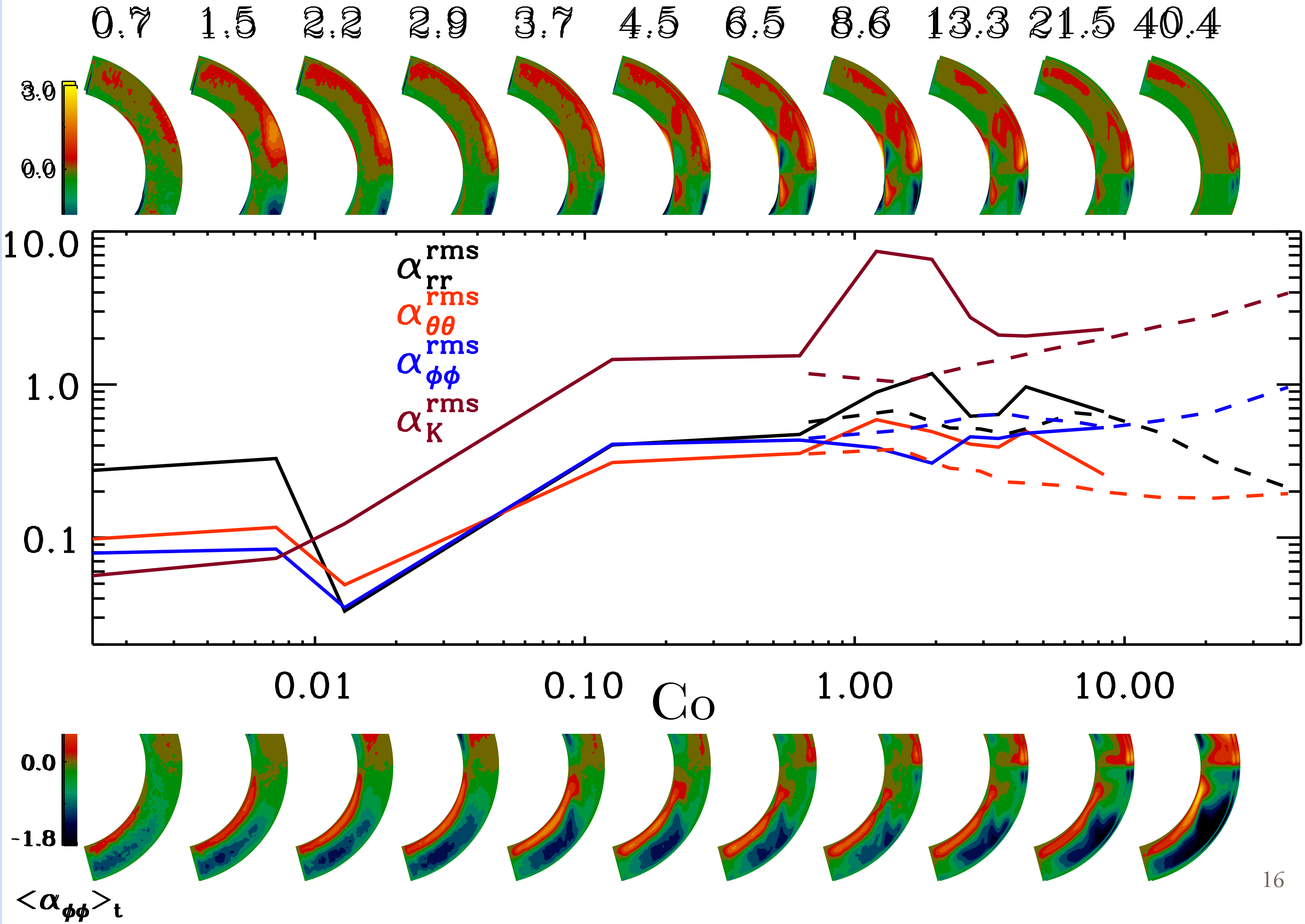
$$\alpha^{\mathrm{V}}_{ij} = \sqrt{\langle \alpha^{\mathrm{v}\,2}_{ij} \rangle_t}\,,$$

Differential rotation



Omega_sol=1 =>E=8x10^{-4}


M0.5
M1
M1.5
M2
M2.5
M3
M4
M5
M7
M10
M15
latitude
time [yr]
$\overline{B}_\phi$[kG]



0.7 1.5 2.2 2.9 3.7 4.5 6.5 8.6 13.3 21.5 40.4
3.0
0.0
10.0
1.0
0.1
α_{rr}^{rms}
$\alpha_{\theta\theta}^{rms}$
$\alpha_{\phi\phi}^{rms}$
α_{K}^{rms}
0.01
0.10
1.00
10.00
Co
0.0
-1.8
$\langle\alpha_{\phi\phi}\rangle_t$

Magnetic helicity fluxes

$$\alpha = -\frac{1}{3}\tau_c \overline{\boldsymbol{\omega}' \cdot \boldsymbol{u}'} + \frac{1}{3}\frac{\tau_c}{\overline{\rho}} \overline{\boldsymbol{J}' \cdot \boldsymbol{B}'} = \alpha_K + \alpha_M,$$

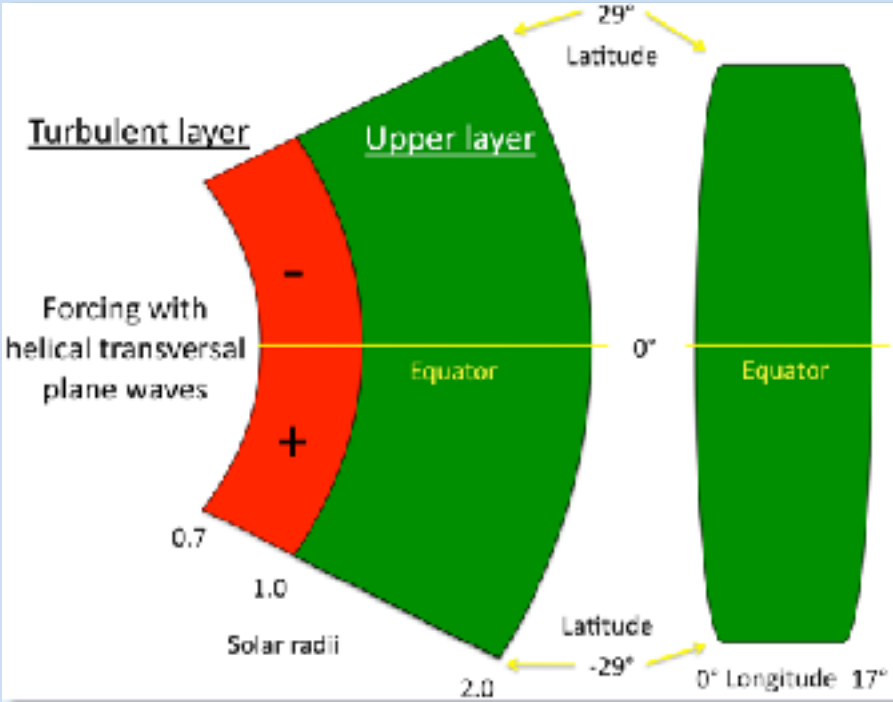
$$\frac{\partial \alpha_M}{\partial t} = -2\eta_t k_f^2 \left(\frac{\overline{\boldsymbol{u}' \times \boldsymbol{B}'} \cdot \overline{\boldsymbol{B}}}{B_{eq}^2} + \frac{\alpha_M}{Re_M} \right) - \boldsymbol{\nabla} \cdot \overline{\boldsymbol{\mathcal{F}}}_{\alpha_M},$$

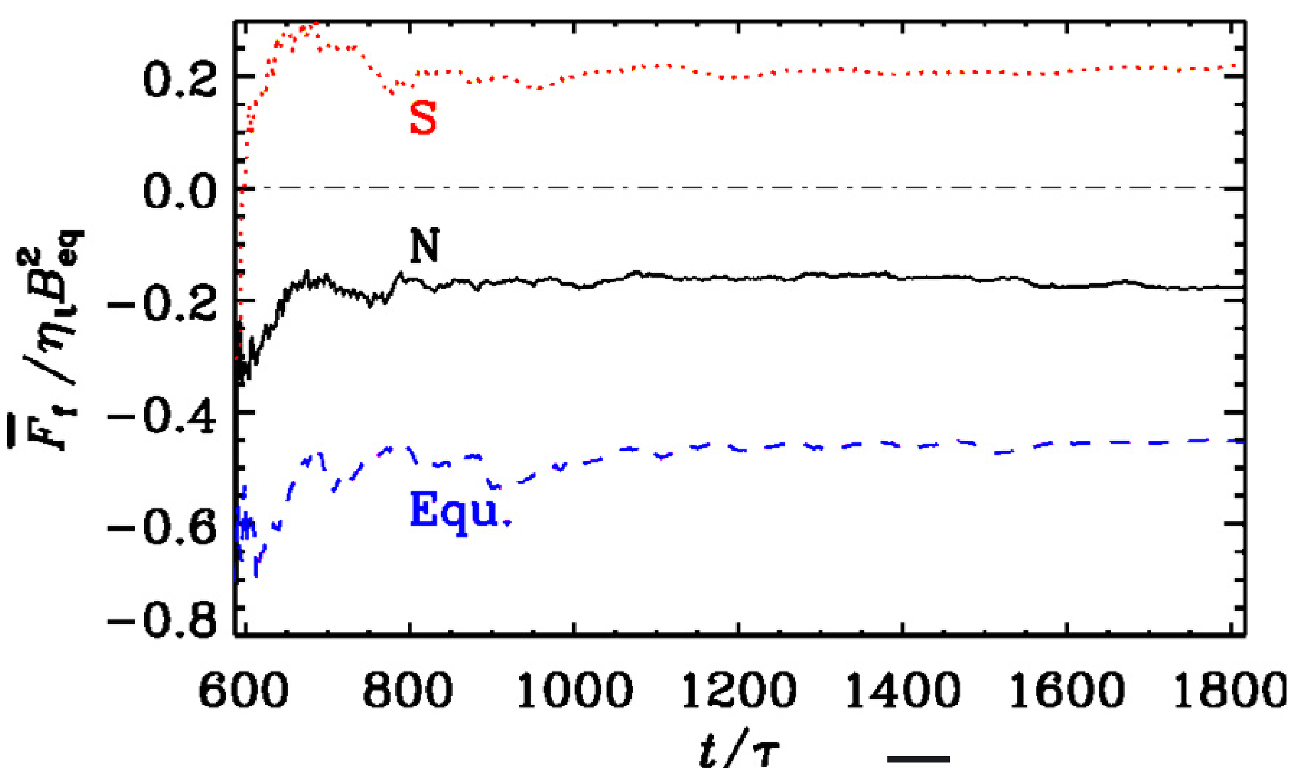
$$\overline{\boldsymbol{\mathcal{F}}}_{\alpha_M} = \frac{\eta_t k_f^2}{B_{eq}^2} \overline{\boldsymbol{\mathcal{F}}}_h^f,$$

$$\alpha = \frac{\alpha_K + Re_M \left(\eta_t \overline{\boldsymbol{J}} \cdot \overline{\boldsymbol{B}} - \frac{1}{2} \boldsymbol{\nabla} \cdot \overline{\boldsymbol{\mathcal{F}}}_h^f \right) / B_{eq}^2}{1 + Re_M \overline{\boldsymbol{B}}^2 / B_{eq}^2}$$

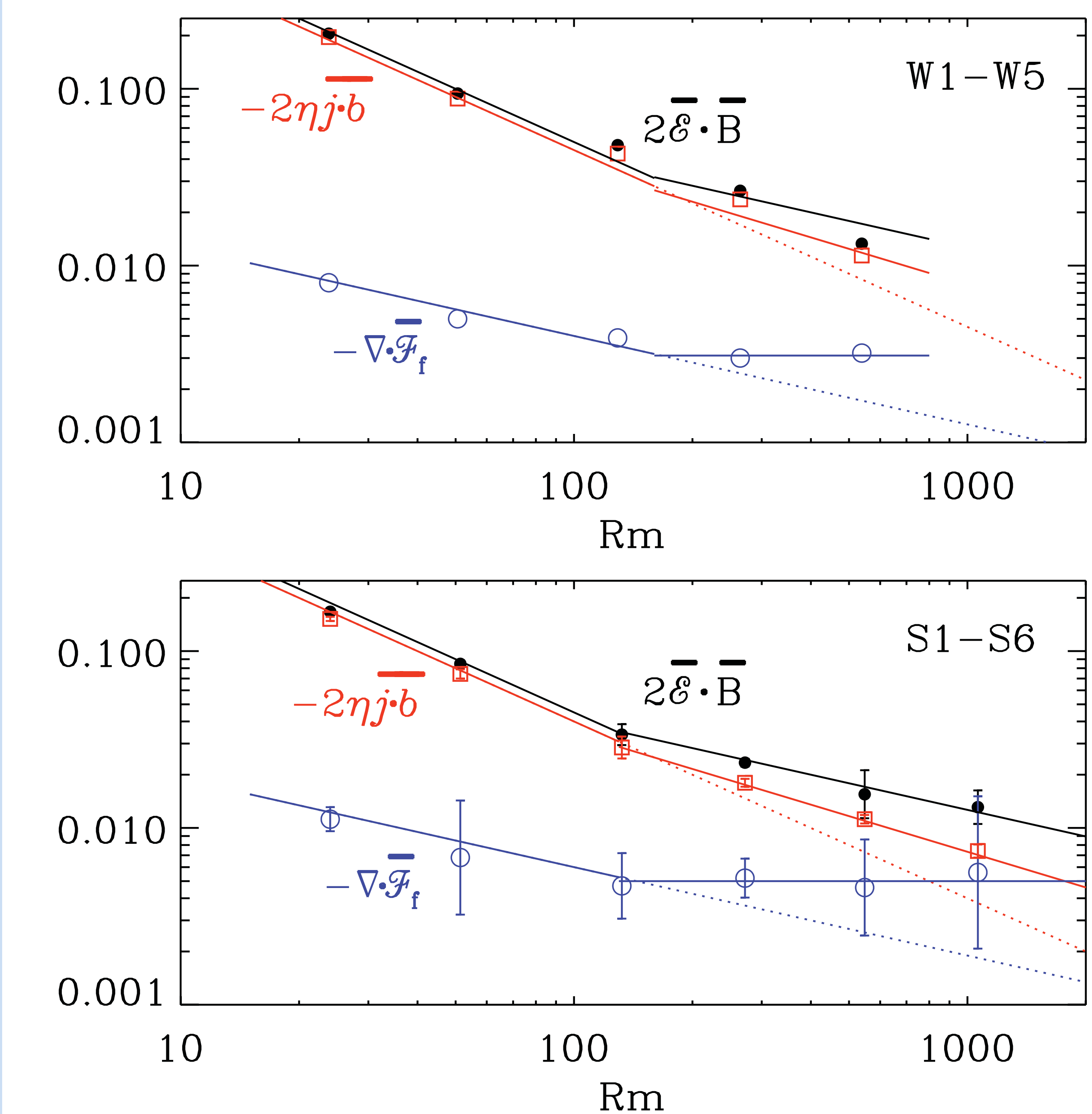
Brandenburg & Subramanian 2005

Helicity fluxes



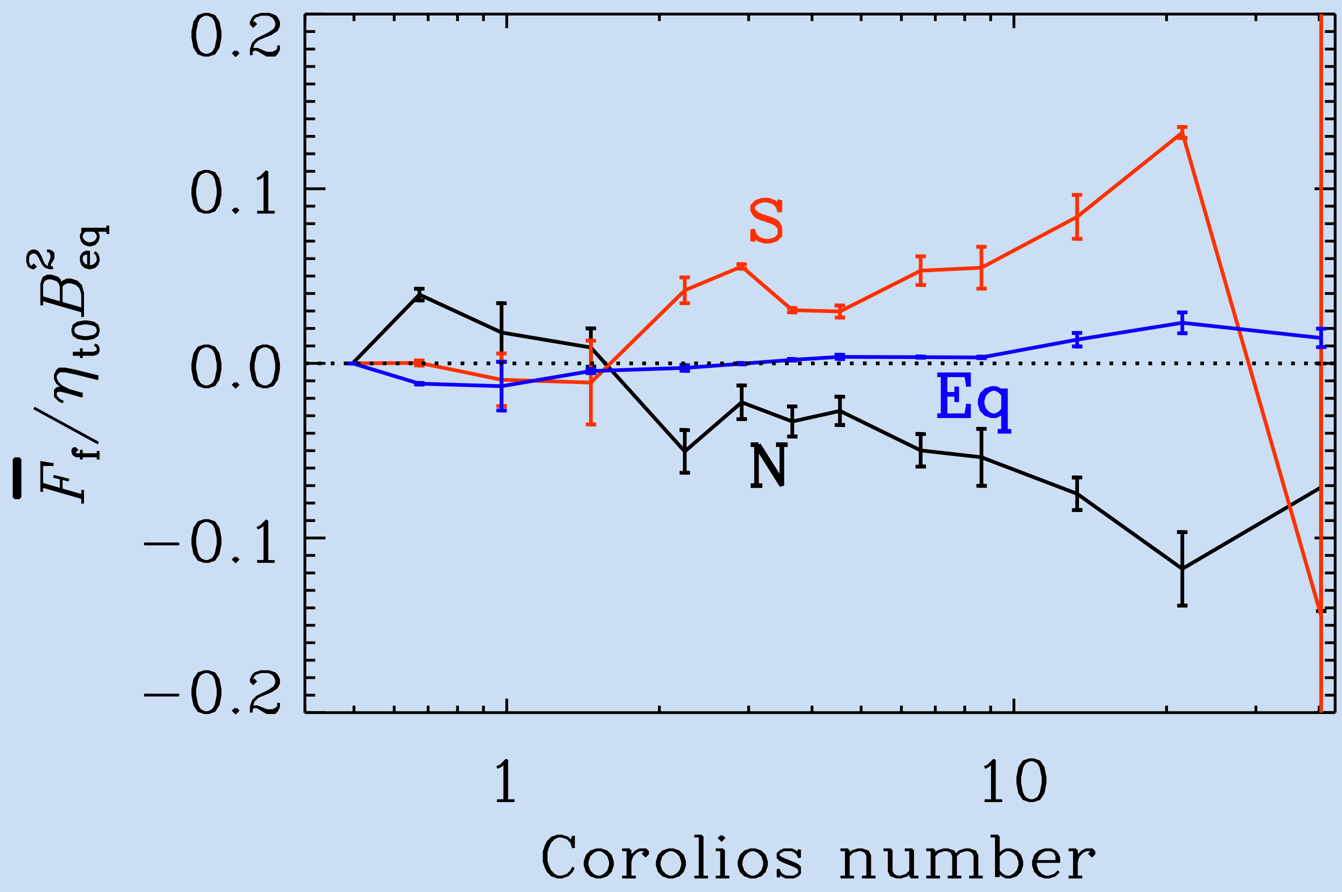


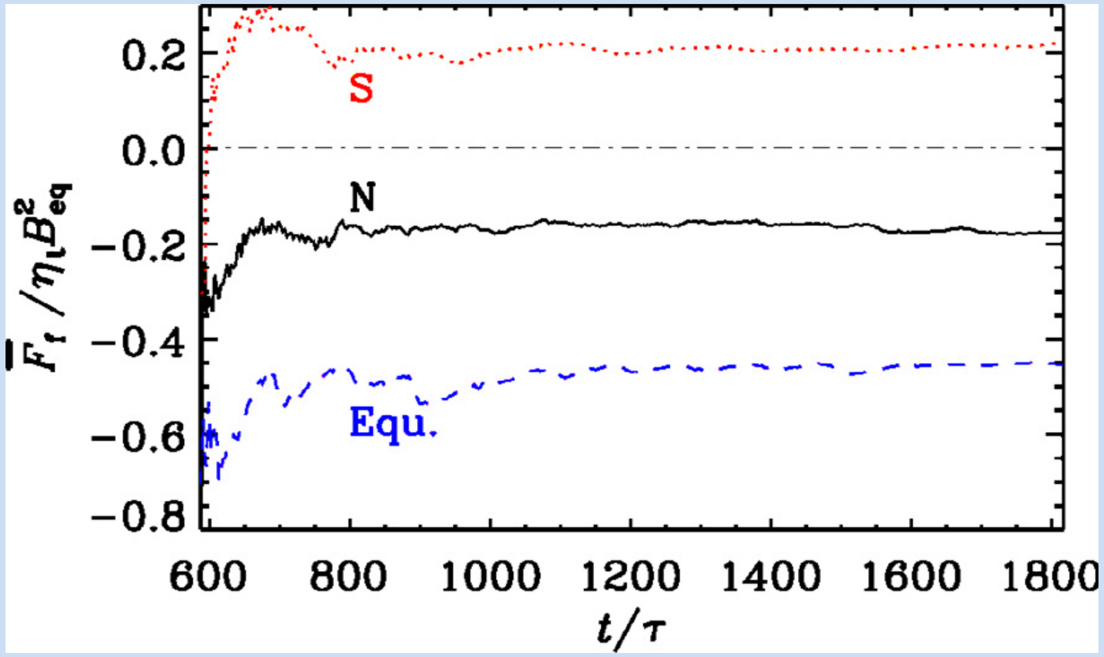
$\overline{\boldsymbol{F}}_f = \overline{\boldsymbol{e} \times \boldsymbol{a}}$

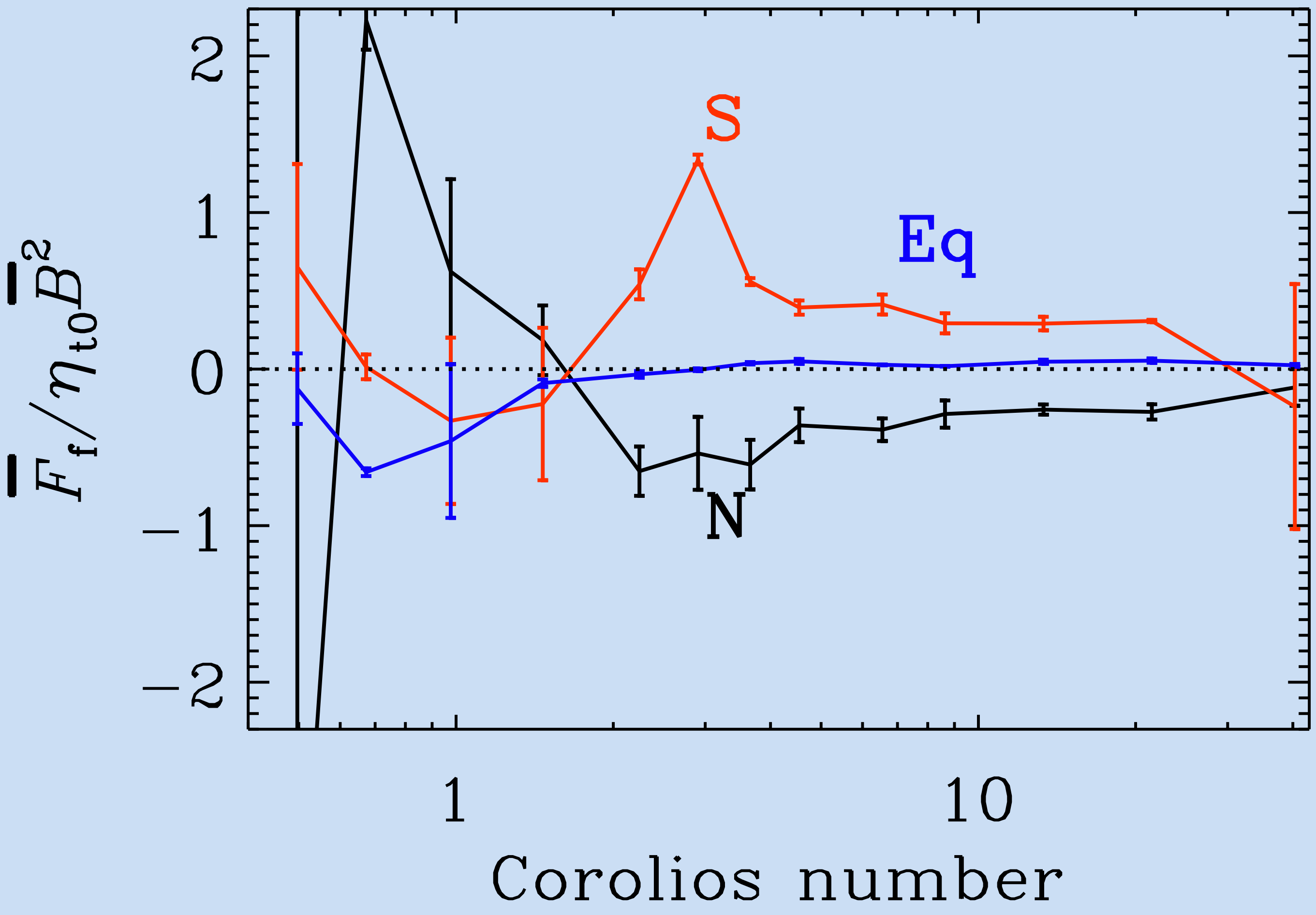


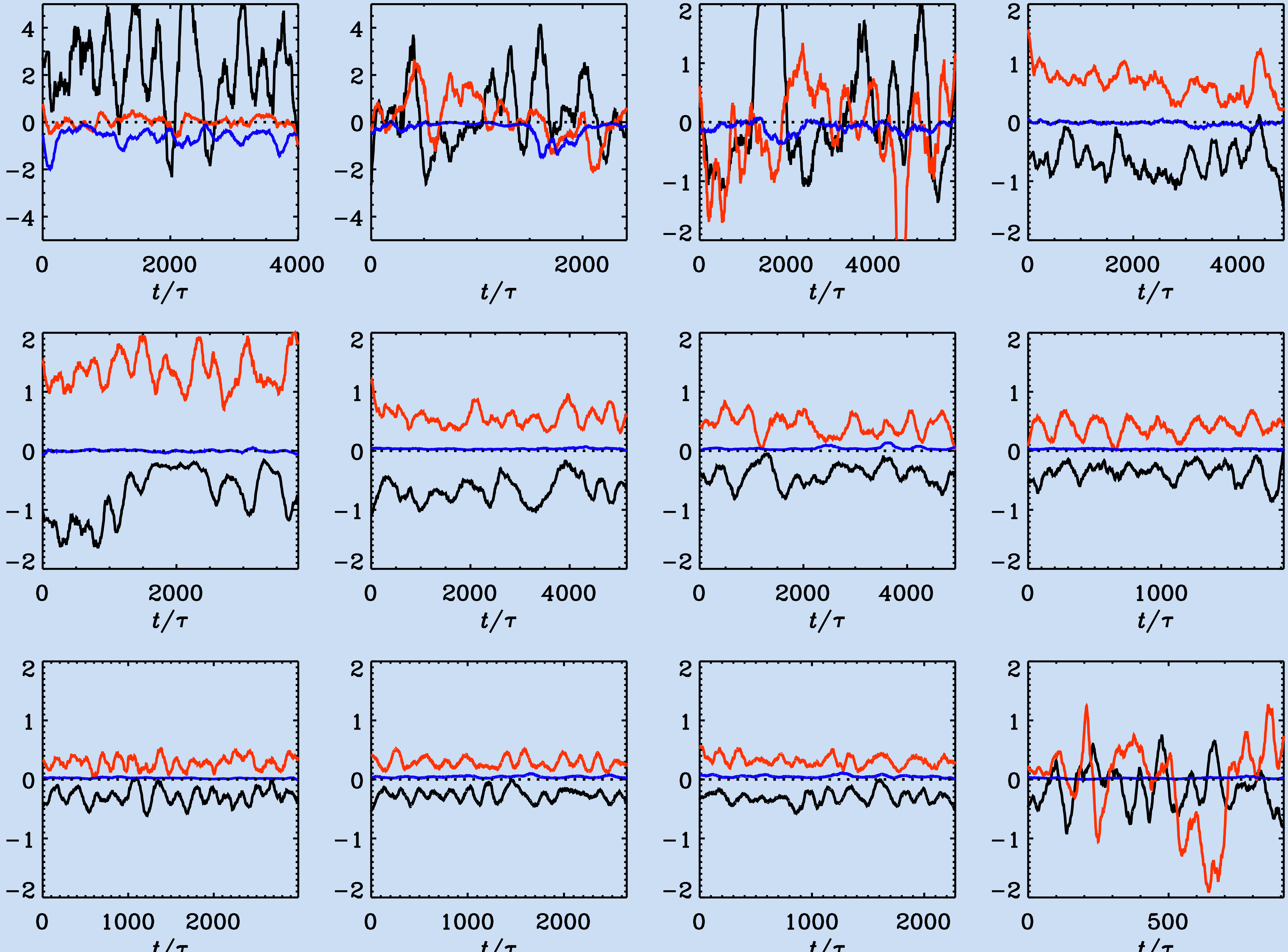
Warnecke et al. 2011

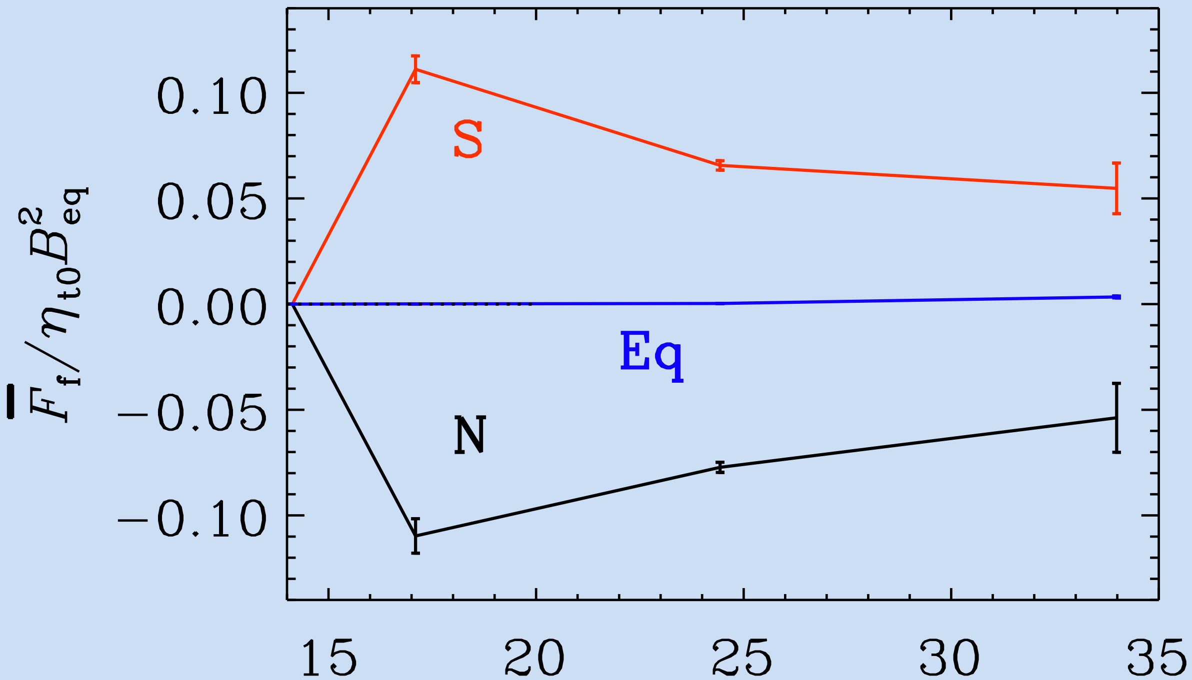
Del Sordo et al. 2013

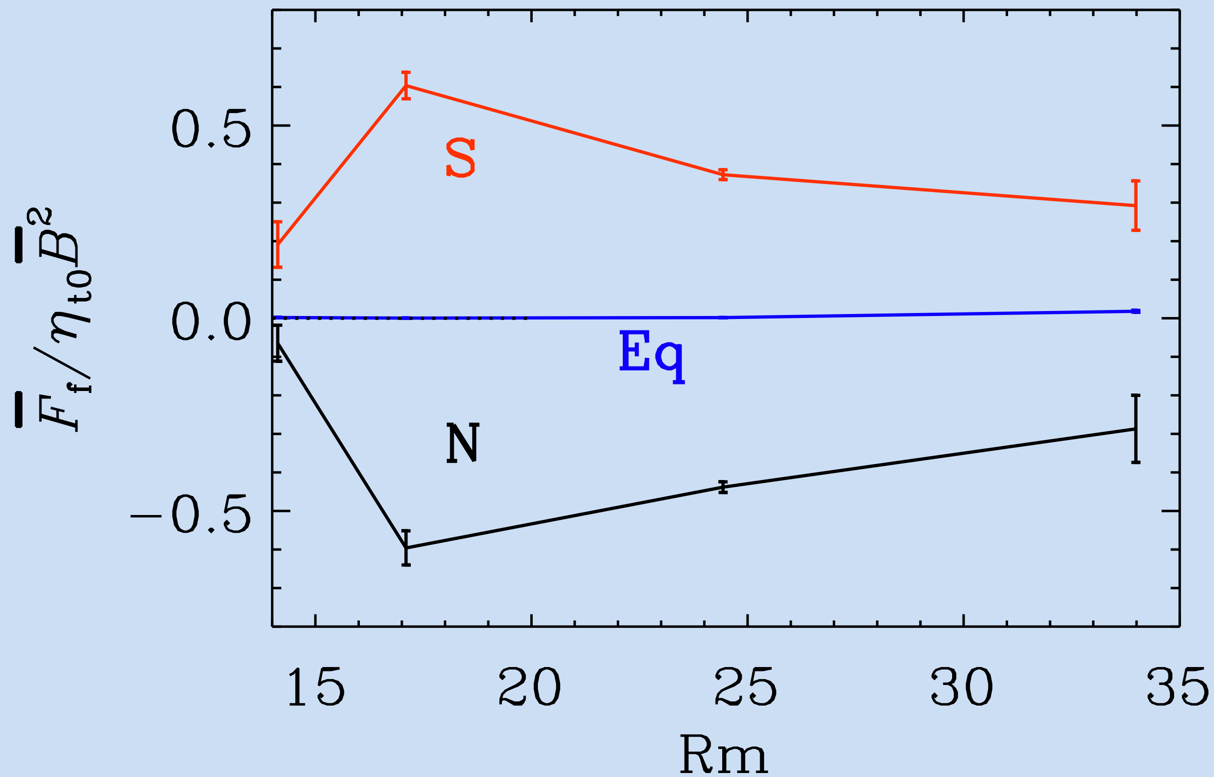

0.2
0.1
0.0
−0.1
−0.2
S
Eq
N
1
10
Corolios number

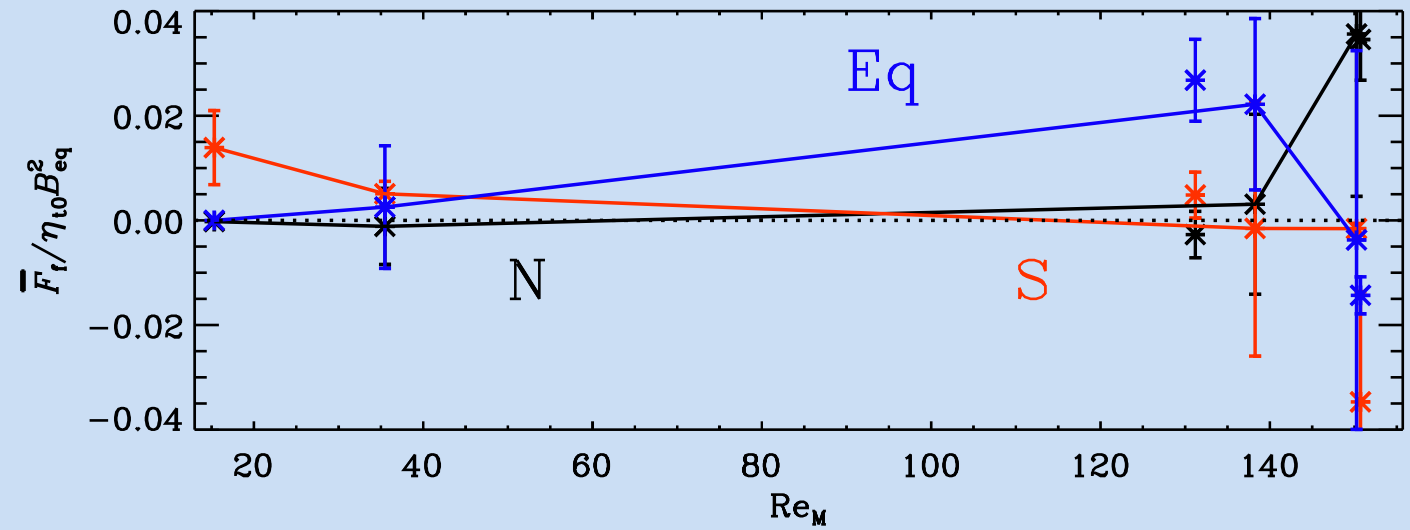

0.2
0.0
−0.2
−0.4
−0.6
−0.8
S
N
Equ.
600
800
1000
1200
1400
1600
1800
t/τ

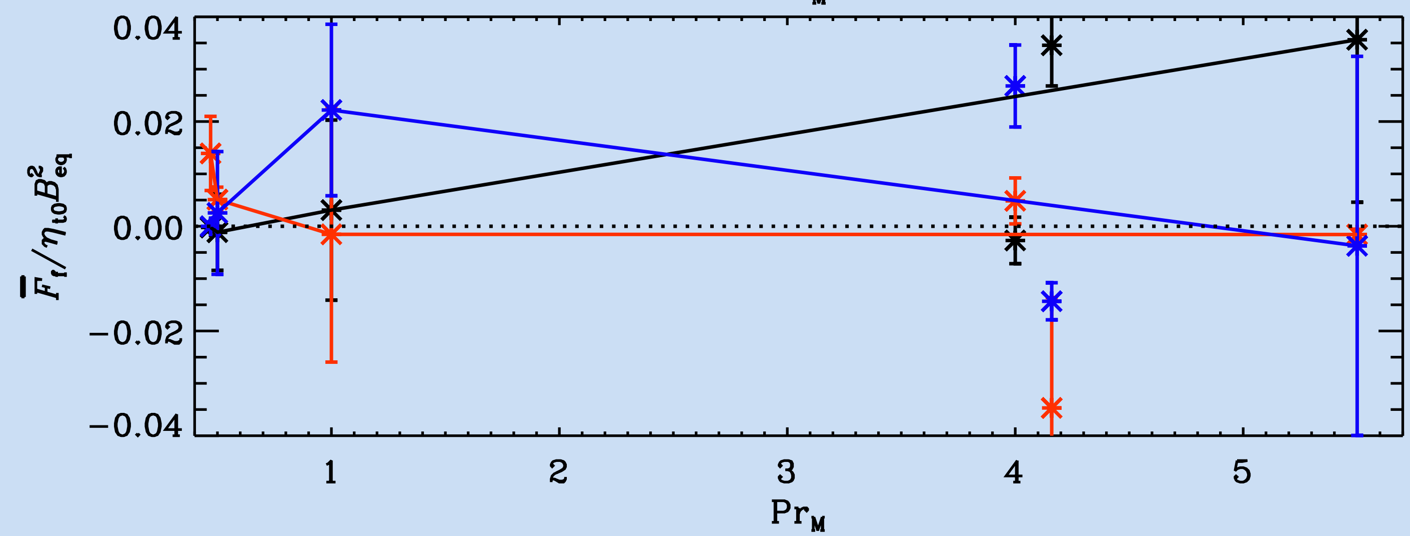

2
1
0
−1
−2
S
Eq
N
1
10
Corolios number

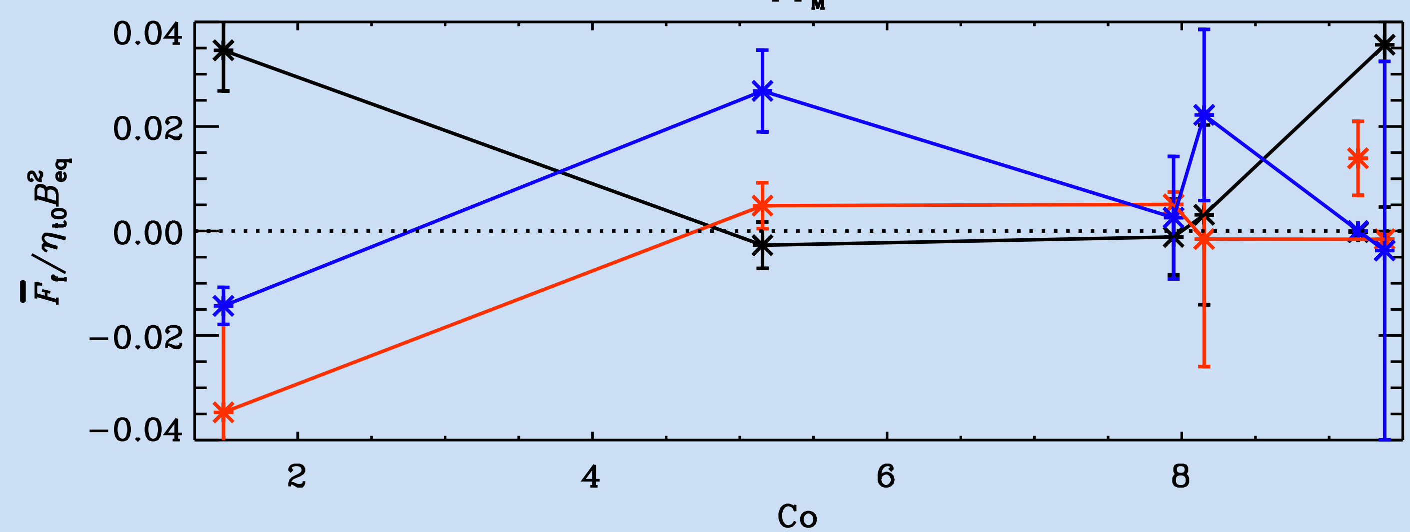

t/τ


S
Eq
N
$\overline{F}_f/\eta_{t0}B^2_{eq}$


S
Eq
N
$\overline{F}_f/\eta_{t0}\overline{B}^2$
Rm


Eq
N
S
$\overline{F}_f/\eta_{t0}B^2_{eq}$
Re_M


$\overline{F}_f/\eta_{t0}B^2_{eq}$
Pr_M


$\overline{F}_f/\eta_{t0}B^2_{eq}$
Co

Coronal model driven by emerging flux simulation

flux-emergence simulation

from / similar to Cheung et al (2010) ApJ 720, 233

– flux rope rises from bottom and breaks through surface

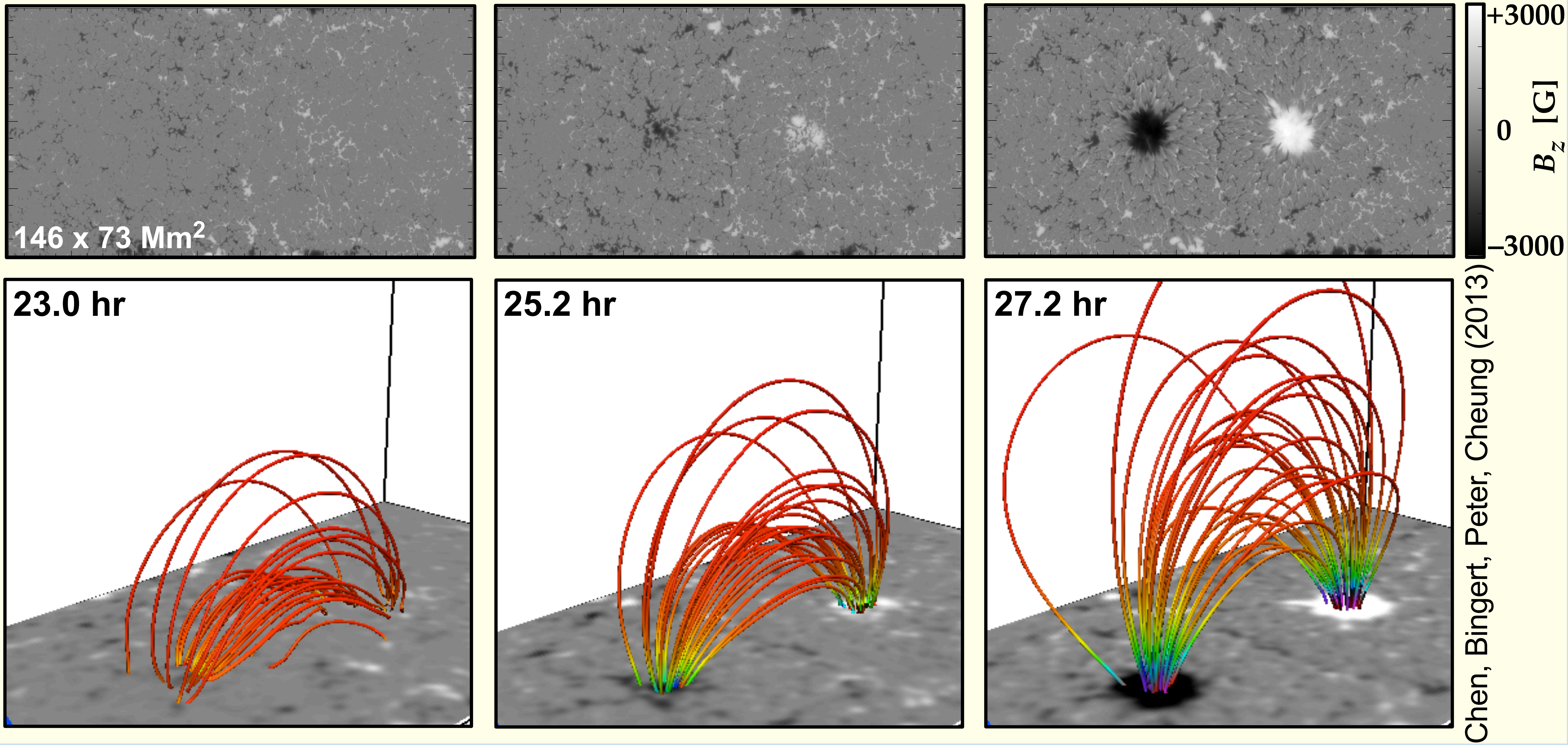
→ pair of sunspots

coronal simulation

– use photospheric layer (T, ρ, v, B) as time-dependent lower boundary

→ magnetic field expands

→ coronal loops form



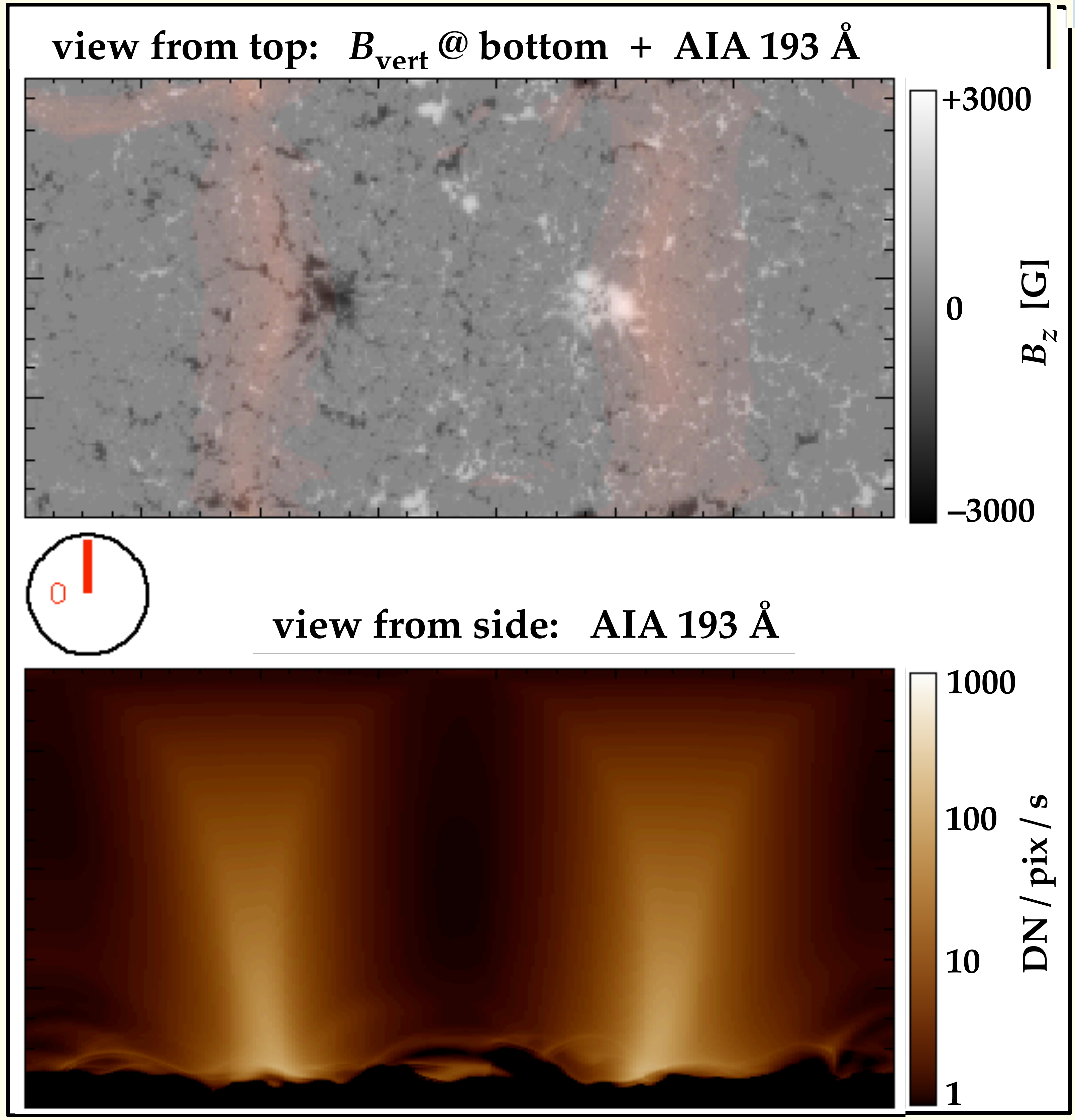
Chen, Bingert, Peter, Cheung (2013)

Coronal model driven by emerging flux simulation

► loops form at different places at different times

► loop footpoints are in sunspot periphery (penumbra)

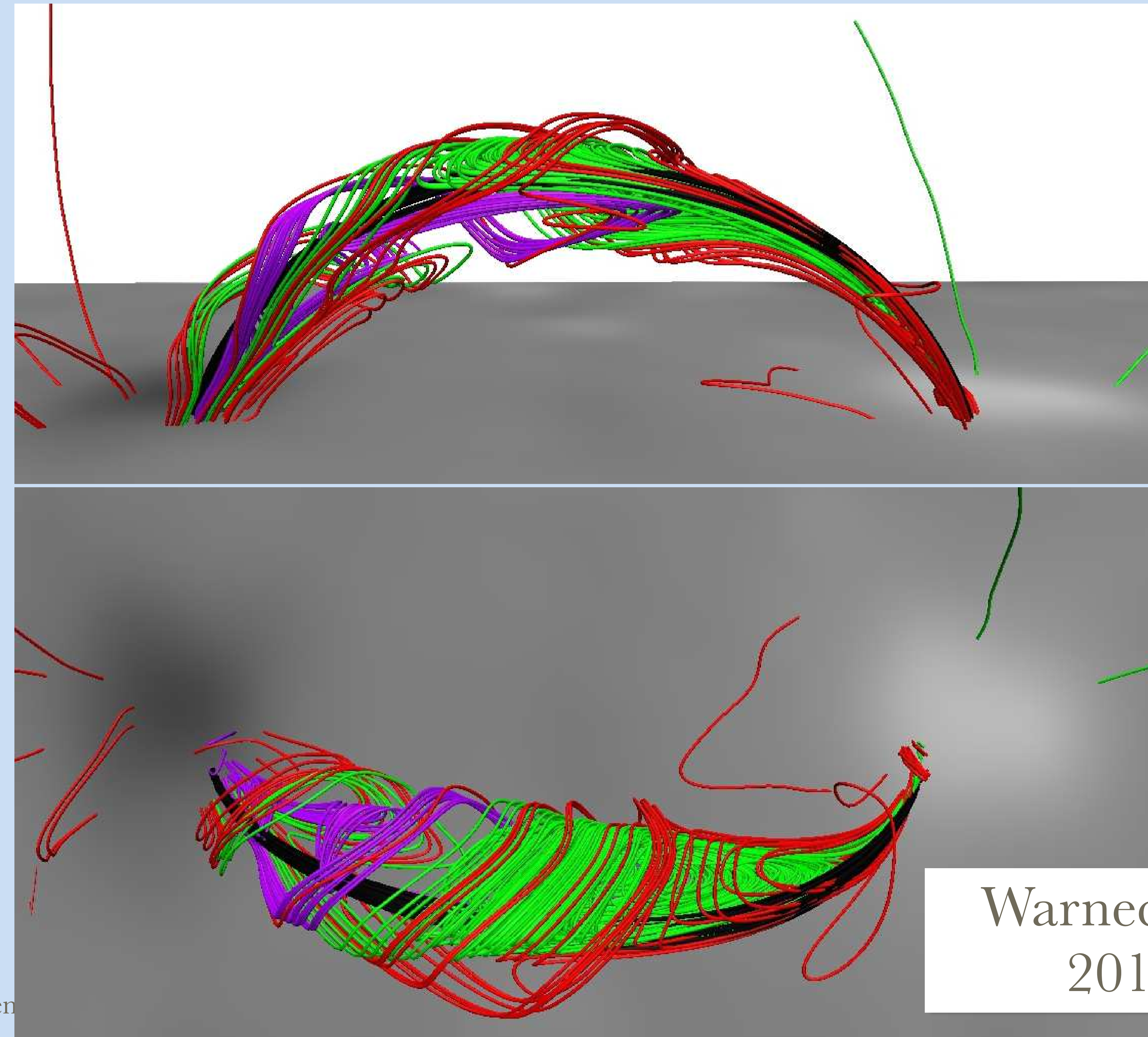
synthesized coronal emission (1.5 10^6 K)



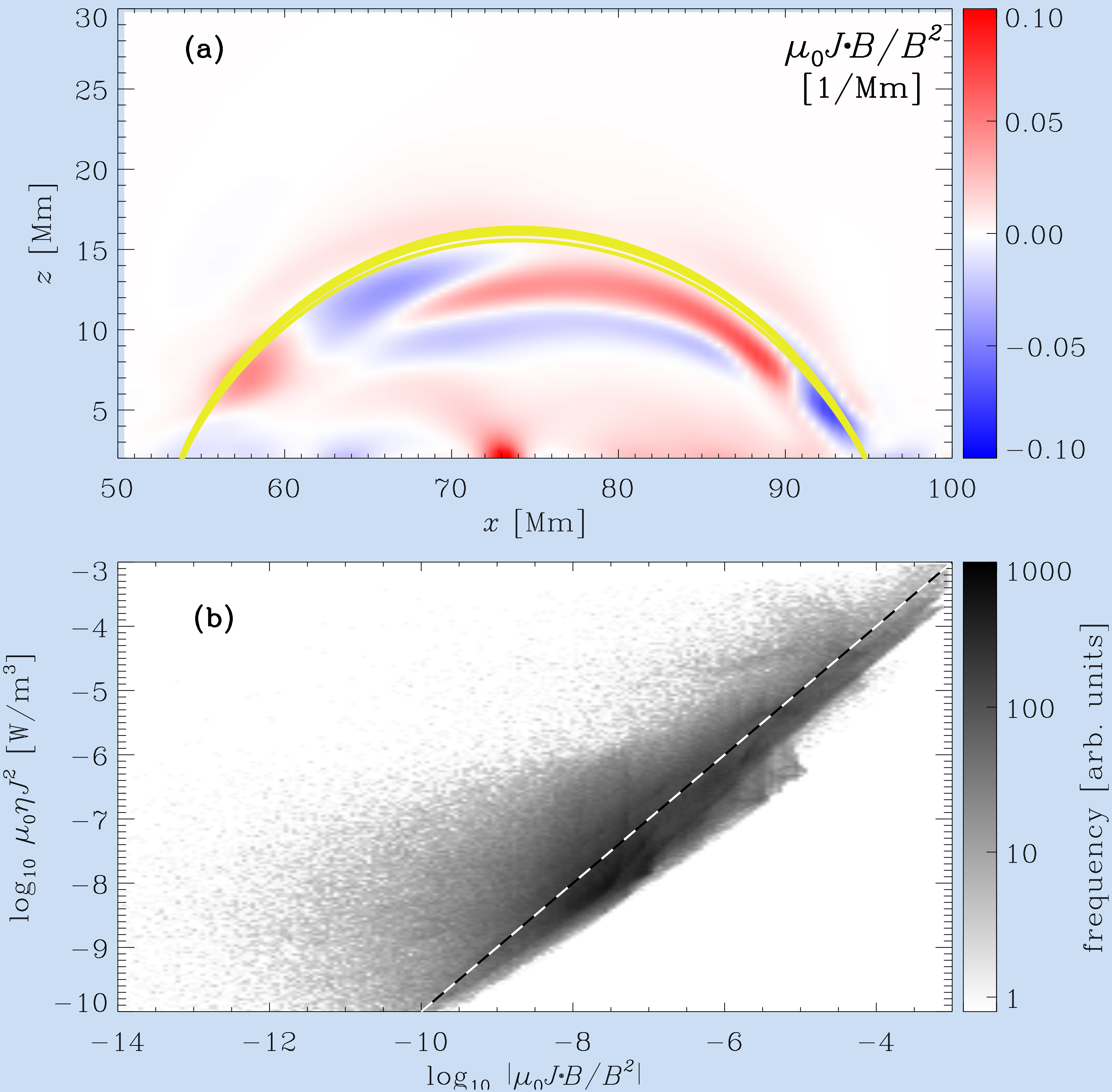
34 min
out of 10 hrs

Chen, Bingert, Peter, Cheung (2013

Helical currents in coronal loops

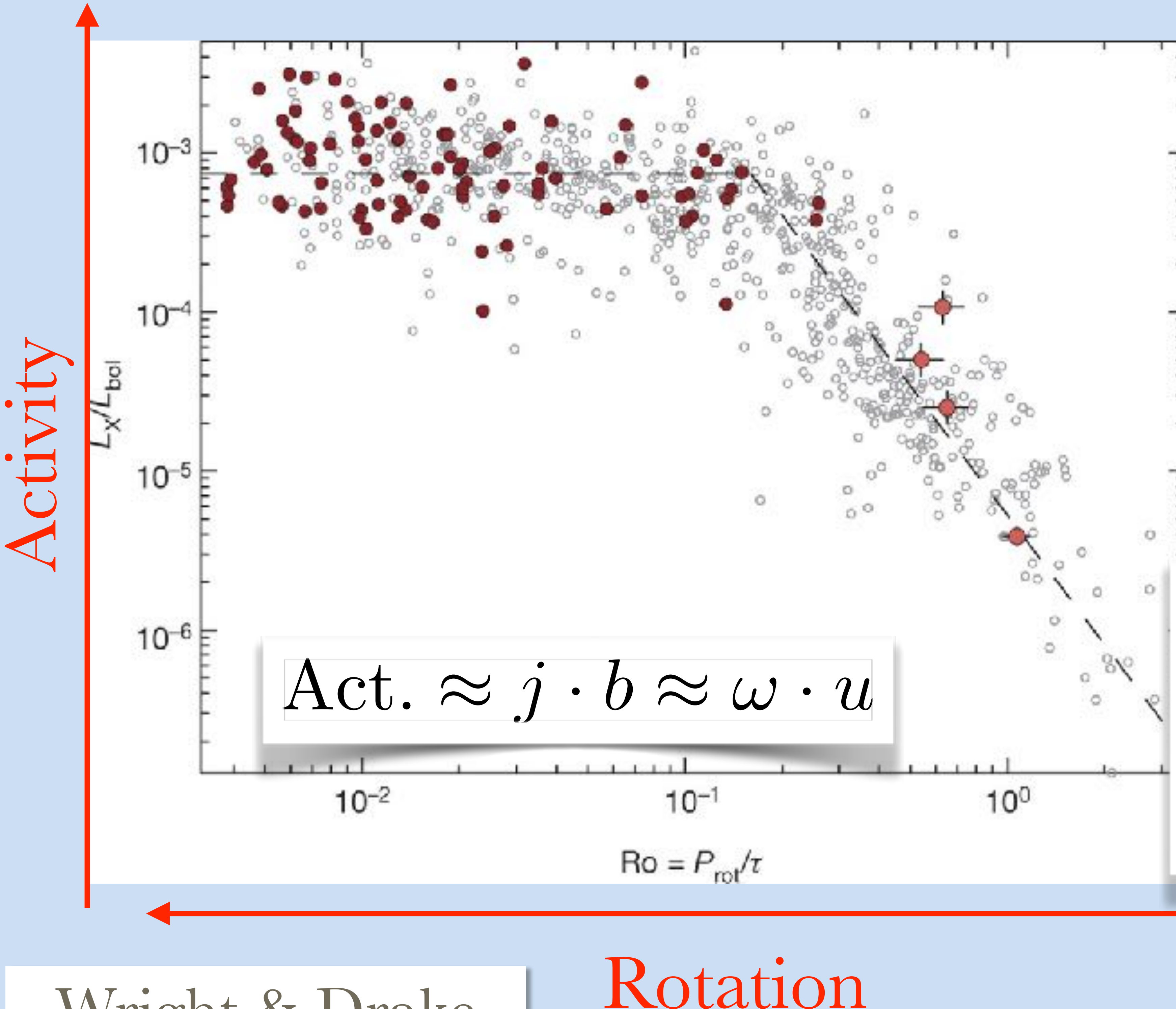


Warnecke et al. 2017b



(a)
$\mu_0 J{\cdot}B/B^2$ [1/Mm]
0.10
0.05
0.00
−0.05
−0.10
z [Mm]
x [Mm]
(b)
$\log_{10}\ \mu_0\eta J^2$ [W/m^3]
$\log_{10}\ |\mu_0 J{\cdot}B/B^2|$
1000
100
10
1
frequency [arb. units]

Rotation Activity Relation



$$\text{Act.} \approx j \cdot b \approx \omega \cdot u$$

$$\nabla\Omega = \text{const}$$

$$\alpha = \frac{\tau_c}{3}\left(-\overline{\boldsymbol{\omega}\cdot\boldsymbol{u}} + \frac{\overline{\boldsymbol{j}\cdot\boldsymbol{b}}}{\overline{\rho}}\right)$$

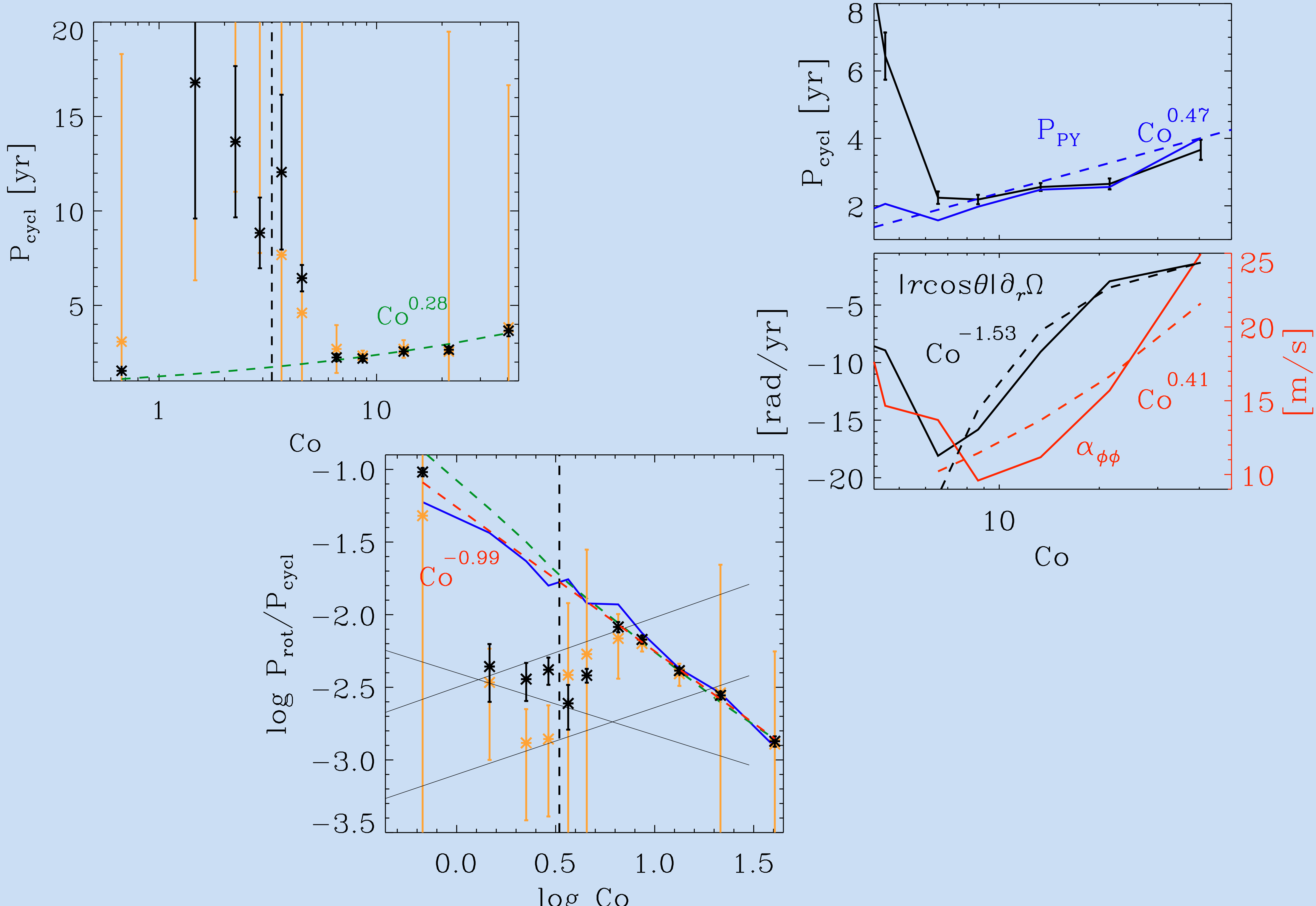
Pouquet et al. 1976

helicity is
a pseudo scalar:
$\alpha \sim \Omega\tau$

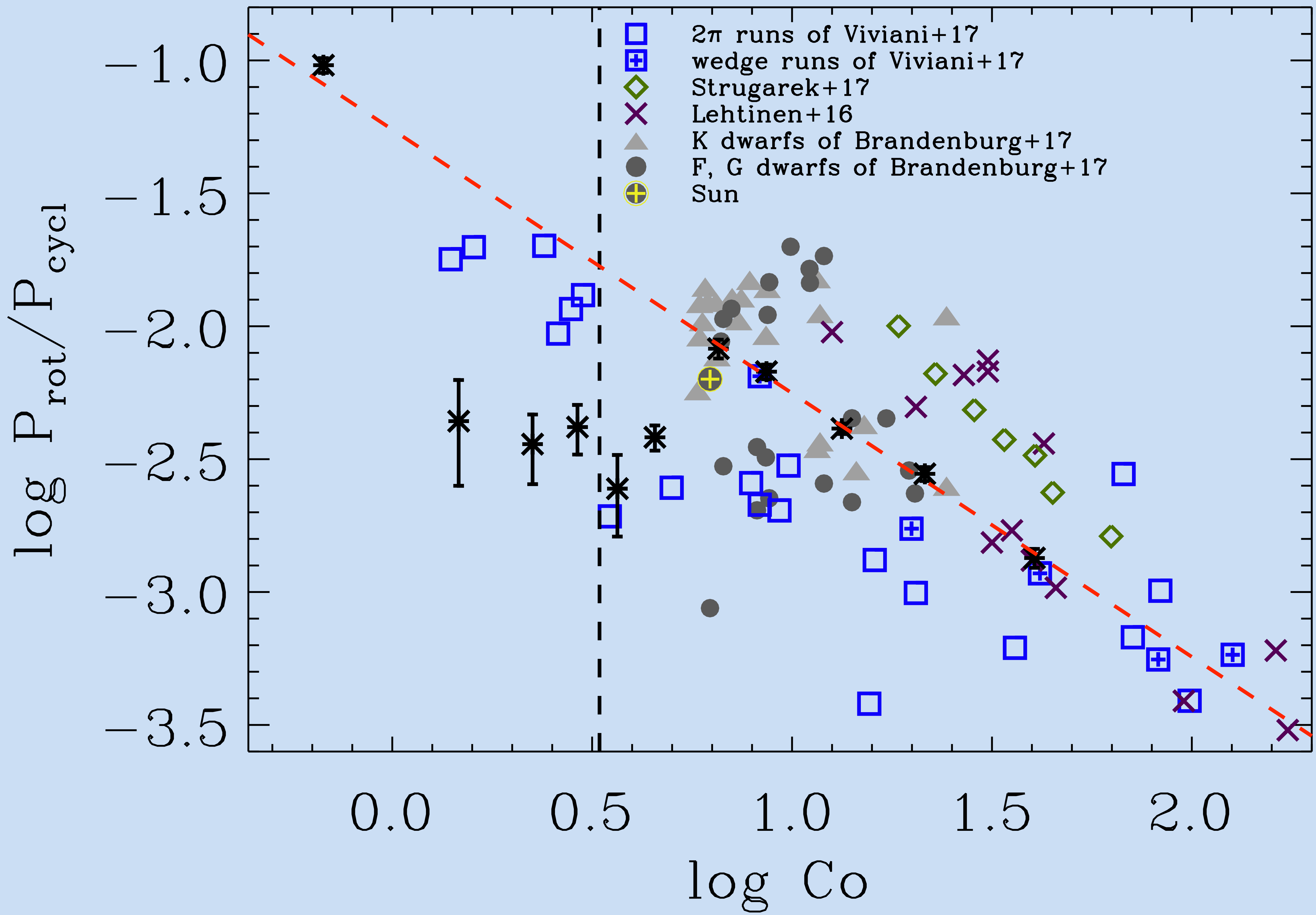
Wright & Drake
2016, Nature

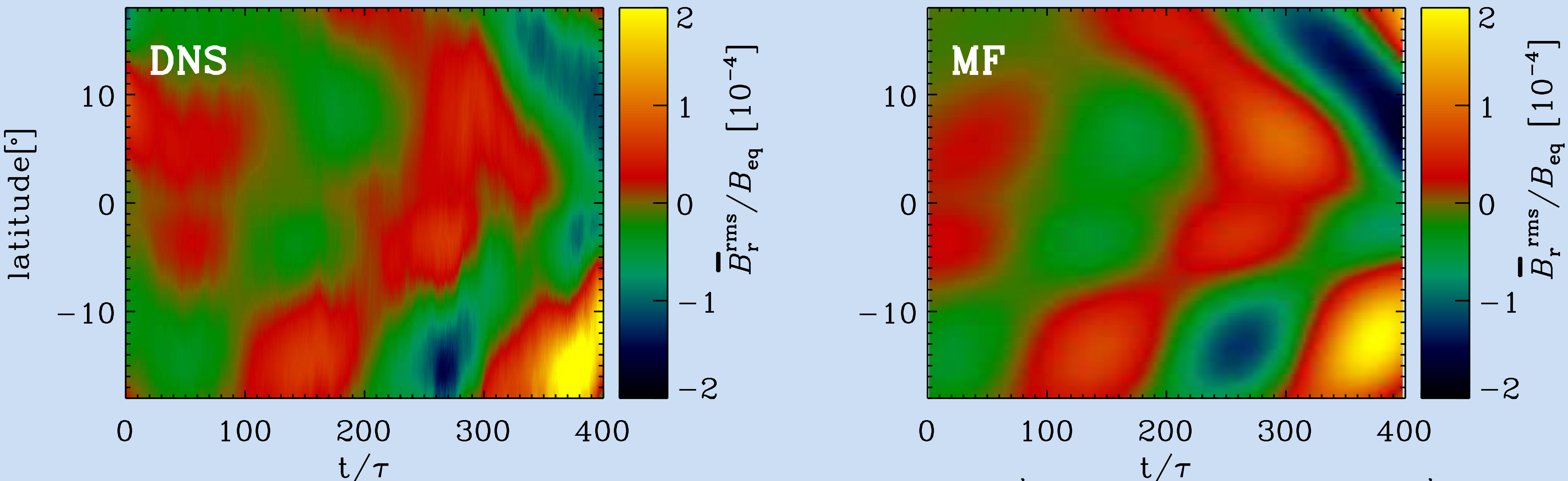
Conclusions

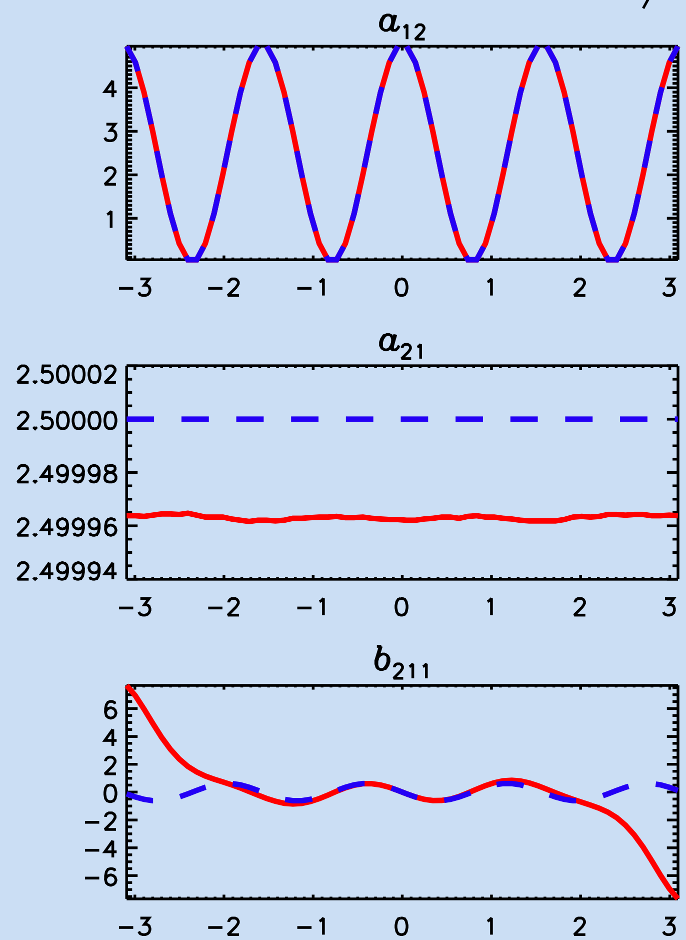
- Alpha effect is more than „just" helicity.
- Alpha becomes highly anisotropic for high rotation.
- Increase of the helicity fluxes with rotation
- Decrease of the helicity fluxes with Rm.
- Helicity flux shown cycle dependency.
- Magnetic helicity important for coronal heating
- Magnetic helicity might play important role for stellar Rotation-Activity-Relation.

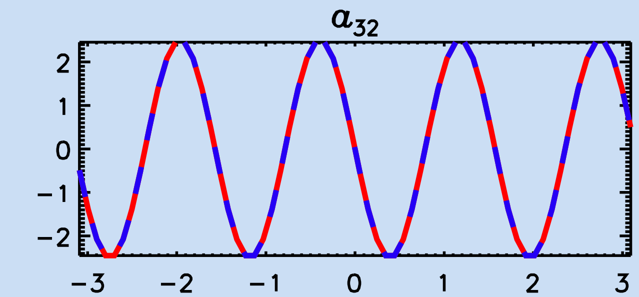


P_{cycl} [yr]
$Co^{0.28}$
Co
log P_{rot}/P_{cycl}
$Co^{-0.99}$
log Co
P_{cycl} [yr]
P_{PY}
$Co^{0.47}$
$|r\cos\theta|\partial_r\Omega$
$Co^{-1.53}$
[rad/yr]
$Co^{0.41}$
$\alpha_{\phi\phi}$
[m/s]
Co


2π runs of Viviani+17
wedge runs of Viviani+17
Strugarek+17
Lehtinen+16
K dwarfs of Brandenburg+17
F, G dwarfs of Brandenburg+17
Sun
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
log P_{rot}/P_{cycl}
0.0
0.5
1.0
1.5
2.0
log Co







Robert flow

$$\boldsymbol{u} = \begin{pmatrix} v_0 \sin k_0 x \cos k_0 y \\ -v_0 \cos k_0 x \sin k_0 y \\ w_0 \cos k_0 x \cos k_0 y \end{pmatrix}$$