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Flow generation by helicity and angular-momentum transport in the Sun

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A NEW SIMPLE DYNAMO MODEL FOR STELLAR ACTIVITY CYCLE

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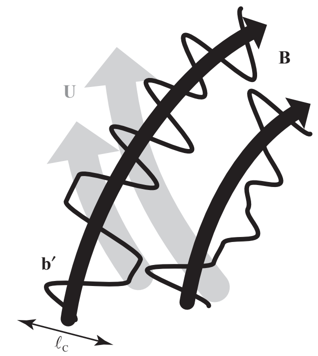

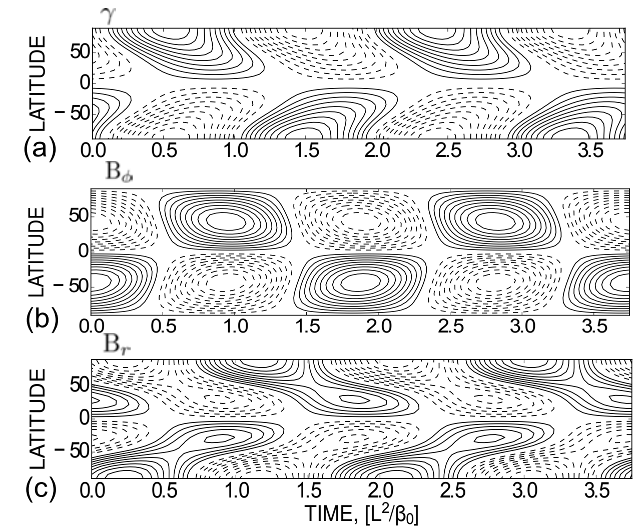

PHYSICS OF PLASMAS 23, 042311 (2016)

Sub-grid-scale description of turbulent magnetic reconnection in magnetohydrodynamics

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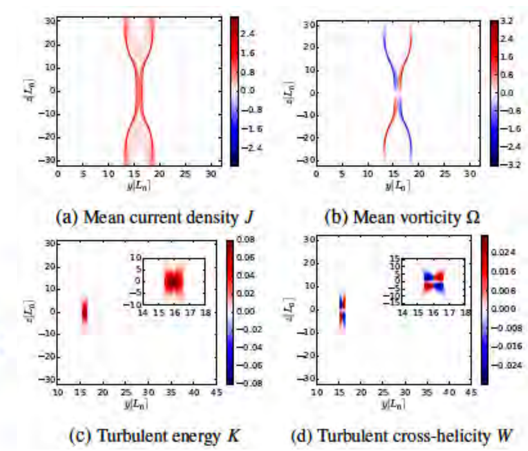

PHYSICS OF PLASMAS 23, 092304 (2016)

Characterizing plasmoid reconnection by turbulence dynamics

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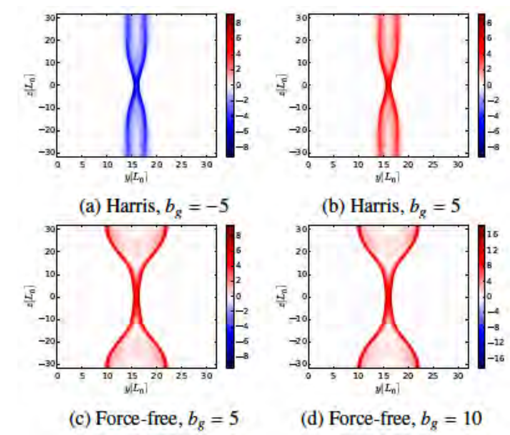

PHYSICAL REVIEW E 93, 033125 (2016)

Large-scale flow generation by inhomogeneous helicity

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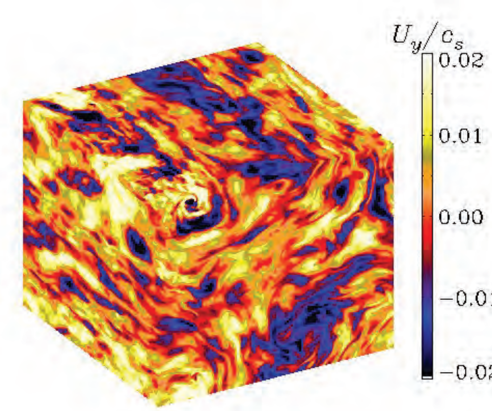


Topics

I. Vortex dynamo and helicity

II. Theoretical formulation and modelling

III. Global flow generation

IV. Stellar convection zone

V. Summary

I. Vortex dynamo and helicity

Vortex generation

Vorticity

$$\boldsymbol{\omega} = \nabla \times \mathbf{u}$$

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \nabla \times (\mathbf{u} \times \boldsymbol{\omega}) + \underbrace{\frac{\nabla \rho \times \nabla p}{\rho^2}}_{\text{baroclinicity}} + \nu \nabla^2 \boldsymbol{\omega}$$

cf., Biermann battery $-\dfrac{\nabla n_e \times \nabla p_e}{n_e^2 e}$

Mean vorticity

$$\boldsymbol{\Omega} = \nabla \times \mathbf{U}$$

$$\frac{\partial \boldsymbol{\Omega}}{\partial t} = \nabla \times (\mathbf{U} \times \boldsymbol{\Omega}) + \nabla \times \underbrace{\langle \mathbf{u}' \times \boldsymbol{\omega}' \rangle}_{\mathbf{V}_{\mathrm{M}}} + \nu \nabla^2 \boldsymbol{\Omega}$$

vortexmotive force

cf., Mean magnetic field

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \nabla \times \underline{\langle \mathbf{u}' \times \mathbf{b}' \rangle} + \eta \nabla^2 \mathbf{B}$$

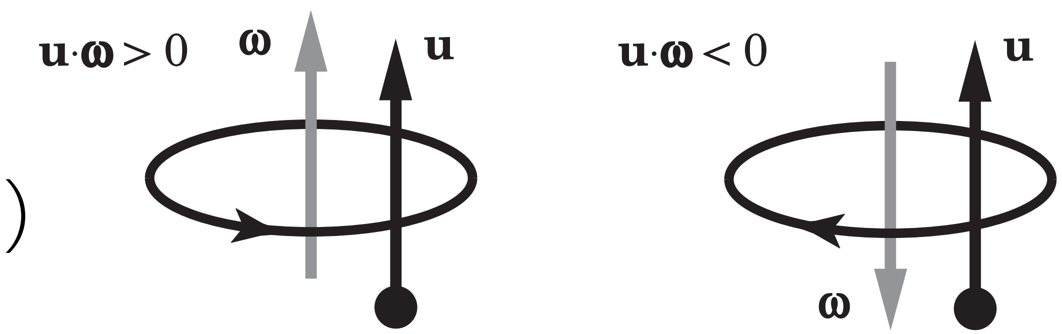
electromotive force

Reynolds stress $\mathcal{R}^{ij} = \langle u'^i u'^j \rangle$

$$V_{\mathrm{M}}^i = -\frac{\partial \mathcal{R}^{ij}}{\partial x^j} + \frac{\partial K}{\partial x^i}$$

Helicity

Pseudo-scalar



Breakage of mirror-symmetry (inversion)

$$x^i \longmapsto \tilde{x}^i = -x^i$$

$$u^i(\mathbf{x}, t) \longmapsto \tilde{u}^i(\tilde{\mathbf{x}}, t) = -u^i(\mathbf{x}, t)$$

$$H(\mathbf{x}, t) \longmapsto \tilde{H}(\tilde{\mathbf{x}}, t) = \left\langle \tilde{u}'^i(\tilde{\mathbf{x}}, t)\tilde{\epsilon}^{ijk}\frac{\partial \tilde{u}'^k(\tilde{\mathbf{x}}, t)}{\partial \tilde{x}^j} \right\rangle$$

$$= \left\langle -u'^i(\mathbf{x}, t)\epsilon^{ijk}\frac{\partial(-u'^k)(\mathbf{x}, t)}{\partial(-x)^j} \right\rangle$$

$$= -\left\langle u'^i(\mathbf{x}, t)\epsilon^{ijk}\frac{\partial u'^k(\mathbf{x}, t)}{\partial x^j} \right\rangle$$

$$= -H(\mathbf{x}, t)$$

mirror-symmetric system

$$F(\mathbf{x}, t) \longmapsto \tilde{F}(\tilde{\mathbf{x}}, t) = F(\tilde{\mathbf{x}}, t)$$

pseudo-scalar

$$F(\mathbf{x}, t) \longmapsto \tilde{F}(\tilde{\mathbf{x}}, t) = -F(\tilde{\mathbf{x}}, t)$$

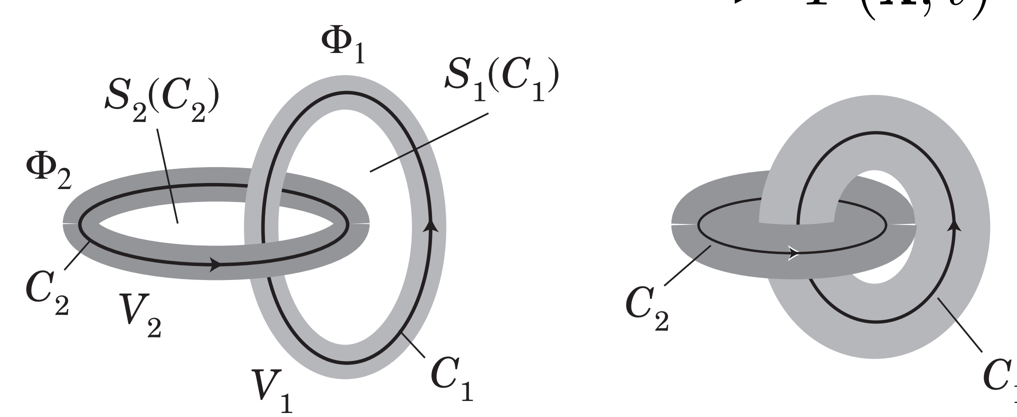
pseudo-scalar
in mirror-symmetric system

$$F(\mathbf{x}, t) = -F(\mathbf{x}, t)$$

$$\longrightarrow F(\mathbf{x}, t) = 0$$

Topological invariant

Knottedness



$$\frac{(\mathbf{u}\cdot\boldsymbol{\omega})^2}{|\mathbf{u}|^2|\boldsymbol{\omega}|^2} + \frac{(\mathbf{u}\times\boldsymbol{\omega})^2}{|\mathbf{u}|^2|\boldsymbol{\omega}|^2} = 1$$

Suppression of turbulent cascade

Scaling, energy cascade

Inverse energy cascade and negative viscosity in helical turbulence

Brissaud, et al. (1973)

Kraichnan (1973)

André & Lesieur (1977)

Recent work on time and length scales in helical turbulence

Ditllevsen & Giuliani (2001)

Kurien (2004)

Chkhetiani & Golbraikh (2008)

Baerenzung, et al. (2008)   Marino, et al. (2013)

Lessinnes, et al. (2011)

Turbulent transport

Symmetry of the Reynolds stress   Krause & Rüdiger (1974); Rüdiger (1980)

Helicity itself is insufficient Λ effect

Further breakage of symmetry is needed

Compressibility   Moiseev, et al. (1983)

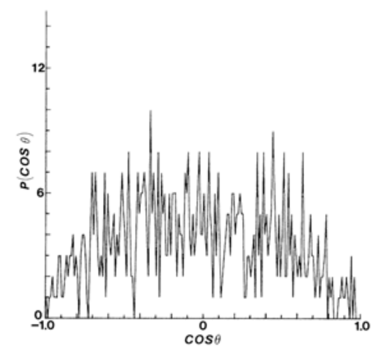
Anisotropy   Frisch, et al. (1987) AKA effect

Mean flow   Gvaramadze, et al. (1989)

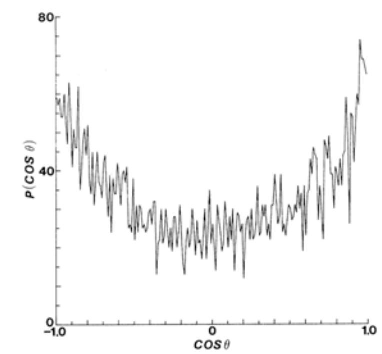
High helicity $\stackrel{?}{=}$ Low dissipation (Pelz, et al., PRL, 1985)

Outer part of the channel

In the region where dissipation is greater than 30% of its maximum value



In the region where dissipation is less than 5% of its maximum value

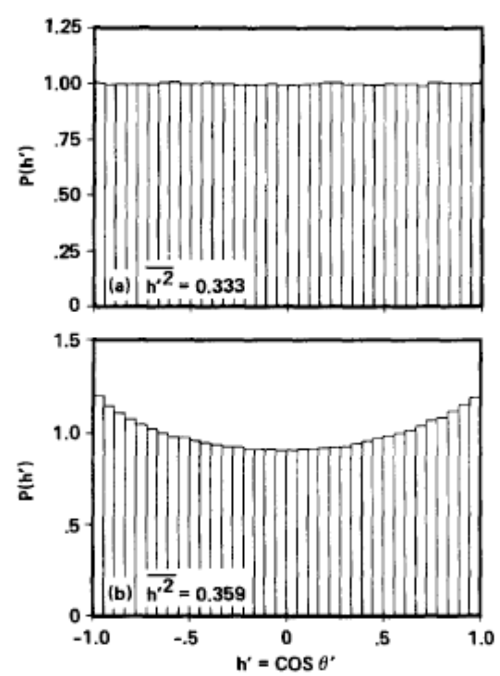


Direct Numerical Simulations of several flows (Rogers & Moin, PoF, 1987)

Isotropic turbulence

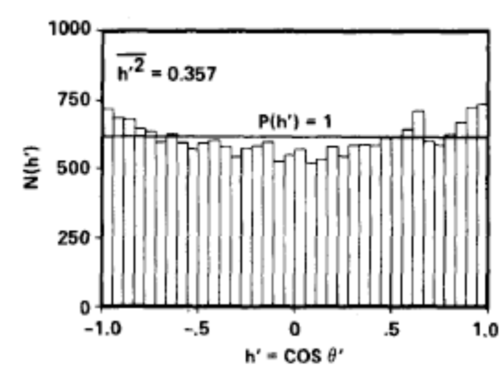
Initial

Developed



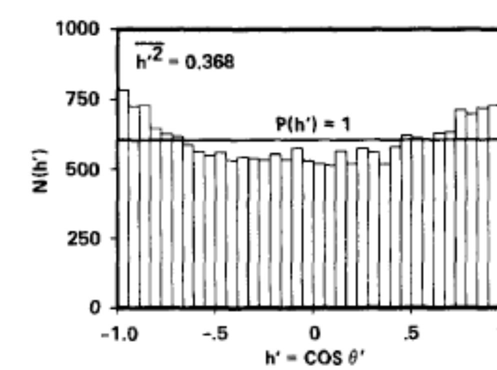
$P(h'|s_{ij}^2 < 0.0075(s_{ij}^2)_{\max})$

Low dissipation



$P(h'|s_{ij}^2 > 0.3(s_{ij}^2)_{\max})$

High dissipation

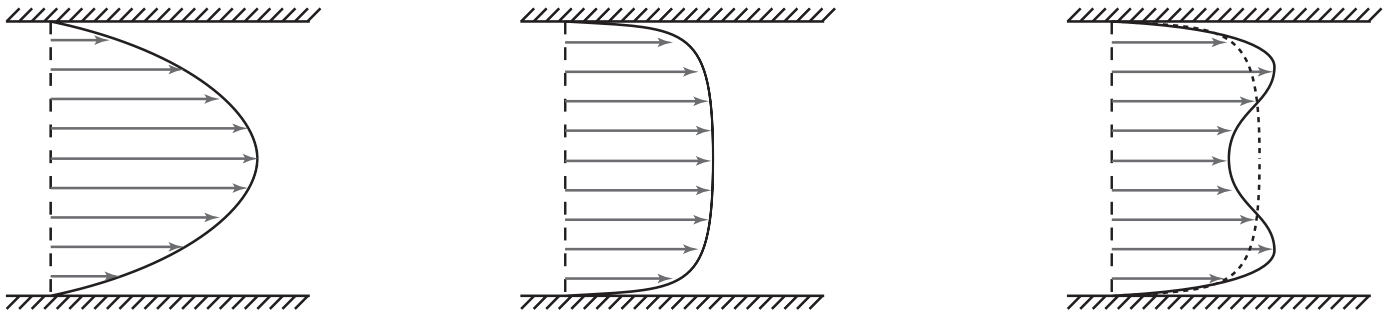


The relationship between helicity density and dissipation is tenuous.

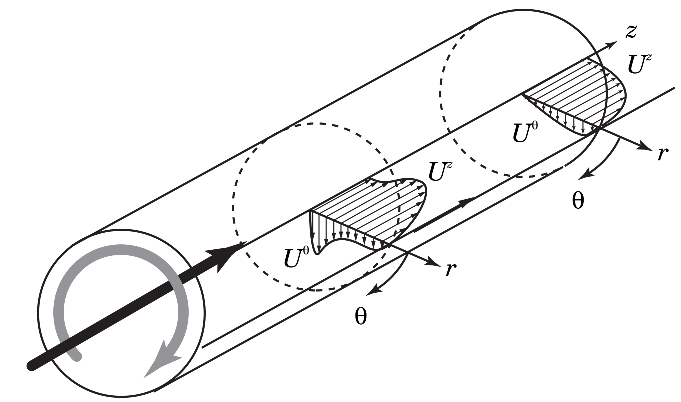
Experimental study confirmed Rogers & Moin (1987)

(Wallace, Balint & Ong, PoF, 1992)

Transport suppression in turbulent swirling flow



Laminar Turbulent Turbulent swirl

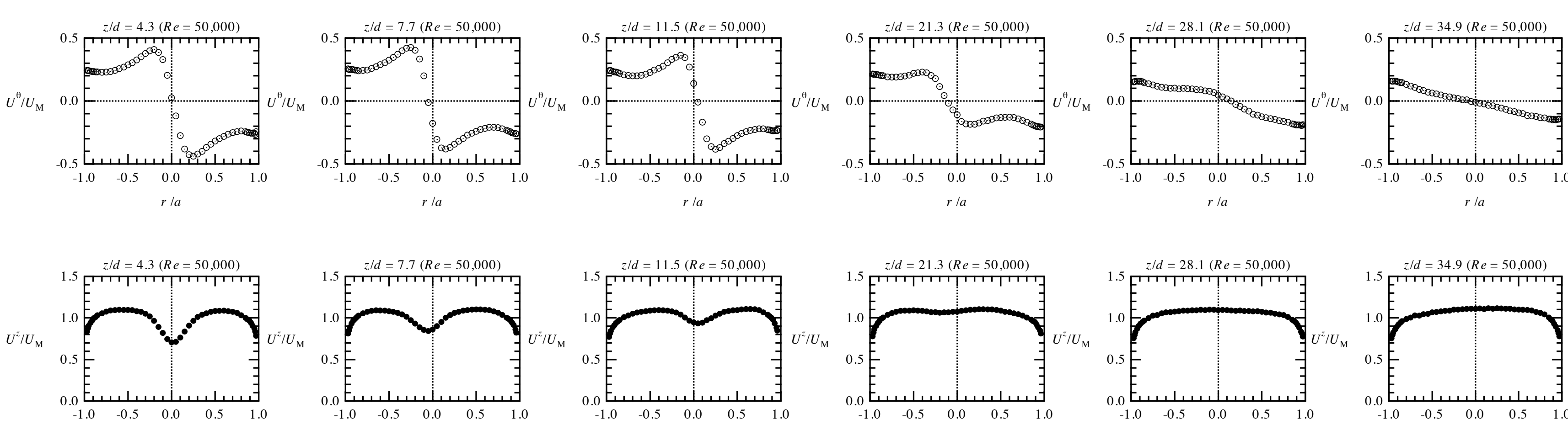


Reynolds stress $$\langle u'_\alpha u'_\beta \rangle = \frac{2}{3} K \delta_{\alpha\beta} - \nu_{\mathrm{T}} \left(\frac{\partial U_\alpha}{\partial x_\beta} + \frac{\partial U_\beta}{\partial x_\alpha} \right)$$

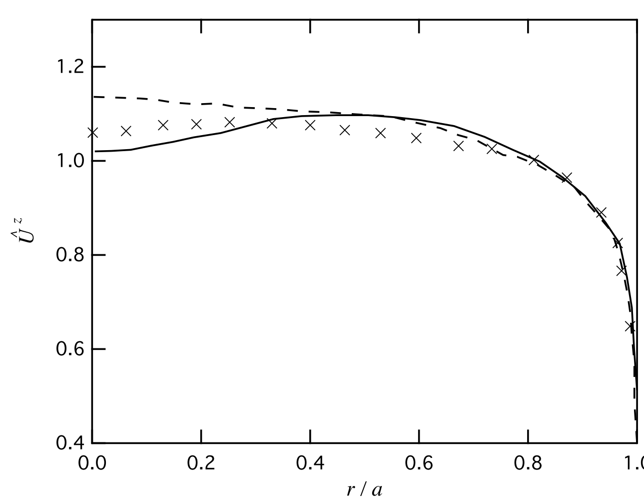
$$\longrightarrow \quad \frac{\partial U_\alpha}{\partial t} + U_a \frac{\partial U_\alpha}{\partial x_a} = - \frac{\partial P}{\partial x_\alpha} + \frac{\partial}{\partial x_a} \left[(\nu + \underline{\nu_{\mathrm{T}}}) \left(\frac{\partial U_\alpha}{\partial x_a} + \frac{\partial U_a}{\partial x_\alpha} \right) \right]$$

Experimental studies (Kitoh, 1991; Steenbergen, 1995)

circumferential



axial



(Yokoi & Yoshizawa, PoF 1993)

II. Theoretical formulation and modelling

Two-Scale Direct-Interaction Approximation (TSDIA)

Yoshizawa, 1984: mirrorsymmetric case
Yokoi & Yoshizawa, 1993: non-mirrosymmetric case

- **DIA** — A closure theory (propagator renormalization) for homogeneous isotropic turbulence
- **Multiple-scale analysis** — Fast and slowly varying fields

- Introduction of two scales
- Fourier transform of the fast variables
- Scale-parameter expansion
- Introduction of the Green's function
- Statistical assumptions on the basic fields
- Calculation of the statistical quantities using the DIA

(i) Introduction of two scales

Fast and slow variables

$$\boldsymbol{\xi} = \mathbf{x},\ \mathbf{X} = \delta\mathbf{x};\ \tau = t,\ T = \delta t$$

Slow variables ***X*** and *T* change only when ***x*** and *t* change much.

$$f = F\left(\mathbf{X}; T\right) + f'\left(\boldsymbol{\xi}, \mathbf{X}; \tau, T\right)$$

$$\nabla = \nabla_{\boldsymbol{\xi}} + \delta \nabla_{\mathbf{X}}; \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} + \delta \frac{\partial}{\partial T}$$

Velocity-fluctuation equation

$$\frac{\partial u'_\alpha}{\partial \tau} + U_a \frac{\partial u'_\alpha}{\partial \xi_a} + \frac{\partial}{\partial \xi_a} u'_a u'_\alpha + \frac{\partial p'}{\partial \xi_\alpha} - \nu \nabla_\xi^2 u'_\alpha$$
$$= \delta \left(-u'_a \frac{\partial U_\alpha}{\partial X_a} - \frac{D u'_\alpha}{DT} - \frac{\partial p'}{\partial X_\alpha} - \frac{\partial}{\partial X_a} \left(u'_a u'_\alpha - R_{a\alpha} + 2\nu \frac{\partial^2 u'_\alpha}{\partial X_a \partial \xi_a} \right) \right)$$
$$+ \delta^2 \left(\nu \nabla_X^2 u'_\alpha \right)$$

$$\frac{\partial u'_a}{\partial \xi_a} + \delta \frac{\partial u'_a}{\partial X_a} = 0$$

where $$\frac{D}{DT} = \frac{\partial}{\partial T} + \mathbf{U} \cdot \nabla_X$$

Theoretical formulation

Basic field: homogeneous isotropic but non-mirror-symmetric

$$\frac{\langle u'_{0\alpha}(\mathbf{k};\tau) u'_{0\beta}(\mathbf{k};\tau)\rangle}{\delta(\mathbf{k}+\mathbf{k}')} = D_{\alpha\beta}(\mathbf{k}) Q_0(k;\tau,\tau') + \frac{i}{2}\frac{k_a}{k^2}\epsilon_{\alpha\beta a} H_0(k;\tau,\tau')$$

Calculation of the Reynolds stress

$$\left\langle {u'}^{\alpha} {u'}^{\beta} \right\rangle = \left\langle {u'_{\mathrm{B}}}^{\alpha} {u'_{\mathrm{B}}}^{\beta} \right\rangle + \left\langle {u'_{\mathrm{B}}}^{\alpha} {u'_{01}}^{\beta} \right\rangle + \left\langle {u'_{01}}^{\alpha} {u'_{\mathrm{B}}}^{\beta} \right\rangle + \cdots$$
$$+ \left\langle {u'_{\mathrm{B}}}^{\alpha} {u'_{10}}^{\beta} \right\rangle + \left\langle {u'_{10}}^{\alpha} {u'_{\mathrm{B}}}^{\beta} \right\rangle + \cdots$$

$$\left\langle {u'}^{\alpha} {u'}^{\beta} \right\rangle_{\mathrm{D}} = -\nu_{\mathrm{T}} \mathcal{S}^{\alpha\beta} + \left[\Gamma^{\alpha} \left(\Omega^{\beta} + 2\omega_{\mathrm{F}}^{\beta} \right) + \Gamma^{\beta} \left(\Omega^{\alpha} + 2\omega_{\mathrm{F}}^{\alpha} \right) \right]_{\mathrm{D}}$$

where $$\mathcal{S}^{\alpha\beta} = \frac{\partial U^{\alpha}}{\partial x^{\beta}} + \frac{\partial U^{\beta}}{\partial x^{\alpha}} - \frac{2}{3} \nabla \cdot \mathbf{U} \delta^{\alpha\beta}$$

mixing length

$$\nu_{\mathrm{T}} \sim \tau u^2 \sim u\ell$$

Eddy viscosity $$\nu_{\mathrm{T}} = \frac{7}{15} \int \mathrm{d}\mathbf{k} \int_{-\infty}^{t} d\tau_1 \; G(k;\tau,\tau_1) Q(k;\tau,\tau_1)$$

Helicity-related coefficient $$\mathbf{\Gamma} = \frac{1}{30} \int k^{-2} \mathrm{d}\mathbf{k} \int_{-\infty}^{t} d\tau_1 \; G(k;\tau,\tau_1) \nabla H(k;\tau,\tau_1)$$

helicity inhomogeneity is essential

Eddy-viscosity + Helicity model

(Yokoi & Yoshizawa, 1993)

Reynolds stress

$$\mathcal{R}_{\alpha\beta} \equiv \langle u'_\alpha u'_\beta \rangle$$

$$= \frac{2}{3} K \delta_{\alpha\beta} - \nu_{\mathrm{T}} \left(\frac{\partial U_\alpha}{\partial x_\beta} + \frac{\partial U_\beta}{\partial x_\alpha} \right) + \eta \left[\Omega_\alpha \frac{\partial H}{\partial x_\beta} + \Omega_\beta \frac{\partial H}{\partial x_\alpha} - \frac{2}{3} \delta_{\alpha\beta} \left(\boldsymbol{\Omega} \cdot \nabla \right) H \right]$$

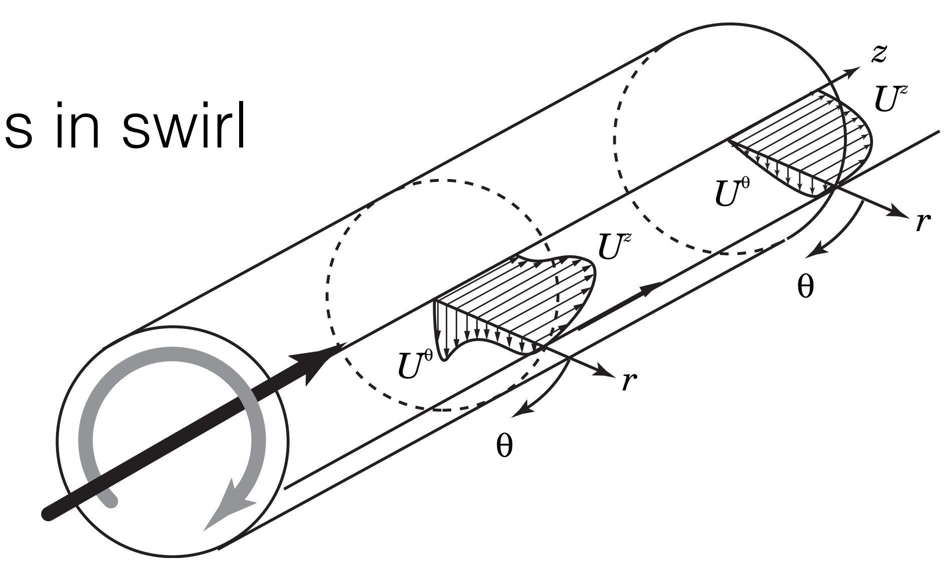
$$\nu_{\mathrm{T}} = C_\nu \tau K, \quad \tau = K/\epsilon, \quad \eta = C_H \tau (K^3/\epsilon^2)$$

Turbulence quantities

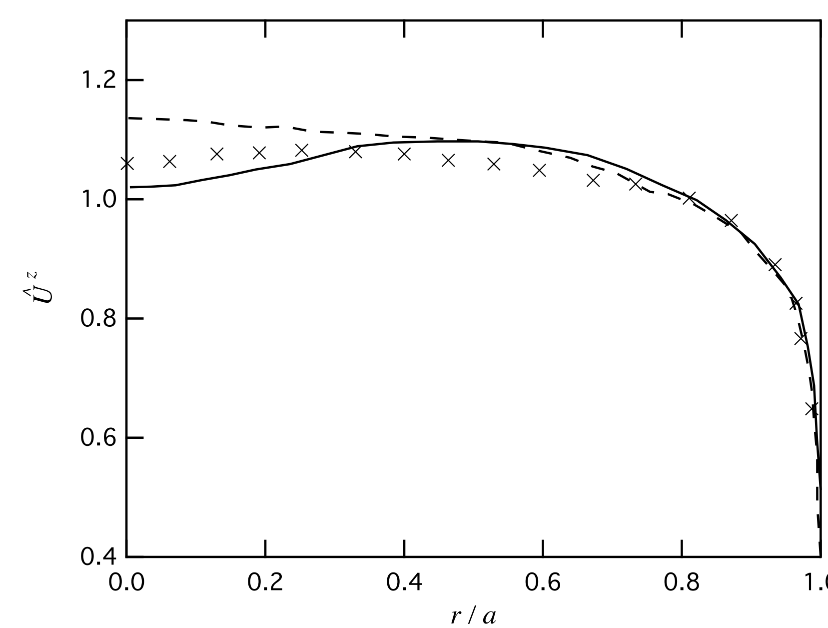
$$K \equiv \frac{1}{2} \langle \mathbf{u}'^2 \rangle, \quad \epsilon \equiv \nu \left\langle \frac{\partial u'_b}{\partial x_a} \frac{\partial u'_b}{\partial x_a} \right\rangle,$$

$$H \equiv \langle \mathbf{u}' \cdot \boldsymbol{\omega}' \rangle, \quad \epsilon_H \equiv 2\nu \left\langle \frac{\partial u'_b}{\partial x_a} \frac{\partial \omega'_b}{\partial x_a} \right\rangle$$

Velocity profiles in swirl

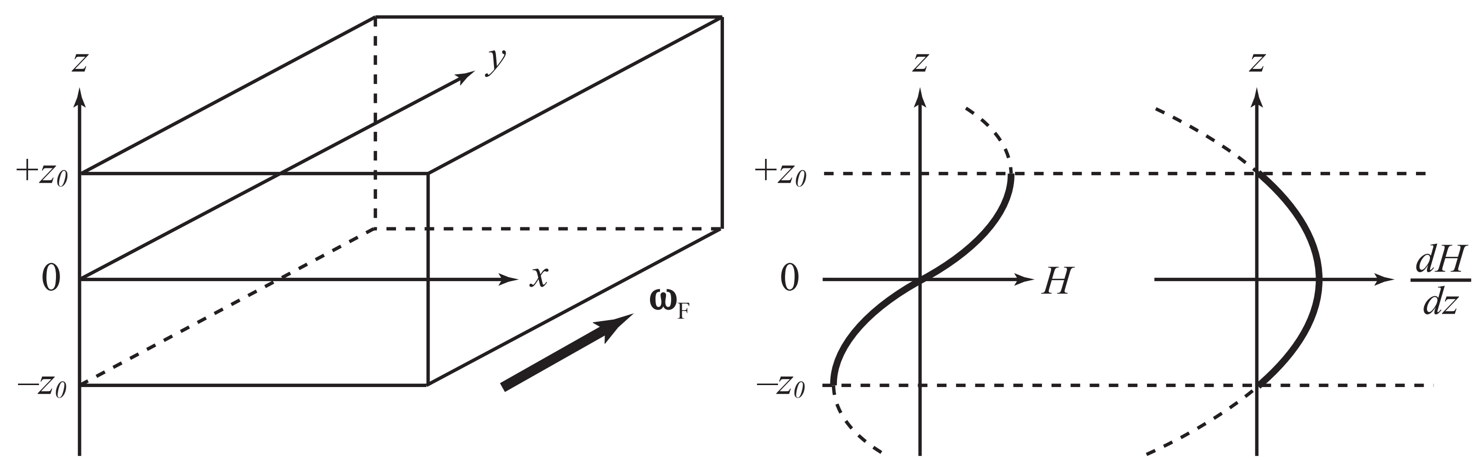


Helicity turbulence model



III. Global flow generation

DNS set-up



Set-up of the turbulence and rotation $\boldsymbol{\omega}_\mathrm{F}$ (left), the schematic spatial profile of the turbulent helicity *H* (= $\langle \mathbf{u}' \cdot \boldsymbol{\omega}' \rangle$) (center) and its derivative *dH/dz* (right).

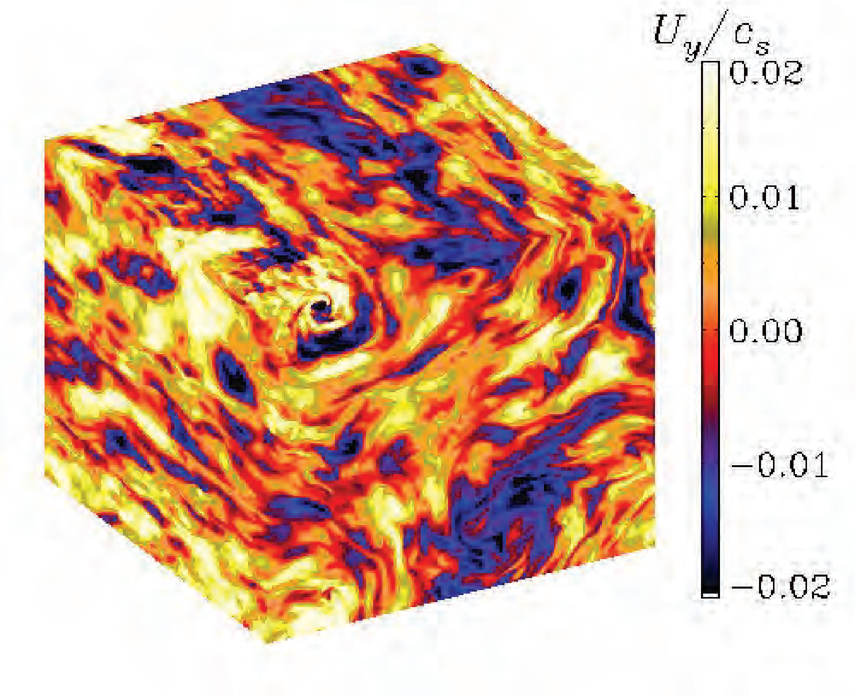
Rotation $$\boldsymbol{\omega}_\mathrm{F} = (\omega_\mathrm{F}^x, \omega_\mathrm{F}^y, \omega_\mathrm{F}^z) = (0, \omega_\mathrm{F}, 0)$$

Inhomogeneous turbulent helicity $$H(z) = -\frac{1}{2} H_0 z (z^2 - 3z_0^2)$$

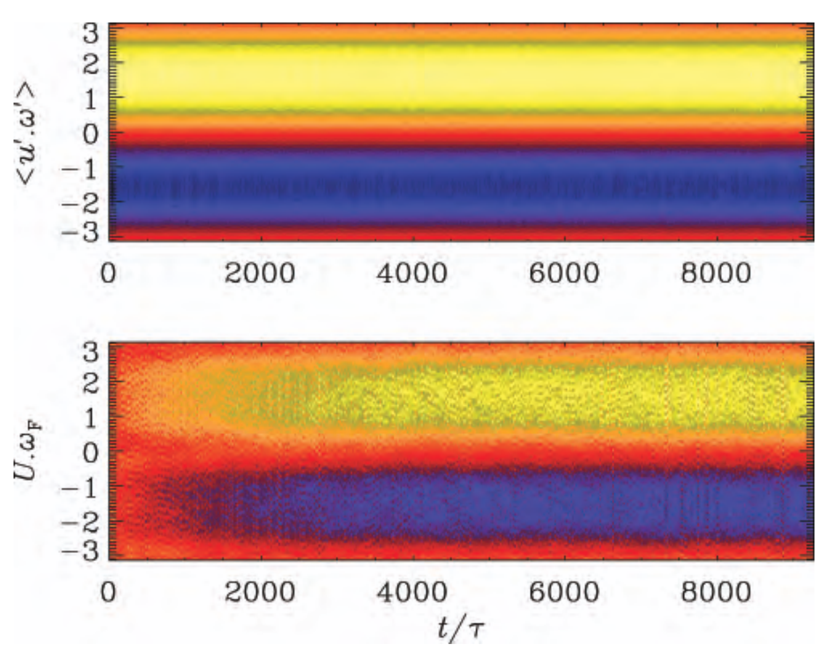
Run	k_f/k_1	Re	Co	$\eta/(\nu_\mathrm{T}\tau^2)$
A	15	60	0.74	0.22
B1	5	150	2.6	0.27
B2	5	460	1.7	0.27
B3	5	980	1.6	0.51
C1	30	18	0.63	0.50
C2	30	80	0.55	0.03
C3	30	100	0.46	0.08

Summary of DNS results

Global flow generation



Axial flow component U^y on the periphery of the domain



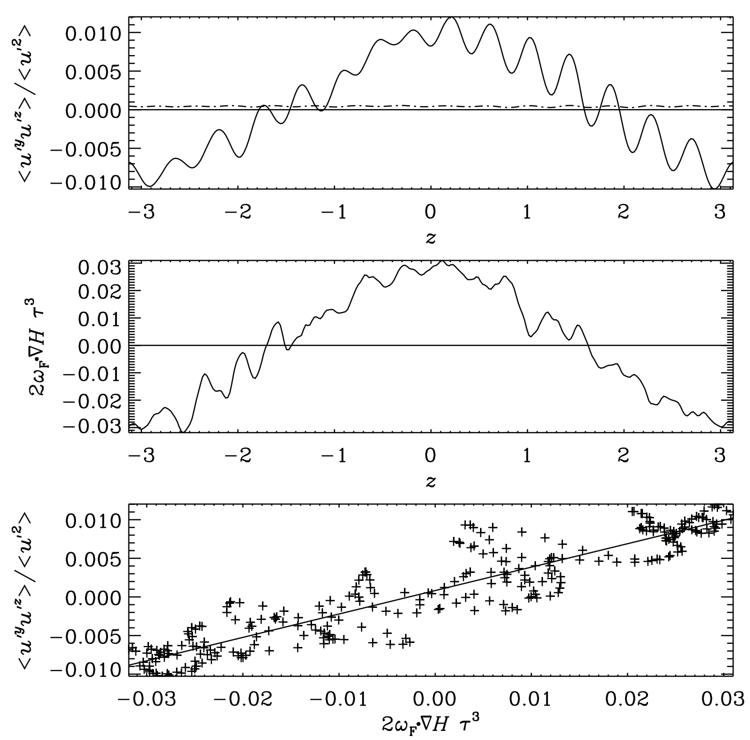
Turbulent helicity $\langle \mathbf{u}' \cdot \boldsymbol{\omega}' \rangle$ (top) and mean-flow helicity $\mathbf{U} \cdot 2\boldsymbol{\omega}_F$ (bottom)

Reynolds stress

$$\left\langle u'^{\alpha} u'^{\beta} \right\rangle_{\rm D} = -\nu_{\rm T} \mathcal{S}^{\alpha\beta} + \left[\Gamma^{\alpha} \left(\Omega^{\beta} + 2\omega_{\rm F}^{\beta} \right) + \Gamma^{\beta} \left(\Omega^{\alpha} + 2\omega_{\rm F}^{\alpha} \right) \right]_{\rm D}$$

Early stage

$$\langle u'^{y} u'^{z} \rangle = \eta 2\omega_{\rm F}^{y} \frac{\partial H}{\partial z}$$

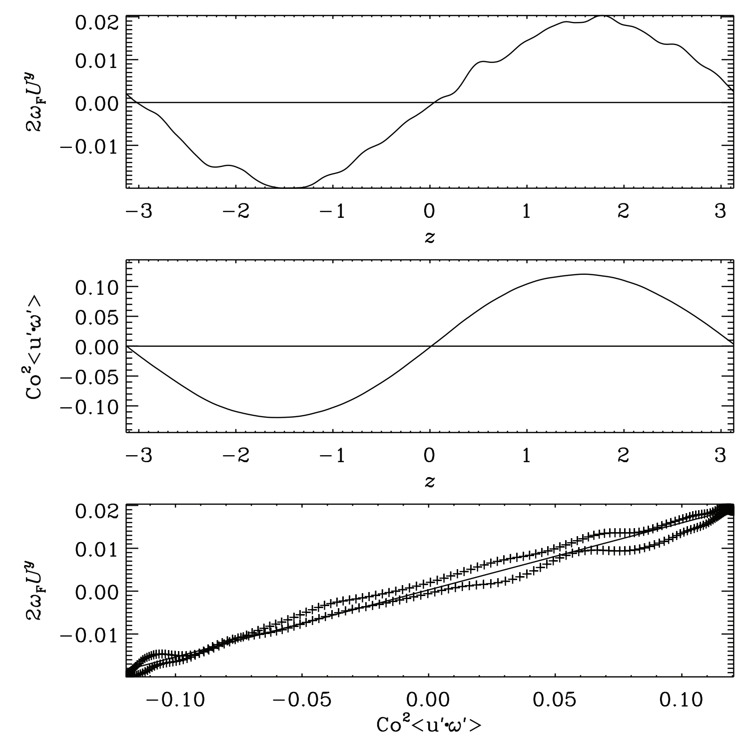


Reynolds stress $\langle u'^{y} u'^{z} \rangle$ (top), helicity-effect term $(\nabla H)^z\, 2\omega_{\rm F}{}^y$ (middle), and their correlation (bottom).

Developed stage

$$\langle u'^{y} u'^{z} \rangle = -\nu_{\rm T} \frac{\partial U^{y}}{\partial z} + \eta 2\omega_{\rm F}^{y} \frac{\partial H}{\partial z}$$

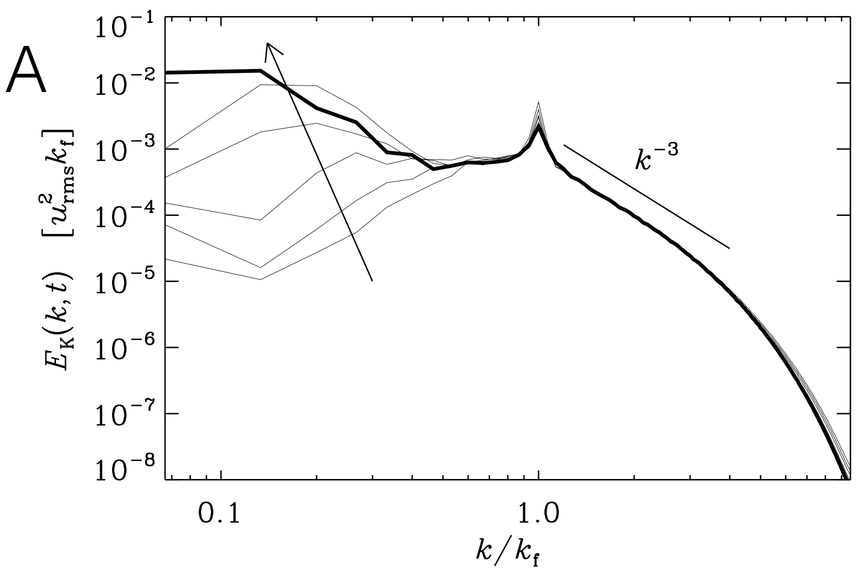
$$U^{y} = (\eta / \nu_{\rm T})\, 2\omega_{\rm F}^{y} H$$



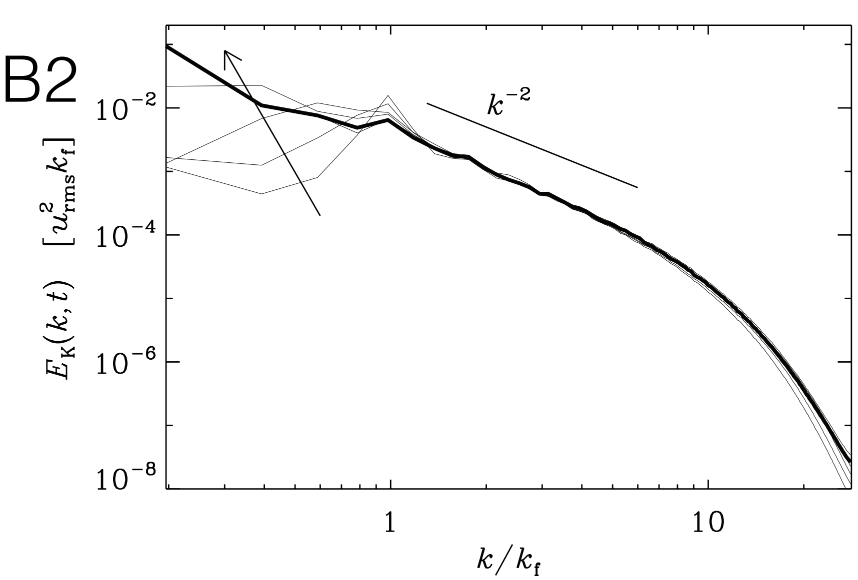
Mean axial velocity U^y (top), turbulent helicity multiplied by rotation $2\omega_{\rm F} H$ (middle), and their correlation (bottom).

Spectra

Run A



Run B2



Physical origin

Reynolds stress $\mathcal{R}^{ij} \equiv \langle u'^i u'^j \rangle$

Vortexmotive force $\mathbf{V}_\mathrm{M} \equiv \langle \mathbf{u}' \times \boldsymbol{\omega}' \rangle$

$$V_\mathrm{M}^i = -\frac{\partial \mathcal{R}^{ij}}{\partial x^j} + \frac{\partial K}{\partial x^i}$$

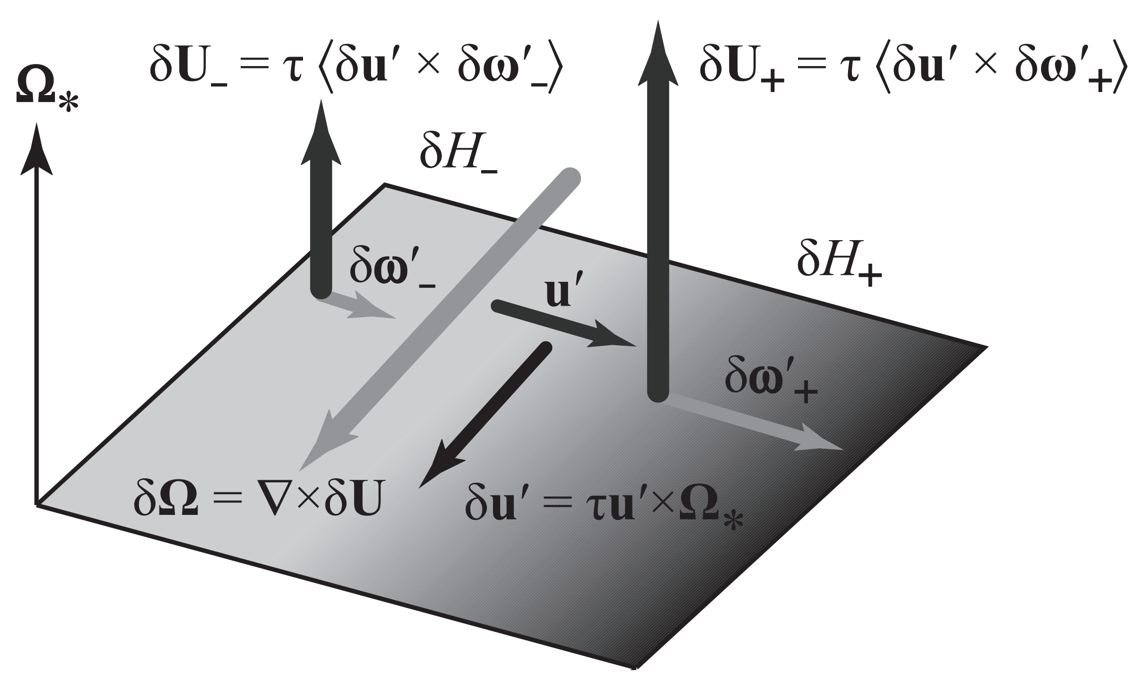
$$\frac{\partial \boldsymbol{\Omega}}{\partial t} = \nabla \times (\mathbf{U} \times \boldsymbol{\Omega}) + \nabla \times \mathbf{V}_\mathrm{M} + \nu \nabla^2 \boldsymbol{\Omega}$$

$$\mathbf{V}_\mathrm{M} = -D_\Gamma 2\boldsymbol{\omega}_\mathrm{F} - \nu_\mathrm{T} \nabla \times \boldsymbol{\Omega}$$

$$D_\Gamma = \nabla \cdot \boldsymbol{\Gamma} \propto \nabla^2 H$$

→ $$\delta \mathbf{U} \sim -(\nabla^2 H)\boldsymbol{\Omega}_*$$

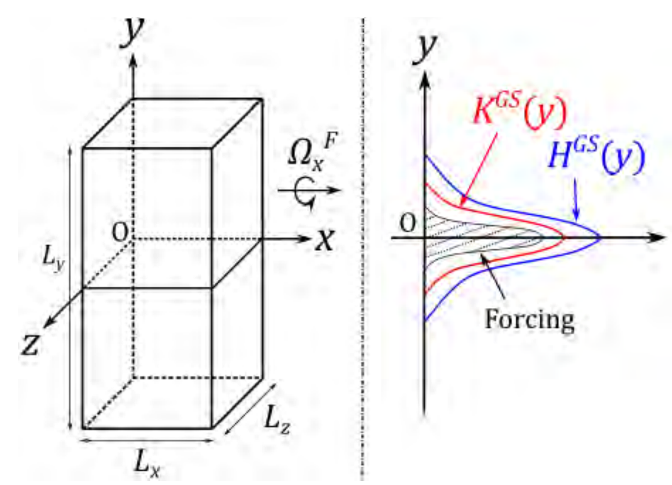
$$\nabla^2 H \simeq -\frac{\delta H}{\ell^2} = -\frac{\langle \mathbf{u}' \cdot \delta\boldsymbol{\omega}' \rangle}{\ell^2}$$

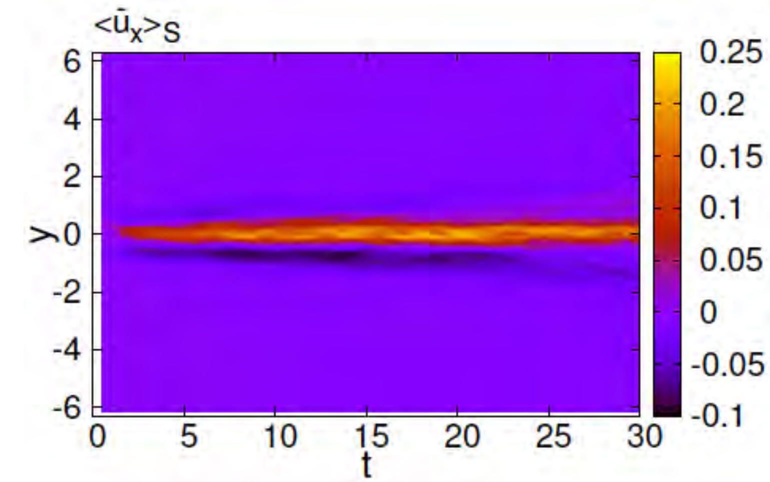


Reynolds stress evolution

(Inagaki, Yokoi & Hamba, submitted to Phys. Rev. Fluids)

Local helical forcing





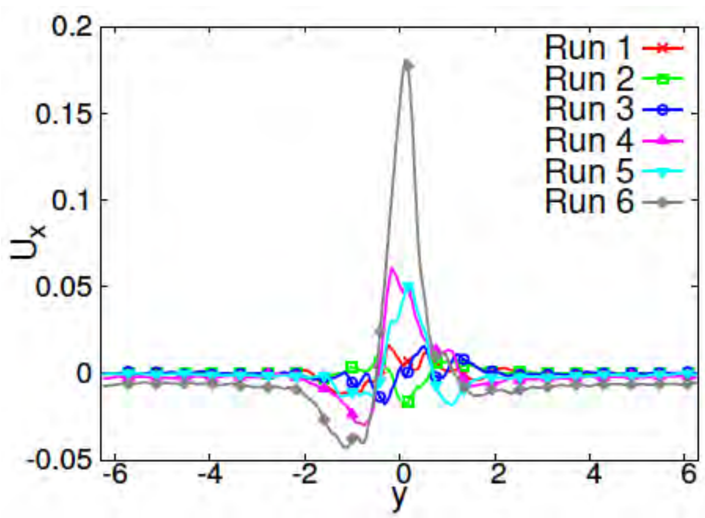
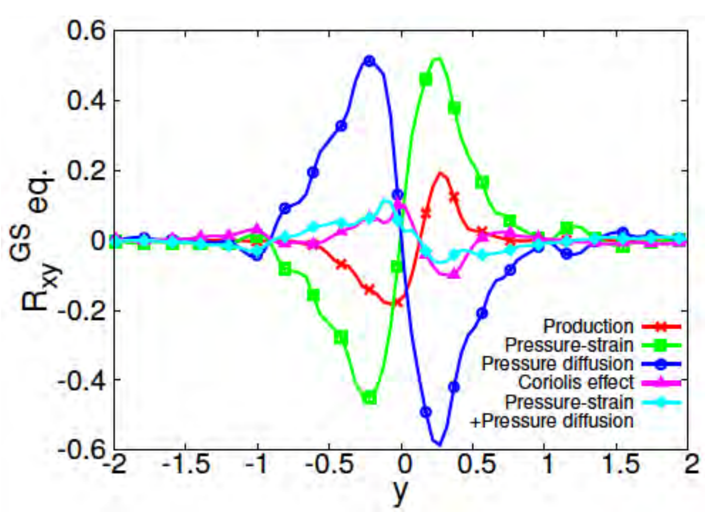


TABLE I. Calculation parameters.

Run	α	Ω_x^F	L_0^{GS}	Ro_0^{GS}
1	0	0	0.506	∞
2	0.5	0	0.547	∞
3	0	5	0.542	0.185
4	0.2	5	0.550	0.182
5	0.5	2	0.544	0.459
6	0.5	5	0.602	0.166

$$\frac{\partial R_{xy}^{GS}}{\partial t} \simeq P_{xy}^{GS} + \Phi_{xy}^{GS} + \Pi_{xy}^{GS} + C_{xy}^{GS} \simeq 0$$



Production $P_{xy}^{GS} = -\frac{2}{3}K^{GS}\frac{\partial U_x}{\partial y} - B_{yy}^{GS}\frac{\partial U_x}{\partial y} - B_{xz}^{GS}\frac{\partial U_z}{\partial y}$

Press. strain $\Phi_{xy}^{GS} = 2\left\langle \overline{p}'\overline{s}'_{xy}\right\rangle$

Press. diff. $\Pi_{xy}^{GS} = -\frac{\partial}{\partial y}\langle \overline{p}'\overline{u}'_x\rangle$ $\quad T_{xy}^{GS} = -\frac{\partial}{\partial y}\left\langle \overline{u}'_x \overline{u}'^{\,2}_y\right\rangle$

Coriolis $C_{xy}^{GS} = 2R_{xz}^{GS}\Omega_x^F$

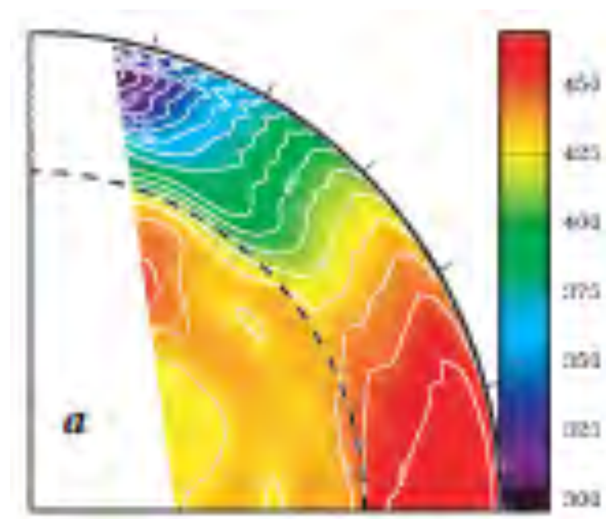
IV. Stellar convection zone

Angular-momentum transport in the solar convection zone

Angular momentum around the rotation axis

$$L = \Gamma r^2 \omega_{\rm F} + \Gamma r U^\phi \qquad \Gamma = \sin\theta$$

$$\frac{\partial}{\partial t}\rho L + \nabla\cdot(\rho \mathbf{F}_L) = 0$$



Miesch (2005) Liv. Rev. Sol. Phys. 2005-1

Vector flux of angular momentum $\mathbf{F}_L$

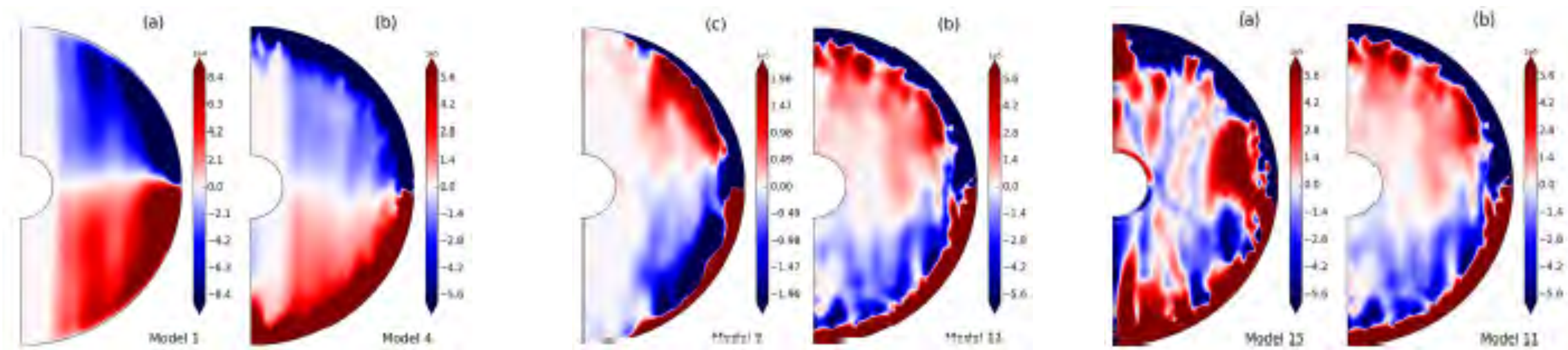
$$F_L^r = L U^r + r\Gamma \mathcal{R}^{r\phi}$$

$$F_L^\theta = L U^\theta + r\Gamma \mathcal{R}^{\theta\phi}$$

Helicity effect $\displaystyle \mathcal{R}_H^{r\phi} = +\frac{\partial H}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r} r U^\theta - \frac{1}{r}\frac{\partial U^r}{\partial \theta}\right)$

$$\mathcal{R}_H^{\theta\phi} = +\frac{1}{r}\frac{\partial H}{\partial \theta}\left(\frac{1}{r}\frac{\partial}{\partial r} r U^\theta - \frac{1}{r}\frac{\partial U^r}{\partial \theta}\right)$$

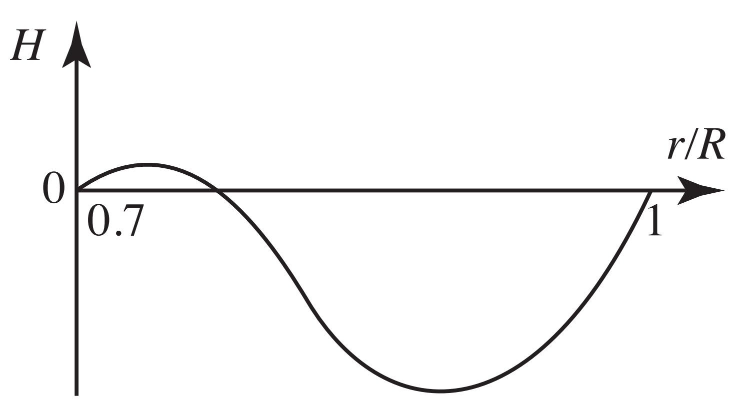
Helicity effect in the stellar convection zone



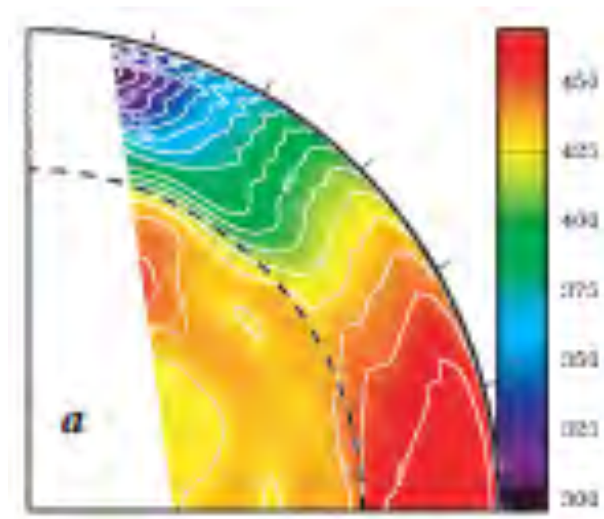
Duarte, et al, (2016) MNRAS **456**, 1708

$$\delta \mathbf{U} \sim -(\nabla^2 H)\mathbf{\Omega}_*$$

Schematic helicity distribution

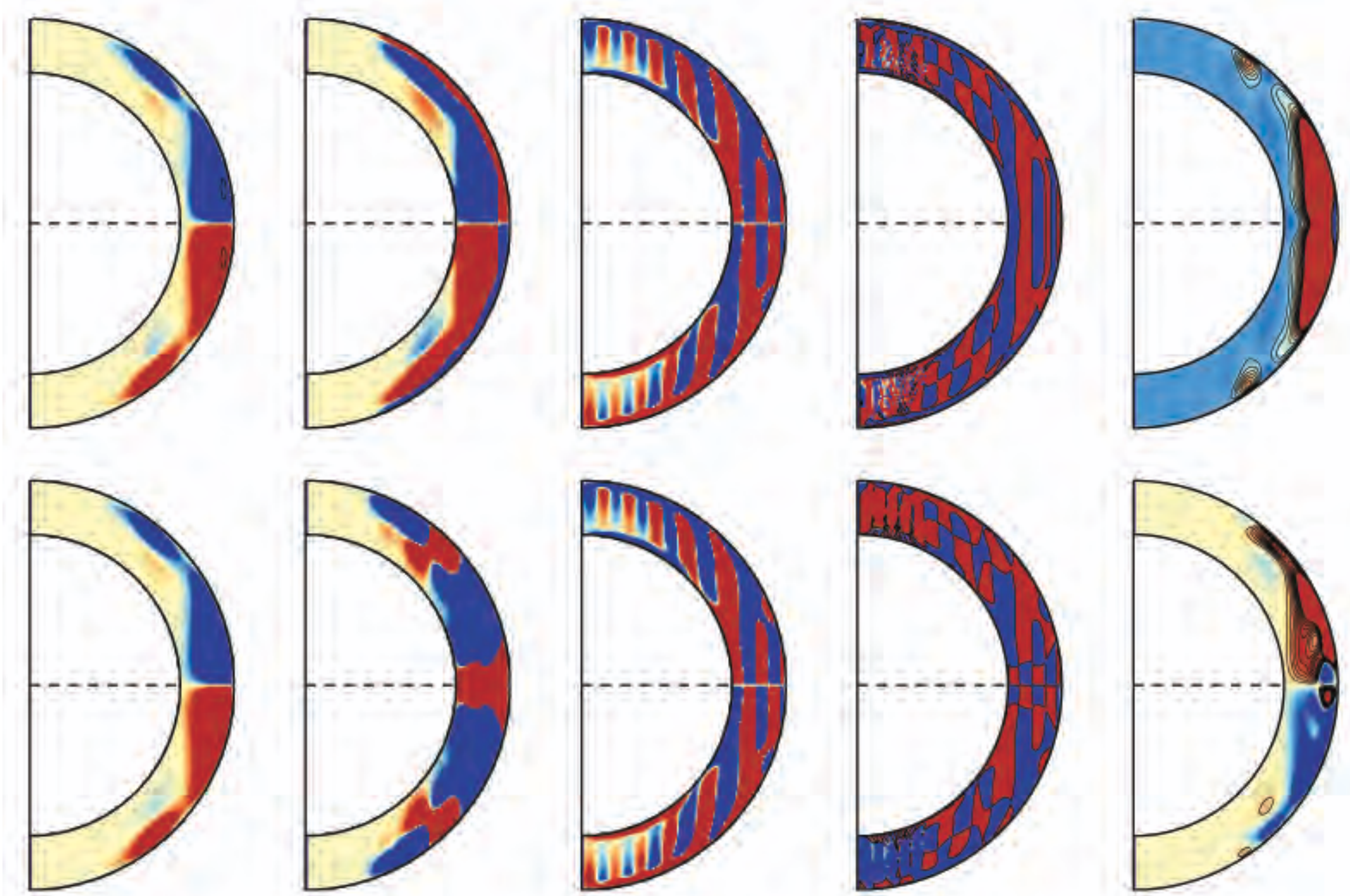


$$\nabla^2 H > 0$$



Helicity effect in the Reynolds stress

Helicity	Helicity Gradient	Azimuthal Vorticity	Helicity effect	Reynolds stress
$\mathbf{u}\cdot\boldsymbol{\omega}$	$\frac{\partial H}{\partial r}$	$\overline{\Omega}^\phi$	$\frac{\partial H}{\partial r}\overline{\Omega}^\phi$	$\overline{u'^r u'^\phi}$



$\mathbf{u}\cdot\boldsymbol{\omega} - \overline{\mathbf{u}}\cdot\overline{\boldsymbol{\omega}}$ $(\equiv H)$	$\frac{1}{r}\frac{\partial H}{\partial \theta}$	$\overline{\Omega}^\phi$	$\frac{1}{r}\frac{\partial H}{\partial \theta}\overline{\Omega}^\phi$	$\overline{u'^\theta u'^\phi}$

(provided by Mark Miesch)

$C_\eta \tau \ell^2 |(\nabla^2 H)\Omega_*|$

Solar parameters

$$v \sim 200\ \mathrm{m\ s^{-1}} = 2\times 10^4\mathrm{cm\ s^{-1}}$$
$$\ell \sim 200\ \mathrm{Mm} = 2\times 10^{10}\mathrm{cm}$$
$$\tau \sim \ell/v \sim 10^6\ \mathrm{s}$$

$r\phi$ component

$$\left|\overline{u'^r u'^\phi}\right| \sim 1.2\times 10^9$$
$$\left|\frac{\partial H}{\partial r}\overline{\Omega}^\phi\right| \sim 9.4\times 10^{-15}$$
$$\tau\ell^2\left|\frac{\partial H}{\partial r}\overline{\Omega}^\phi\right| \sim 10^{12} \longrightarrow 10^9$$

with $C_\eta = O(10^{-3})$

$\theta\phi$ component

$$\left|\overline{u'^\theta u'^\phi}\right| \sim 5.6\times 10^8$$
$$\left|\frac{1}{r}\frac{\partial H}{\partial \theta}\overline{\Omega}^\phi\right| \sim 2.6\times 10^{-15}$$
$$\tau\ell^2\left|\frac{1}{r}\frac{\partial H}{\partial \theta}\overline{\Omega}^\phi\right| \sim 10^{11} \longrightarrow 10^8$$

Magnitude same as the Reynolds stress

V. Summary

Summary

In turbulent momentum transport in hydrodynamics

$$\left\langle u'^{\alpha} u'^{\beta} \right\rangle_{\mathrm{D}} = -\nu_{\mathrm{T}} \mathcal{S}^{\alpha\beta} + \left[\Gamma^{\alpha} \left(\Omega^{\beta} + 2\omega_{\mathrm{F}}^{\beta} \right) + \Gamma^{\beta} \left(\Omega^{\alpha} + 2\omega_{\mathrm{F}}^{\alpha} \right) \right]_{\mathrm{D}}$$

Mean velocity strain (symmetric part of velocity shear)
+ Energy
⟶ Transport enhancement (structure destruction)

Mean absolute vorticity (antisymmetric part of velocity shear)
+ (Inhomogeneous) Helicity
⟶ Transport suppression (structure formation)

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N. Yokoi & A. Yoshizawa, Phys. Fluids A **5**, 464 (1993)

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