Measurements of the Absorption and Scattering Cross Sections for the Interactions of Solar Acoustic Waves with Sunspots

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Abstract

The solar acoustic waves are modified by the interaction with sunspots. The interaction can be treated as a scattering problem: an incident wave propagating toward a sunspot is scattered by the sunspot into different modes. The absorption cross section and scattering cross section are two important parameters in the scattering problem. In this study, we use the wavefunction of the scattered wave, measured with a deconvolution method, to compute the absorption cross section σ_{ab} and the scattering cross section σ_{sc} for the radial order $n = 0-5$ for two sunspots, NOAAs 11084 and 11092. In the computation of the cross sections, the random noise and dissipation in the measured acoustic power are corrected. For both σ_{ab} and σ_{sc}, the value of NOAA 11092 is greater than that of NOAA 11084, but their overall n dependences are similar: decreasing with n. The ratio of σ_{ab}s of two sunspots is approximately equal to the ratio of sunspot radii for all ns, while the ratio of σ_{sc}s of two sunspots is greater than the ratio of sunspot radii and increases with n. This suggests that σ_{ab} is approximately proportional to the sunspot radius, while the change of σ_{sc} with radius is faster than the linear increase.

Data and Preliminary Data Reduction

- HMI 4096×4096 full-disk Dopplergrams (one image/45 seconds)
- Two sunspot regions studied: NOAA 11084 (12402 frames) and NOAA 11092(12118 frames)
- Remap to coordinate of (φ, θ) (or (x, y)) centered at the sunspot center; a area of 60°×60° (1024×1024 pixels) is selected– see **Figure 1**.
- Differential rotation is remove.
- Radius of penumbra: NOAA 11084 is 15pixels and NOAA 11092 is 20 pixels

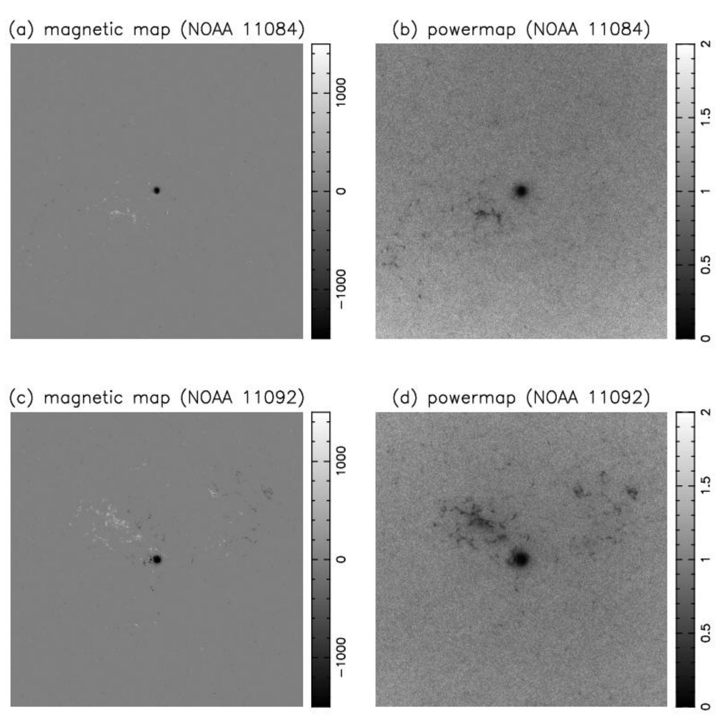

Figure1. Magnetic maps(left panels) and acoustic-power maps (right panels) of NOAAs 11084 and 11092.

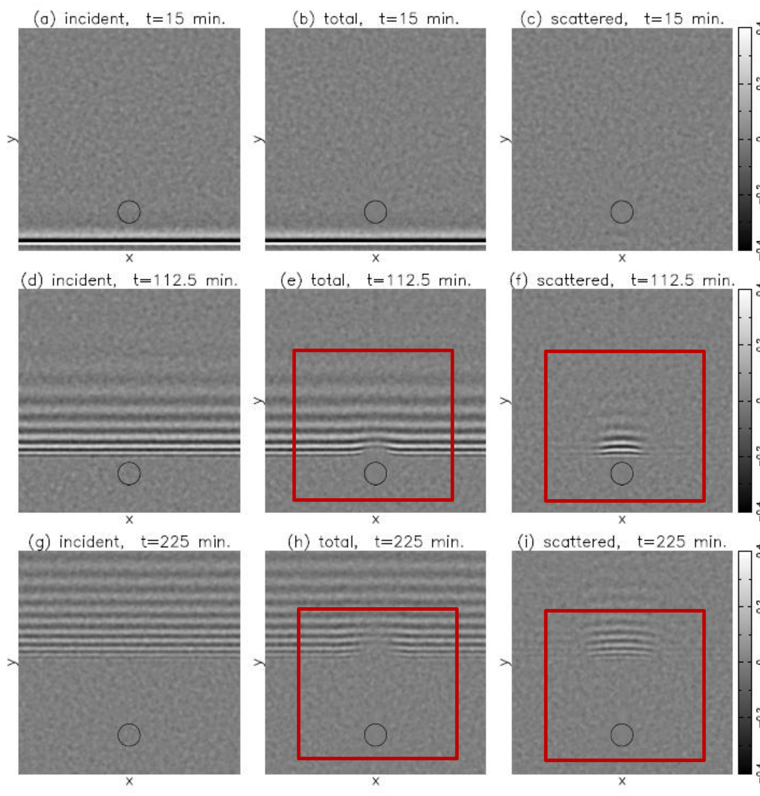

Figure 2. Wavefunctions of incident wave (left panels), total wave (middle panels), and scattered wave (right panels) of NOAA 11092 using the radial order n=2, at three different times. The amplitude of the incident wave on the reference line at t=0 is set to unity. The circle indicates the location and size of the penumbra.

Measurements of Acoustic Wavefunctions

- The incident wave of the scattering problem is a plane wave packet propagating toward the sunspot, formed by a ridge filter and averaged over a reference line at $y=y_0$ (70 pixels from the sunspot center), denoted as $\overline{\psi}(y_0,t)$.
- A deconvolution method developed by Zhao et al. (2011) is used to measure the wavefunction of total wave Ψ(x, y, t) around the sunspot as

$$\Psi(x,y,t) = A^{-1}\sum_{\omega}\frac{\overline{\psi}^*(y_0,\omega)\psi(x,y,\omega)}{|\overline{\psi}(y_0,\omega)|}e^{-i\omega t} = A^{-1}\sum_{\omega}|\psi(x,y,\omega)|\,e^{i[\varepsilon(x,y,\omega)-\bar{\varepsilon}(y_0,\omega)]}e^{-i\omega t} \quad (1)$$

where $\varepsilon(x,y,\omega)$ and $\bar{\varepsilon}(y_0,\omega)$ are the phases of $\psi(x, y, \omega)$ and $\overline{\psi}(y_0,\omega)$, respectively.
- The same method applied to the quiet Sun to obtain the wavefunction of incident wave $\Psi_0(x, y, t)$. The scattered wavefunction is $\Psi_s(x, y, t)=\Psi(x, y, t)-\Psi_0(x, y, t)$. An example is shown in **Figure 2**.

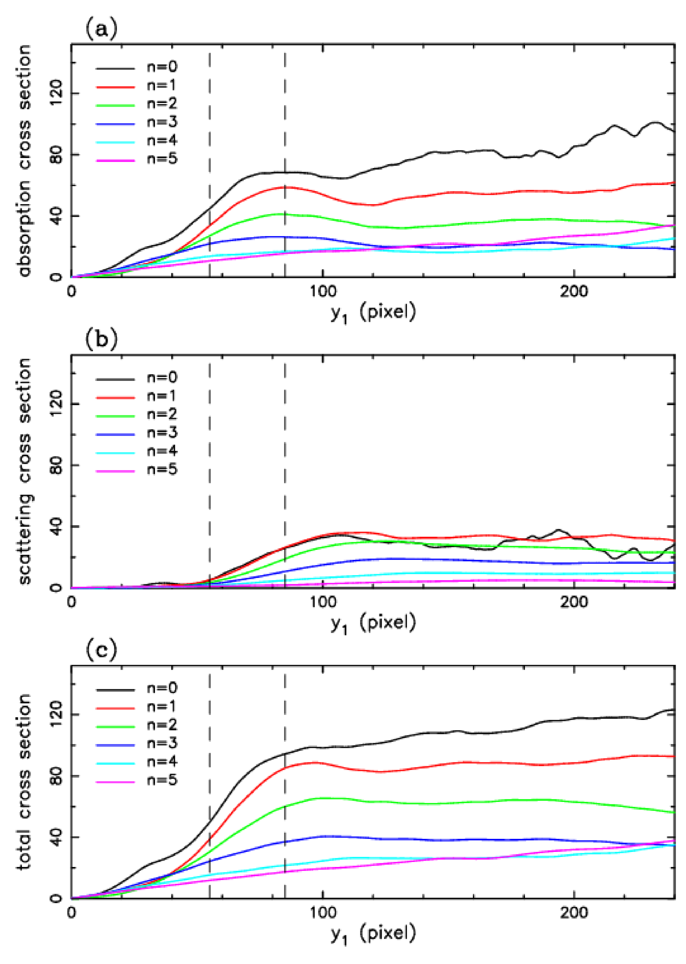

Figure 3. Absorption (panel(a)), scattering (panel(b)), and total cross sections (panel(c)) as a function of y_1 in units of pixel for NOAA 11084. The center of sunspot is located a y_1=70, and two vertical dashed lines indicate the range of penumbra.

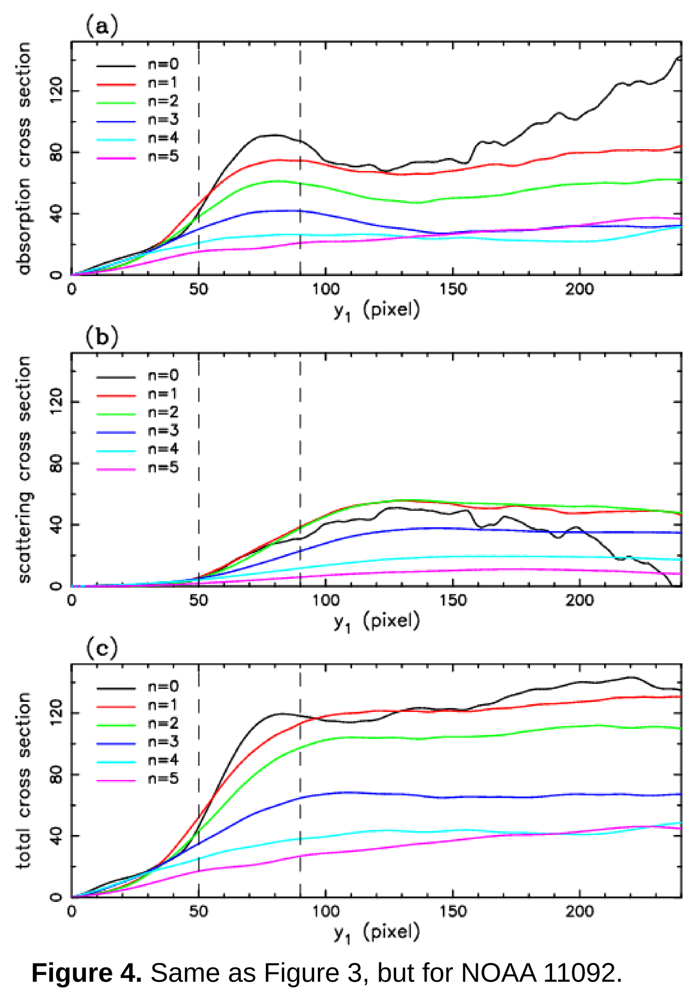

Figure 4. Same as Figure 3, but for NOAA 11092.

Measurements of Absorption and Scattering Cross Sections

1. Definitions

The one-dimensional absorption cross section $\sigma_{ab}(\omega)$ can be expressed in terms of the total wavefunction $\Psi_t(x, y, \omega)$ and the incident wavefunction $\overline{\Psi}_0(y_0,\omega)$ as

$$\sigma_{ab}(\omega) = \frac{-\oint_{S_1}|\Psi_t(x,y,\omega)|^2\hat{v}_t\cdot\hat{s}\,ds}{|\overline{\Psi}_0(y_0,\omega)|^2} \quad (2)$$

where $\hat{v}_t$ is the unit vector in the direction of the wave vector of $\Psi_t(x, y, \omega)$, and $\hat{s}$ is the unit vector of the surface element ds pointing outward from the surface. The integral is over a close contour S_1, a red rectangular in **Figure 2**. The upper boundary is the reference line, and the lower boundary $y=y_1$ is outside the sunspot.

The one-dimensional scattering cross section $\sigma_{sc}(\omega)$ can be expressed in terms of the wavefunction of scattered wave $\Psi_s(x, y, \omega)$ and the incident wavefunction $\overline{\Psi}_0(y_0,\omega)$ as

$$\sigma_{sc}(\omega) = \frac{\oint_{S_1}|\Psi_s(x,y,\omega)|^2\hat{v}_s\cdot\hat{s}\,ds}{|\overline{\Psi}_0(y_0,\omega)|^2} \quad (3)$$

where $\hat{v}_s$ is the unit vector in the direction of the wave vector of $\Psi_s(x, y, \omega)$. Due to the low S/N, we can compute only the cross section of n instead of each ω. From Parseval's theorem, each power in equations **(2)** and **(3)** is replaced by the power summed over time, denoted with a tilde sign.

2. Correction of Random Noise

We assume that the power of random noise in the active region equals to that in the quiet region. The power of random noise in the quiet Sun is approximately equal to $\sum_x|\tilde{\Psi}^{(qs)}(x,y_1)|^2 - N_x|\tilde{\overline{\Psi}}^{(qs)}(y_1)|^2$.

3. Correction of Dissipation

We assume that the rate of decrease in power in the active region is the same as that in the quiet Sun. Thus, to correct dissipation, the power in the active region is normalized by the power in the quiet Sun.

4. Computation of absorption and scattering cross sections

After the random noise and dissipation corrections , the σ_{ab} and σ_{sc} for each n can be expressed in terms of three measured quantities as

$$\sigma_{ab} \approx \frac{\sum_x|\tilde{\Psi}^{(qs)}(x,y_1)|^2 - \sum_x|\tilde{\Psi}_t^{(ar)}(x,y_1)|^2}{|\tilde{\overline{\Psi}}^{(qs)}(y_1)|^2} \quad (4)$$

$$\sigma_{sc} \approx N_x + \frac{\sum_x|\tilde{\Psi}_s^{(ar)}(x,y_1)|^2 - \sum_x|\tilde{\Psi}^{(qs)}(x,y_1)|^2}{|\tilde{\overline{\Psi}}^{(qs)}(y_1)|^2} \quad (5)$$

5. Results

A. The values of σ_{ab} and σ_{sc} vs. y_1 are shown in **Figures 3** and **4**. The fact that they are approximately constant in y_1 suggests our random noise and dissipation corrections work reasonably well.
B. The values of σ_{ab} and σ_{sc} for each n are computed by averaging over y_1=140-200 in **Figures 3** and **4**; the results are shown in **Figure 5**.
C. The values of σ_{ab} and σ_{sc} normalized by the sunspot radius are shown in **Figure 6**.
D. The σ_{ab} ratio and σ_{sc} ratio of two sunspots are shown in **Figure 7**.

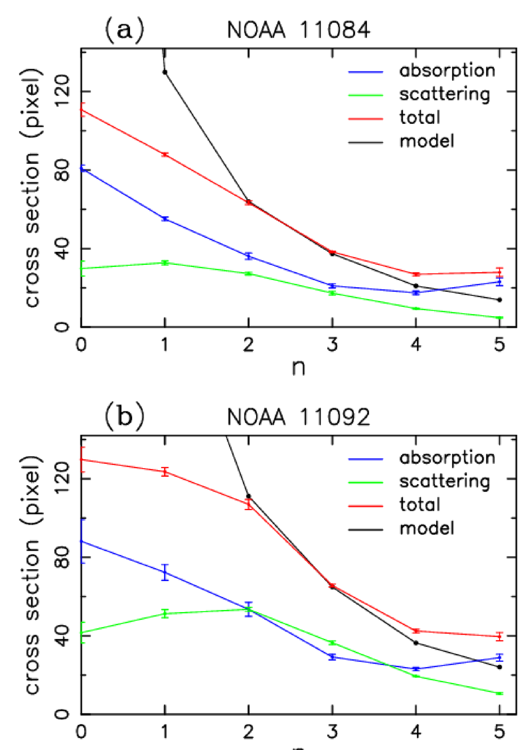

Figure 5. Panel(a): average of absorption, scattering, and total cross sections over y_1=140-200 in Figure 3 of NOAA 11084. Panel(b): results of NOAA 11092.

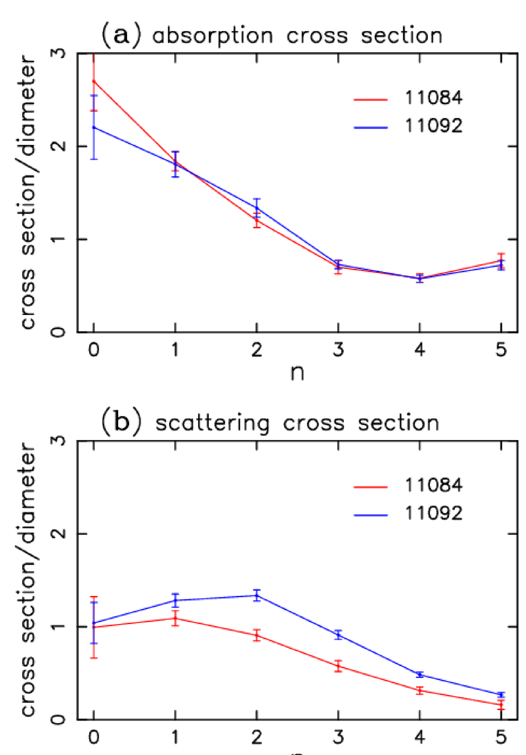

Figure 6. Panel(a): absorption cross section normalized by the sunspot radius for NOAAs 11084 and 11092. Panel(b): scattering cross section normalized by the sunspot radius.

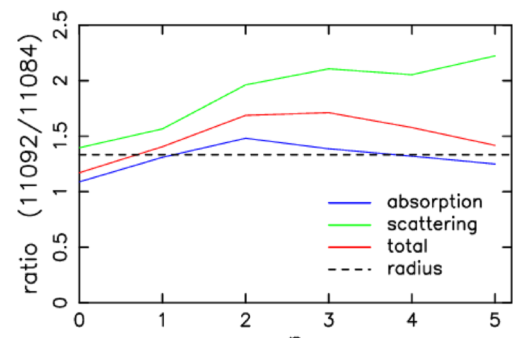

Figure 7. Ratio of the cross section of NOAA 11092 to that of NOAA 11084 for the absorption cross section, the scattering cross section, and the total cross section. The ratio of penumbra radii is shown as the dashed line.

Conclusions

- The mechanisms generating σ_{ab} and σ_{sc} are different; the regions responsible for two mechanisms may be different.
- The corrections of random noise and dissipation are necessary in measuring σ_{ab} and σ_{sc}.
- Both σ_{ab} and σ_{sc} decreases with n for two sunspots.
- The values of σ_{ab} and σ_{sc} of the larger sunspot are greater than those of the smaller sunspot.
- The value of σ_{ab} normalized by the sunspot radius is similar for two sunspots for all n, while the normalized σ_{sc} of the larger sunspot is greater than that of the smaller sunspot (**Figure 6**).
- The σ_{ab} ratio of two sunspots is approximately equal to the sunspot radius ratio for all n, while the σ_{sc} ratio is greater than the radius ratio and increases with n (**Figure 7**).
- The above results suggest that σ_{ab} increases linearly with the sunspot radius, while the change of σ_{sc} with radius is faster than the linear increase.

References

1. **Zhao, H., Chou, D.-Y., & Yang, M.-H. 2011, ApJ, 740, 56**
2. **Zhao, H. & Chou, D.-Y. 2013, ApJ, 778,4**