Convective full-sphere cross-helicity dynamo

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Dynamo equation:

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\langle \mathbf{U} \rangle \times \langle \mathbf{B} \rangle + \boldsymbol{\mathcal{E}})$$

Effect of turbulence is represented by the mean electromotive force:

$$\boldsymbol{\mathcal{E}} = \langle \mathbf{u}' \times \mathbf{b}' \rangle$$

The cross-helicity density is: $\langle \gamma \rangle = \langle \mathbf{u}' \cdot \mathbf{b}' \rangle$

Having: $(\mathbf{u}' \cdot \mathbf{b}')^2 + (\mathbf{u}' \times \mathbf{b}')^2 = \mathbf{u}'^2 \mathbf{b}'^2$

The more $\langle \mathbf{u}' \cdot \mathbf{b}' \rangle$ the less $\langle \mathbf{u}' \times \mathbf{b}' \rangle$

Example, nonlinear α effect

B-field prevents motions across by factor B^{-1} in each direction, Proctor (2007), increasing $\langle \mathbf{u} \cdot \mathbf{b} \rangle$

Coriolis force

B- quenching

$$\alpha \sim -\tau_c \frac{\langle \boldsymbol{u}' \cdot \boldsymbol{\nabla} \times \boldsymbol{u}' \rangle}{1 + \left(\frac{B}{B_{eq}} \right)^2} \sim -\tau_c \left(\mathbf{G} \cdot \boldsymbol{\Omega} \right) \frac{\left\langle \boldsymbol{u}'^2 \right\rangle}{1 + \left(\frac{B}{B_{eq}} \right)^2}$$

Arguments “pro”

(see Yokoi 2013)

The origin of the turbulent generation effects, e.g., $\boldsymbol{\mathcal{E}} = \alpha \mathbf{B} + \ldots$ is related to the large-scale parameters: global rotation,stratification, etc.

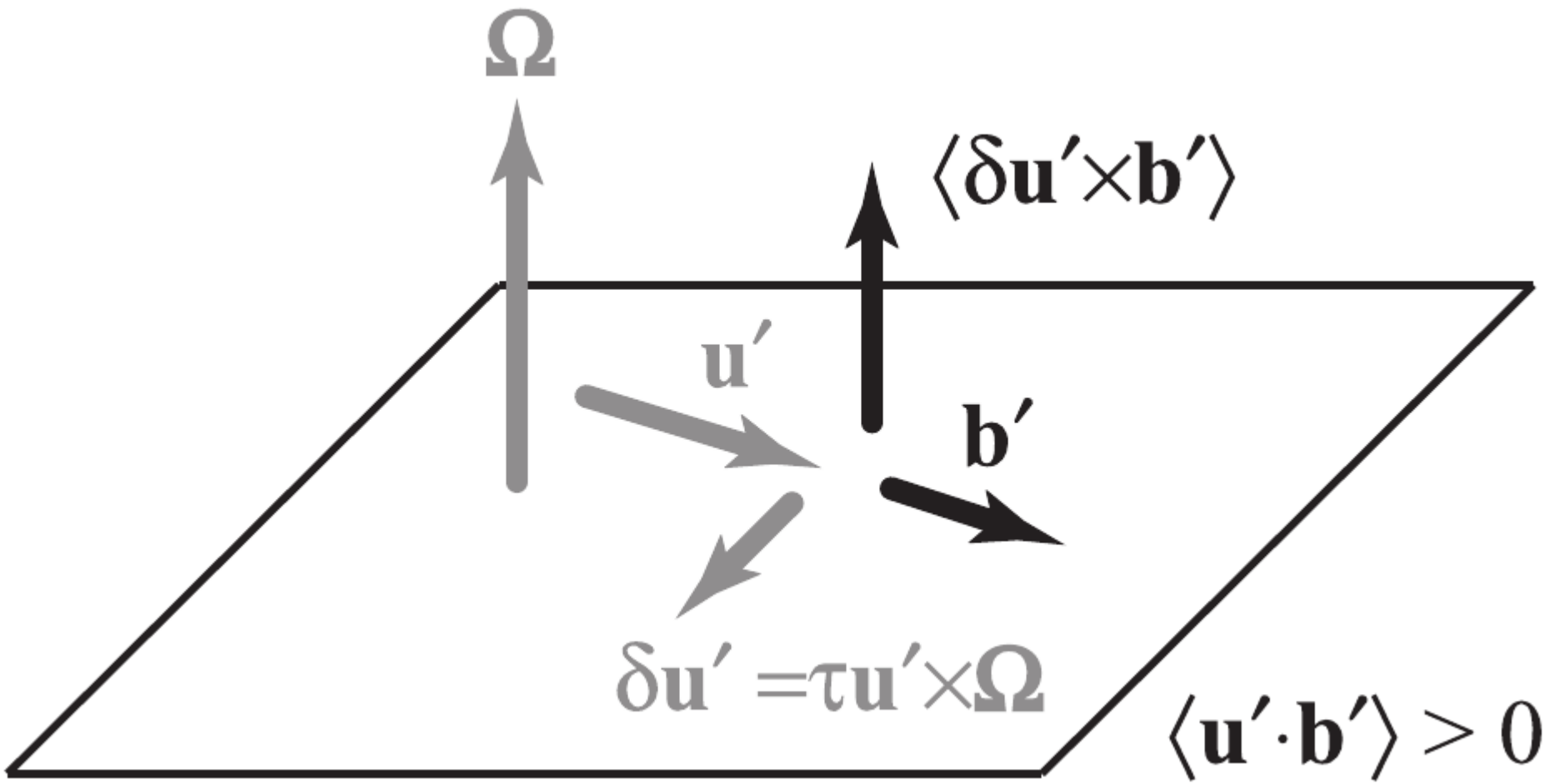
Amplitude of turbulent generation effects is only a few percents of turbulent energy, i.e., $\alpha \leq 0.1 \left|\mathbf{u}'\right|$ Parker(1955)

Thefore the balance is not critical $\left(\mathbf{u}' \cdot \mathbf{b}'\right)^2 + \left(\mathbf{u}' \times \mathbf{b}'\right)^2 = \mathbf{u}'^2 \mathbf{b}'^2$

In stellar turbulence the $\left\langle \mathbf{u}' \cdot \mathbf{b}' \right\rangle$ can be produced without Lorentz force. In this case it contribute to the mean EMF:
(Yoshizawa&Yokoi 1996; Yokoi 2013)

$$\boldsymbol{\mathcal{E}} = \hat{\alpha} \circ \overline{\mathbf{B}} + \boldsymbol{V}^{(p)} \times \overline{\mathbf{B}} + \gamma \Omega \tau_c - \hat{\eta} \circ \left(\nabla \times \overline{\mathbf{B}}\right) + \ldots$$

Yokoi (2013)



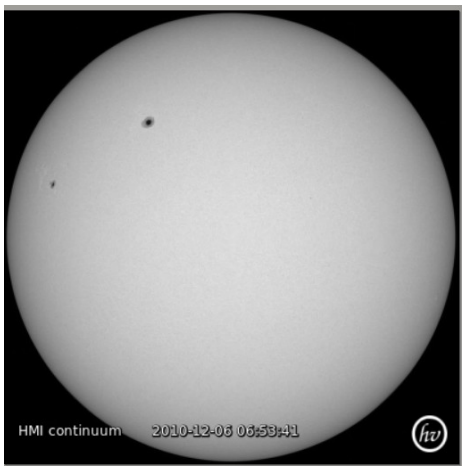
Contribution to the turbulent EMF is parallel to the mean vorticity if the cross helicity is positive in turbulence.

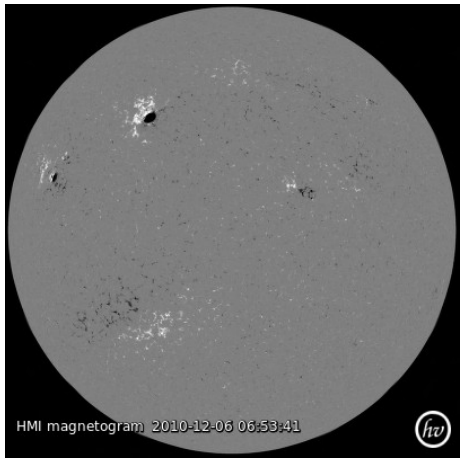
Observations

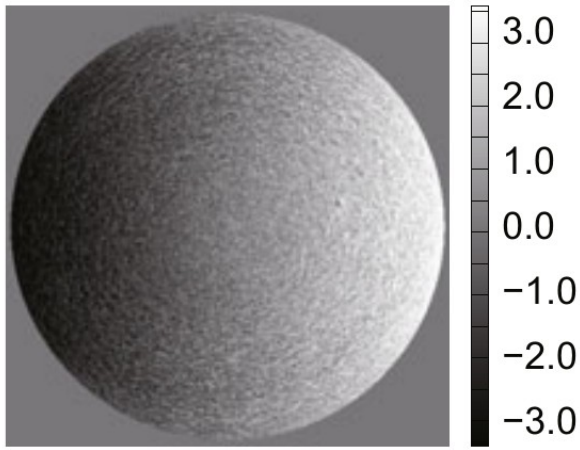
- Correlation of the magnetic network and supercranulation patterns on the solar surface
- Sunspots

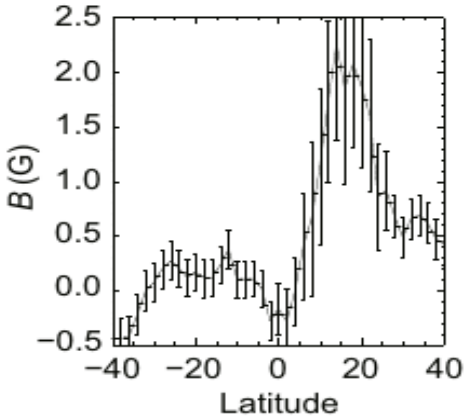
Example of the cross-helicity analysis of Zhao, Wang & Zhang (2014)

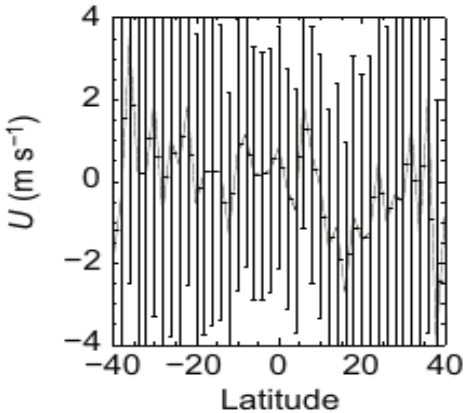
$$max\,(\mathbf{u}\cdot\mathbf{b}) \sim 10^3 \mathrm{G\ km/s} \qquad |\langle \mathbf{u}\cdot\mathbf{b}\rangle| \sim 1\mathrm{G\ km/s}$$

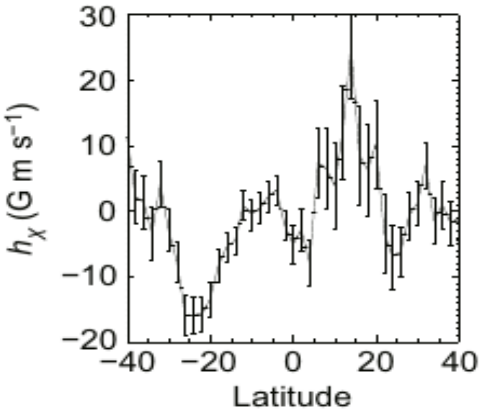




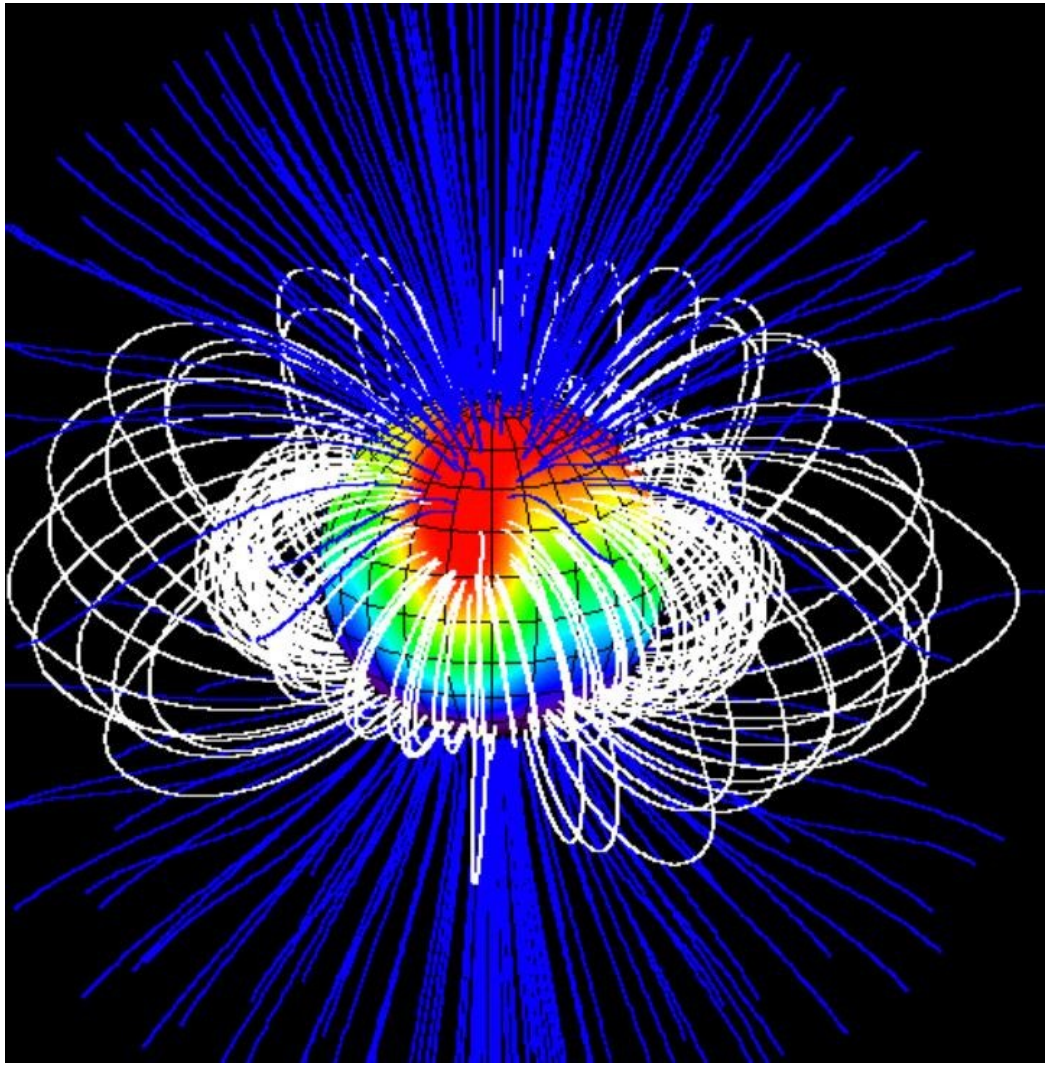








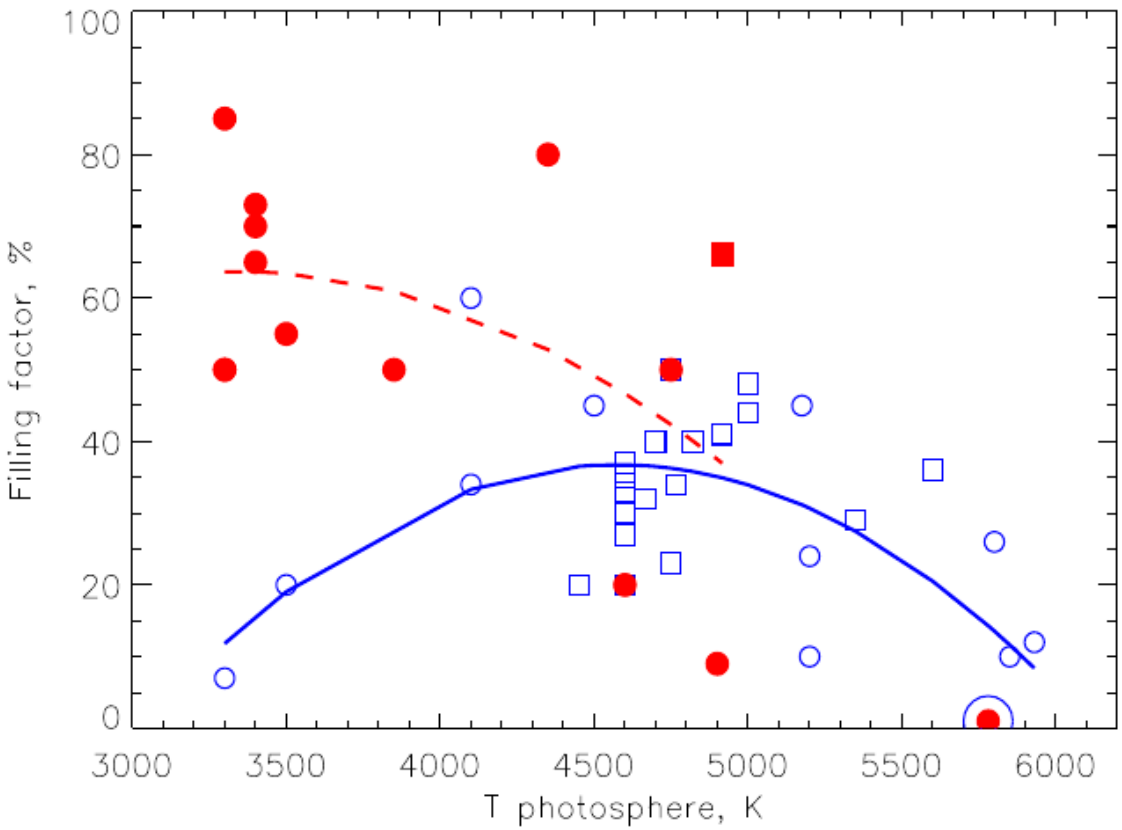
Substantial part of M-dwarf's is covered by the strong magnetic field



V374 Peg, 0.3Ms
rotating with period
of a half day,
Br~2kG is shown by color

(Donati et al 2009)

Spottness and magnetic filling factor on active stars



Berdyugina (2005)

Filling factors of spots (open symbols) and magnetic fields (filled symbols) on the surfaces of active dwarfs (circles) and giants (squares) versus the photosphere temperature. A big circle emphasises the sunspot umbra (f ?.1%)

From solar-stellar analogy we expect the mean cross-helicity of strength 10^3 G km/s (guess!) covering substantial portions of stellar surface.

Dynamo model

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times \left(\langle \mathbf{U} \rangle \times \langle \mathbf{B} \rangle + \boldsymbol{\mathcal{E}} \right)$$

The mean electro-motive force:

(also, Pipin 2017)

$$\boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{E}}^{\alpha,\eta,V} + \boldsymbol{\mathcal{E}}^{\gamma},$$

$$\boldsymbol{\mathcal{E}}^{\gamma} = C_{\gamma} \frac{\boldsymbol{\Omega}}{\Omega} \langle \gamma \rangle f_{\gamma}\left(\Omega^{*}\right) \psi_{\gamma}\left(\beta\right)$$

(poloidal, can't produce ax $\mathbf{B}^{(P)}$!)

Heuristic setup: $f_{\gamma}\left(\Omega^{*}\right) = \dfrac{\arctan\left(\Omega^{*}\right)}{1+\Omega^{*}}, \ \Omega^{*} = 2\tau_{c}\Omega_{\star}$

$$\beta = \langle |\mathbf{B}| \rangle / \sqrt{4\pi\overline{\rho}\mathrm{u}'^{2}}, \quad \psi_{\gamma} = \psi_{\alpha} \sim \beta^{-2}$$

Cross-helicity evolution:

$$\frac{\partial \langle \gamma \rangle}{\partial t} = \frac{1}{3\overline{\rho}} \left(\langle \mathbf{B} \rangle \cdot \nabla \right) \overline{\rho} \left\langle \mathbf{u}'^{2} \right\rangle \underbrace{- 2\boldsymbol{\mathcal{E}} \cdot \boldsymbol{\Omega}}_{\text{excluded}} + \eta_{T} \Delta \langle \gamma \rangle$$

see, Pipin et al (2011)
Yokoi (2013)

Decomposition to AX and NAX modes

$$\begin{aligned}
\langle \mathbf{B} \rangle &= \overline{\mathbf{B}} + \tilde{\mathbf{B}} \\
\overline{\mathbf{B}} &= \hat{\boldsymbol{\phi}}\mathrm{B} + \nabla \times \left(\frac{\mathrm{A}\hat{\boldsymbol{\phi}}}{r \sin\theta} \right) \\
\tilde{\mathbf{B}} &= \boldsymbol{\nabla} \times (\hat{\mathbf{r}}\mathrm{T}) + \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times (\hat{\mathbf{r}}\mathrm{S}),
\end{aligned}$$

$$\langle \gamma \rangle = \overline{\gamma} + \tilde{\gamma}$$

The non-axisymmetric (NAX) modes are treated via the SPH decomposition (ℓ=20, m=10), nonlinear terms are calculated in real space

Dynamo scenarios

Axysymmetric solar-type:

Coupling by DR and α-effect (Parker,1955):

$$\mathbf{B}^{(P)} \rightarrow \mathbf{B}^{(T)} \rightarrow \mathbf{B}^{(P)} \ldots$$

Coupling by α-effect and cross-helicity (Yoshizawa&Yokoi 1996; Yokoi 2013, Yokoi et al 2016, without DR!) :

$$\mathbf{B}^{(P)} \rightarrow \langle \gamma \rangle \rightarrow \mathbf{B}^{(T)} \rightarrow \mathbf{B}^{(P)} \rightarrow \langle \gamma \rangle \ldots$$

Dynamo scenarios: M-dwarfs

- Coupling by α^2-effect, or $\alpha^2\Omega$?
  - In saturated stage (strong **B**, fast rotation), models gives the solid body rotation and the α^2 generates nonaxisymmetric **B**-field because they are unstable prior axisymmetric **B**-field: Kuker & Ruedeger, 1998; Elstner D., Rudiger G., 2007, the DNS Dobler et al 2006, Browning 2008).
  - The origin of the strong dipolar **B**-field (like V374 Peg) is not clear.

- New scenario: coupling by γ^2-effect.

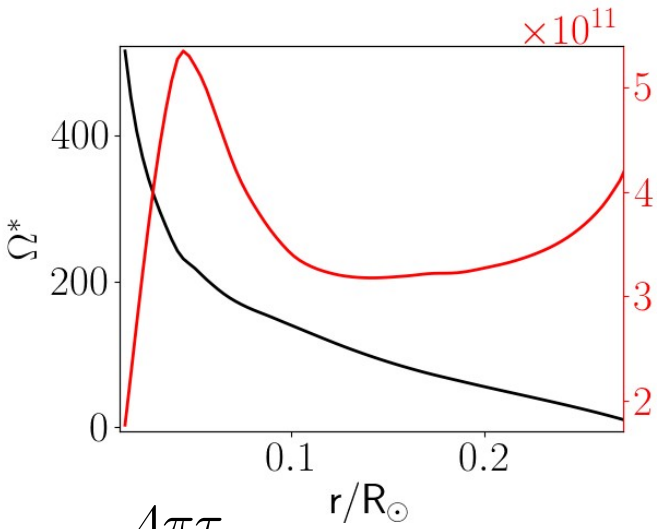
$$\tilde{\mathbf{B}}^{(P)} \rightarrow \tilde{\gamma} \rightarrow \begin{cases} \tilde{\mathbf{B}}^{(P)} & \rightarrow \tilde{\gamma} \\ \tilde{\mathbf{B}}^{(T)} & \\ \overline{\mathbf{B}}^{(T)} & (\mathrm{nonlinear}) \end{cases} \quad \ldots$$

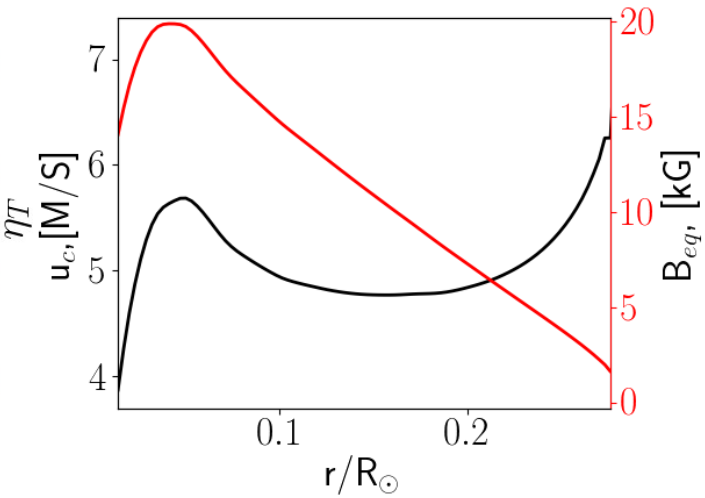
- The other are $\alpha^2\gamma^2$ and $\alpha^2\gamma^2\Omega$:

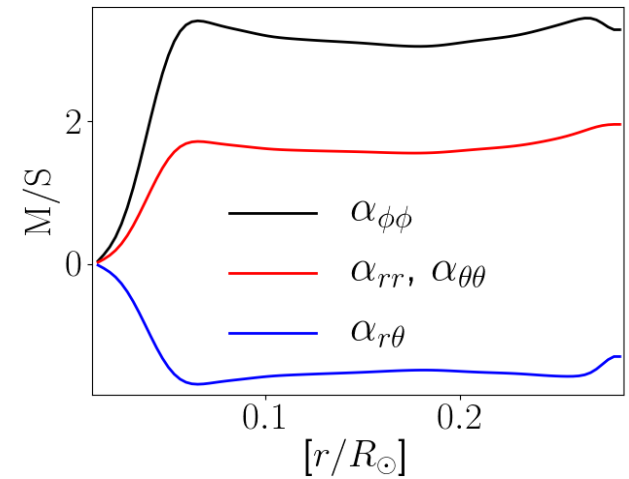
M dwarf of 0.3Ms, rotating with period of 10 days

The reference model of the star is from MESA code (mesa.sourceforge.net)

Effect of rotation on the enthropy profile is calculated from mean-field model of the angular momentum and heat transport (Pipin, 2017).







$$\Omega^* = \frac{4\pi\tau_c}{P^*}$$ - Coriolis number

Anisotropic α-effect from Pipin(2008)

Convective RMS u' is govern by mean entropy profile:

$$\mathrm{u}' = \frac{\ell}{2}\sqrt{-\frac{\mathrm{g}}{2\mathrm{c_p}}\frac{\partial \overline{\mathrm{s}}}{\partial \mathrm{r}}},$$

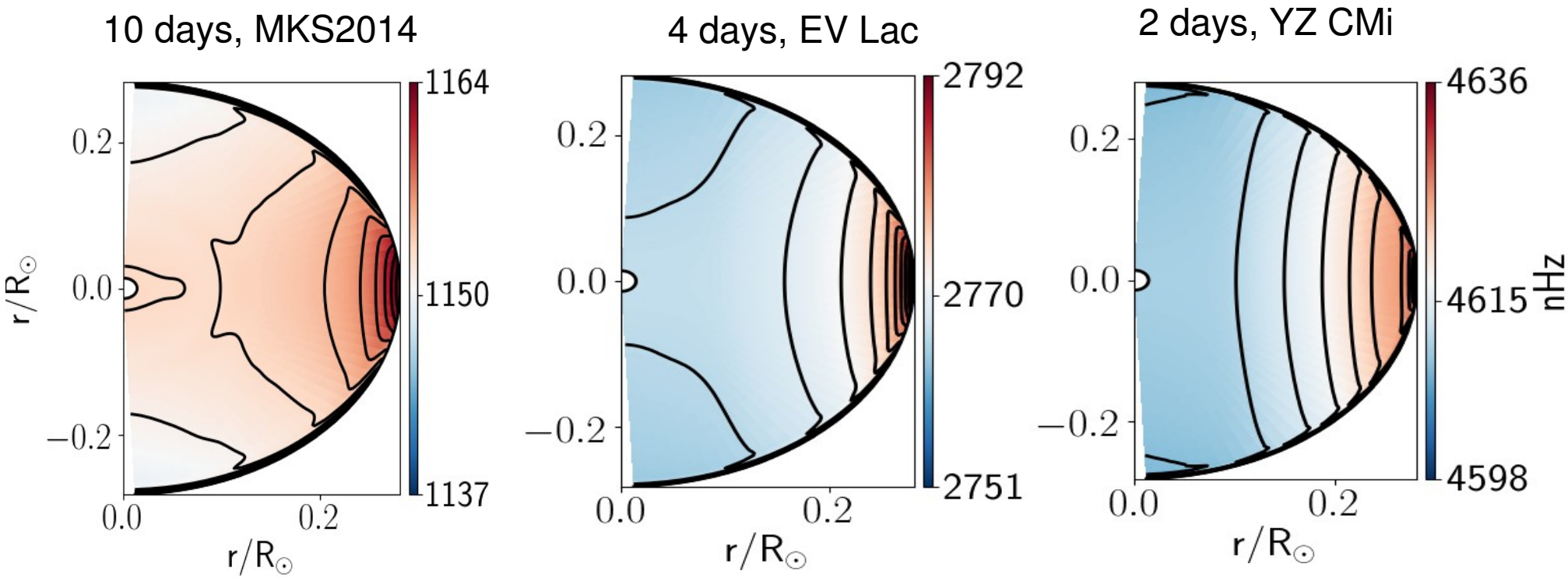
$$\alpha_{ij} \sim \alpha_0 \left(\delta_{ij} - \frac{\Omega_i \Omega_j}{\Omega^2}\right)$$

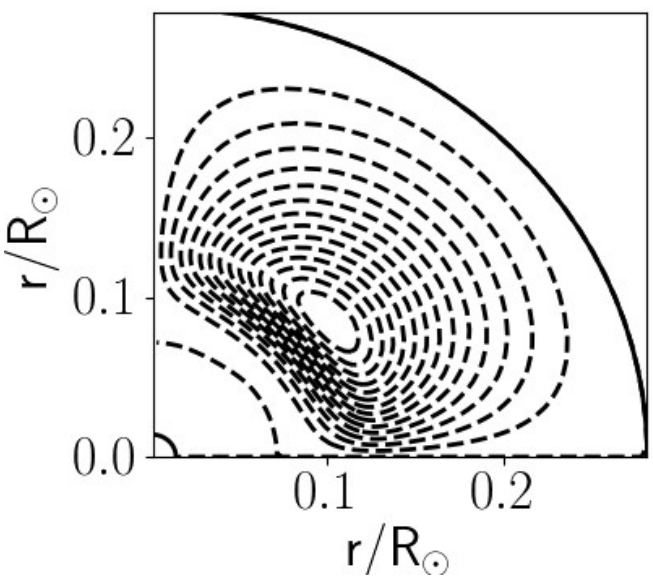
Boundary conditions, zero **B** and γ near the center.

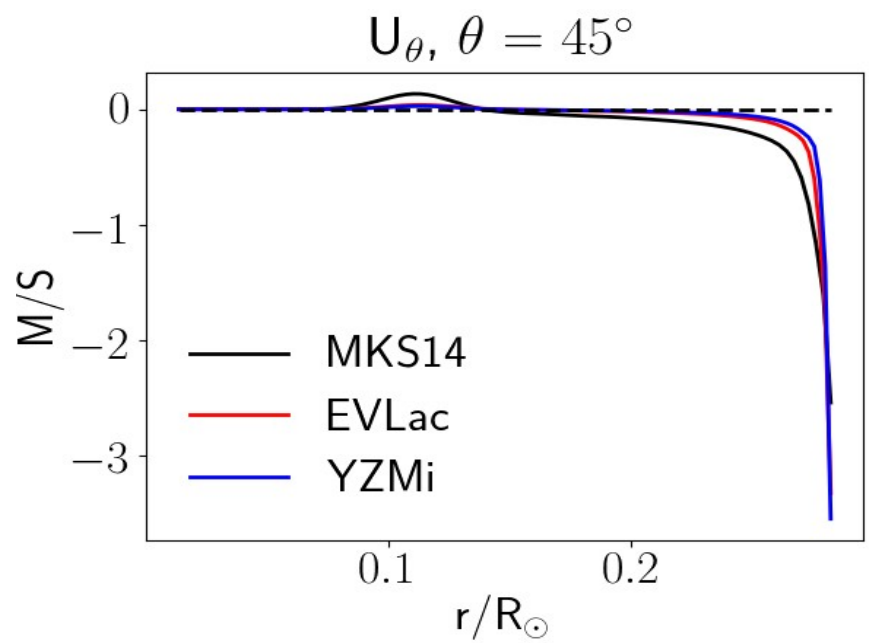
At the top zero $\mathbf{B}^{(T)}$ and $\partial_r \gamma$,

Outside the star PSSF with $R_{SS}=2R_\odot$

Model of global flows

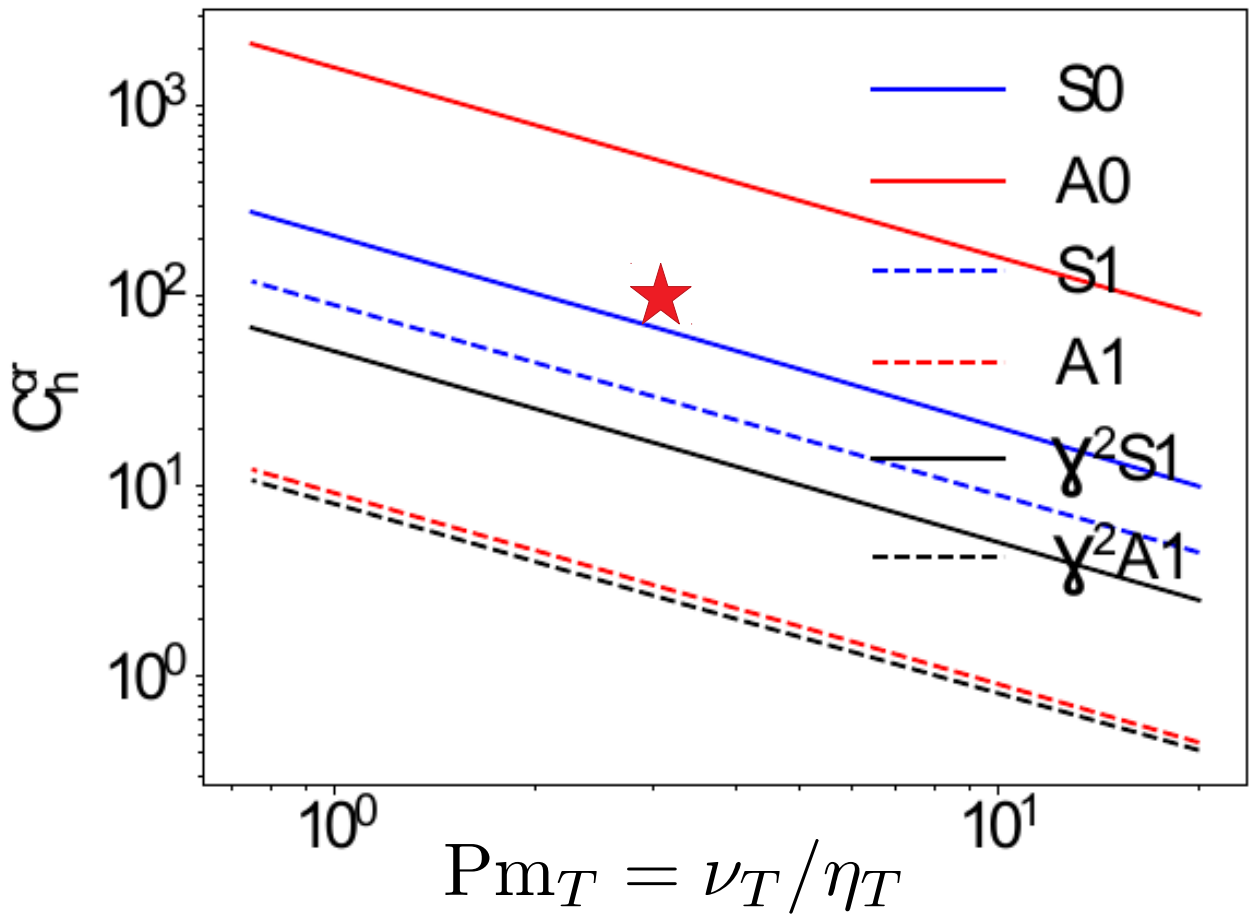






Linear stability of $\omega^2\gamma^2$ and γ^2

Solid body rotation of MKS14, constant parameters along radius, $C_h = \Omega^* C_\gamma$, τ_c=1yr



S0 -symmetric about equator m=0 mode,

A0 - antisymmetric about equator m=0 mode ….

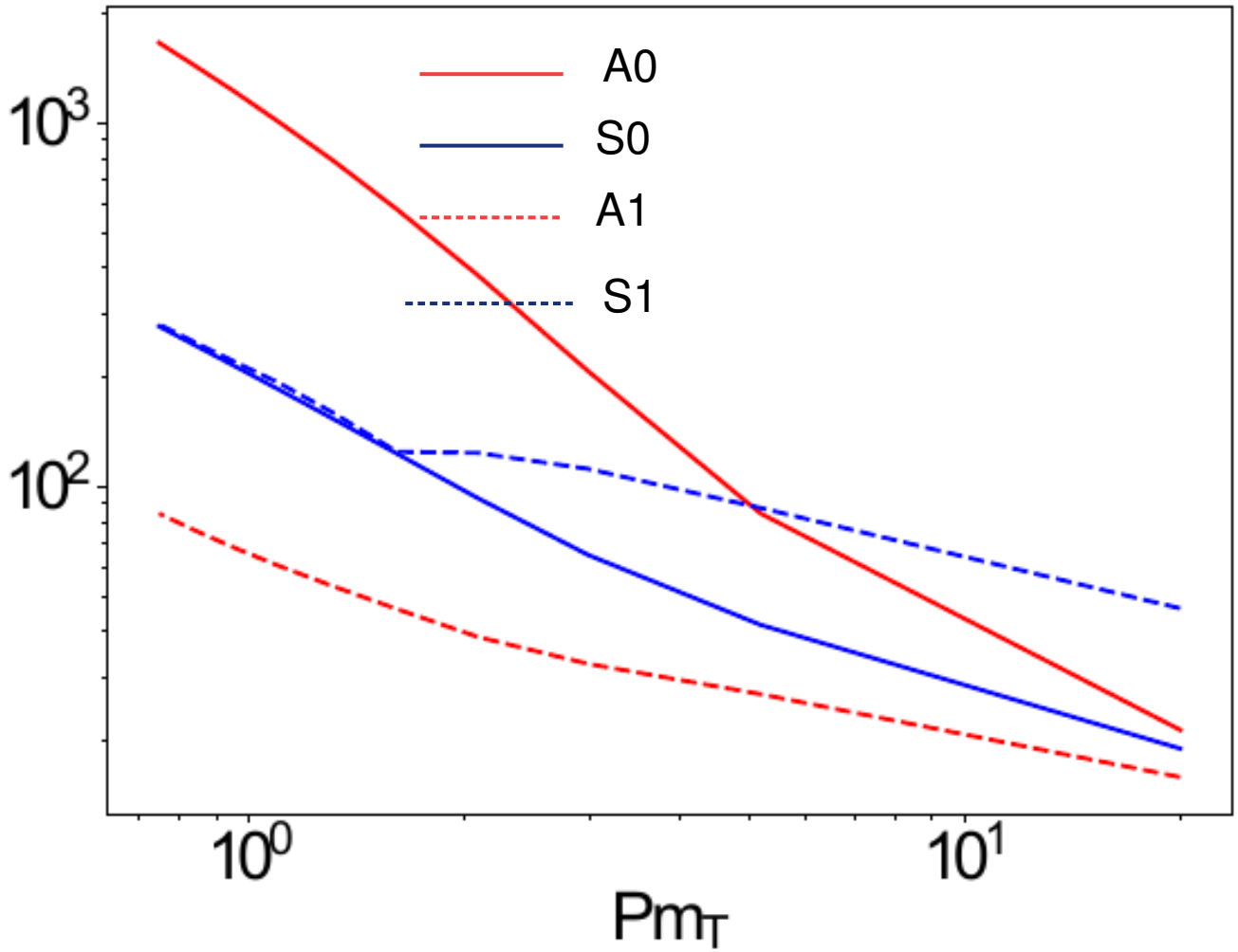
The non-axisymmetric modes are most unstable both in $\omega^2\gamma^2$ and γ^2 regimes

Note that Pm_T controls cross-helicity production:

$$\frac{\partial \langle \gamma \rangle}{\partial t} = \frac{1}{3\overline{\rho}} \left(\langle \mathbf{B} \rangle \cdot \nabla \right) \overline{\rho} \left\langle \mathbf{u}^2 \right\rangle$$

Linear stability of $\lambda^2\gamma^2\Omega$

DR of star rotating with period of 10 days (MKS14) other parameters are constant.



Thresholds of non-axisymmetric and axisymmetric modes are getting close with increase of Pm_T

Nonlinear regime λ^2

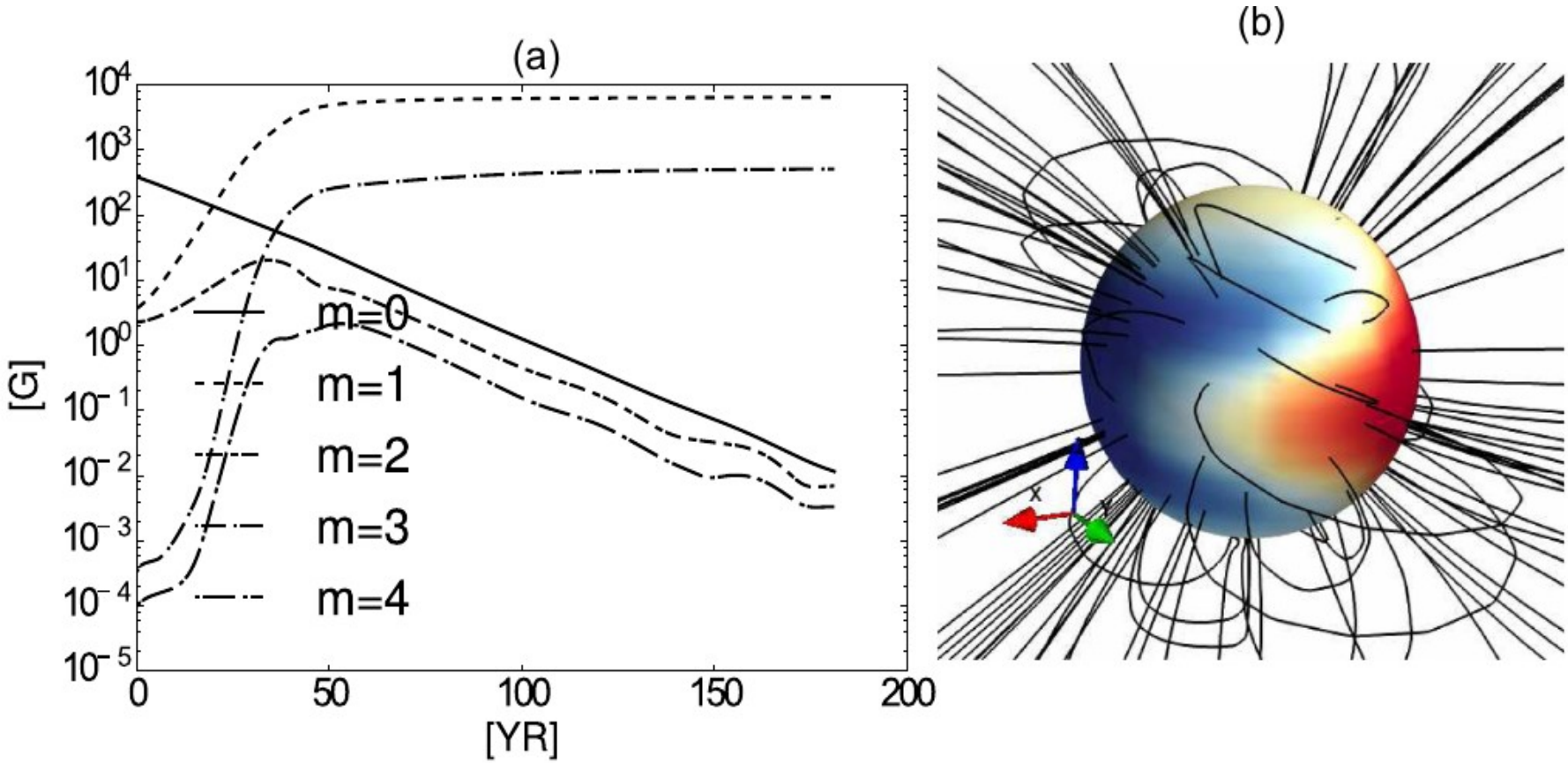
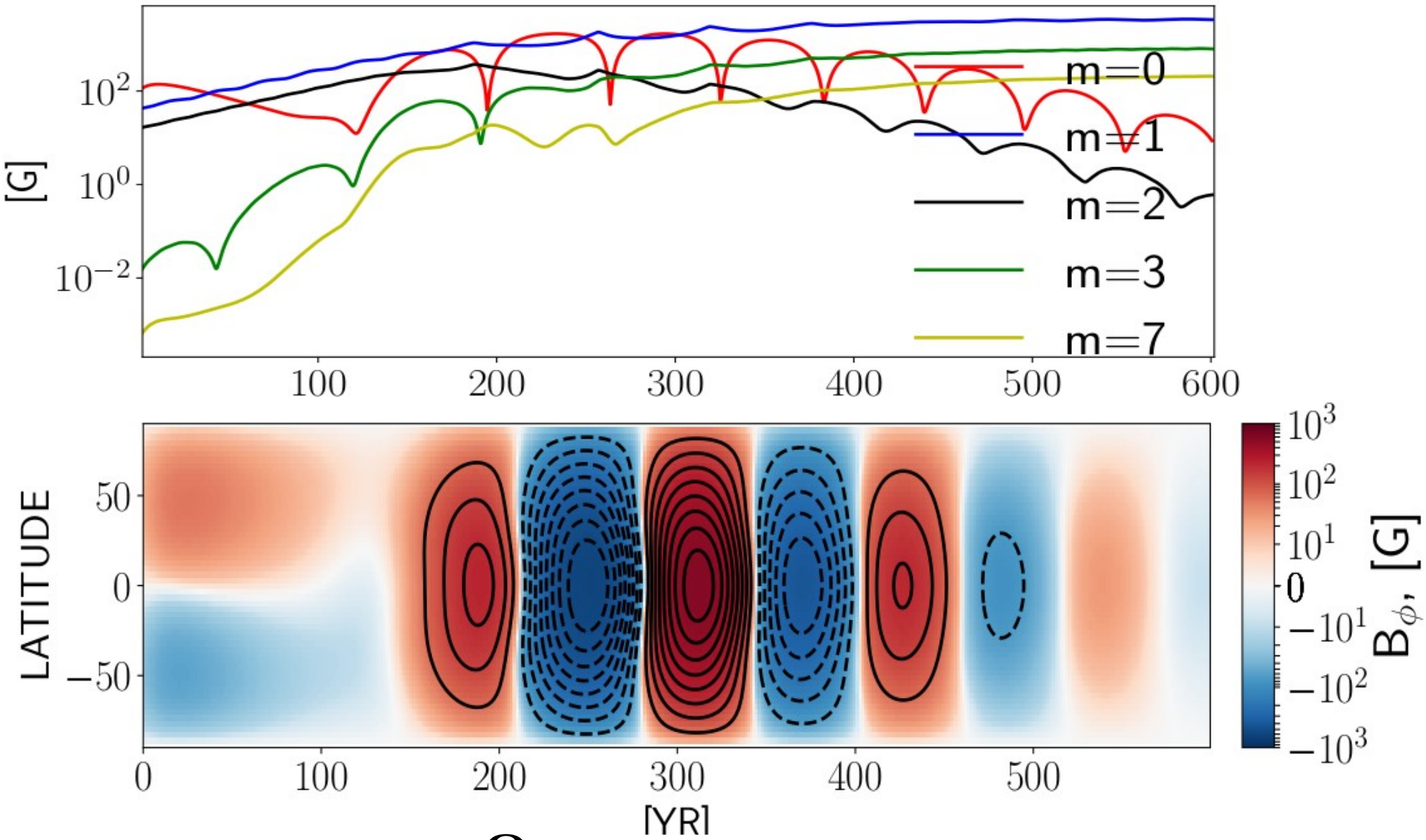


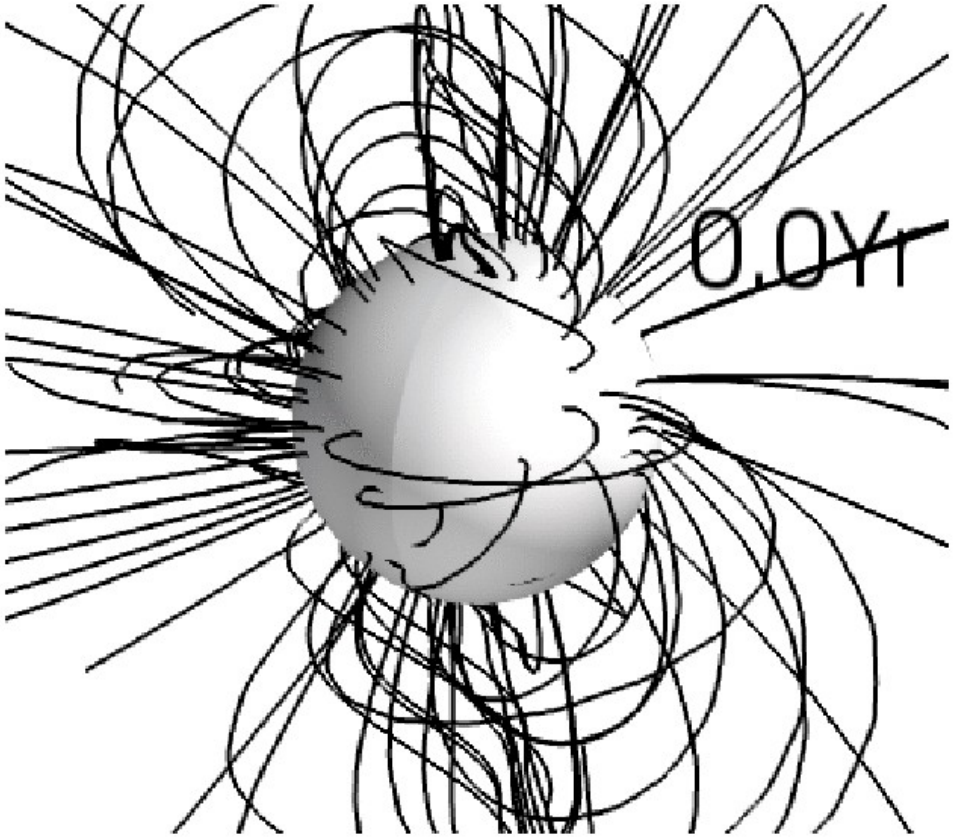
Figure 3. The model M0: (a) evolution of the mean strength of the first five partial modes of the toroidal magnetic field at the $r = \frac{3}{4}R_\star$; (b) snapshot of the large-scale magnetic field lines out of the star and the background image shows the radial magnetic field within range of ± 1500 G.

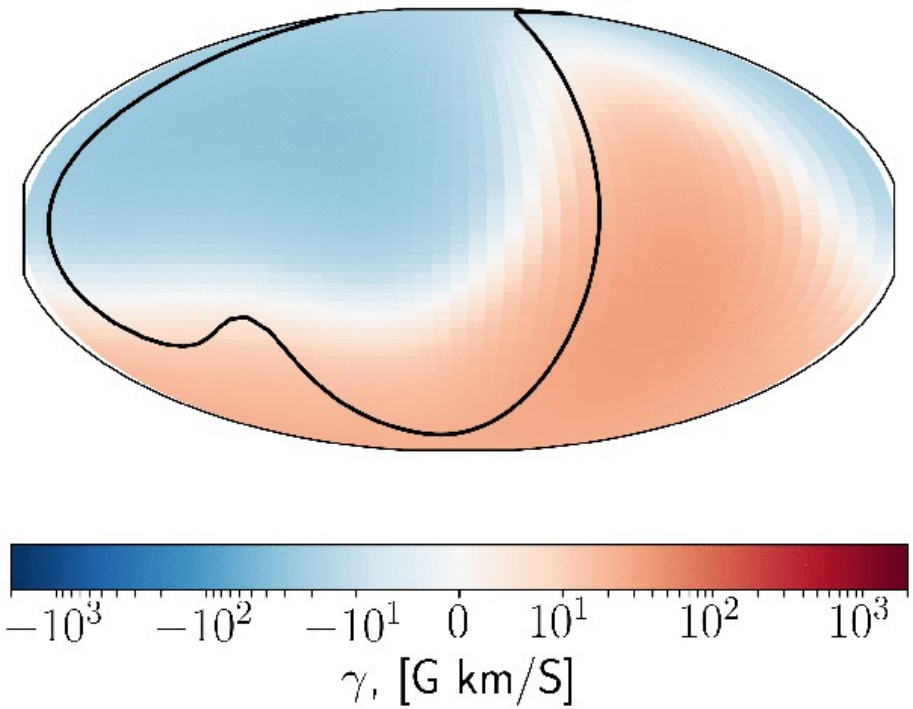
Nonlinear regime γ^2



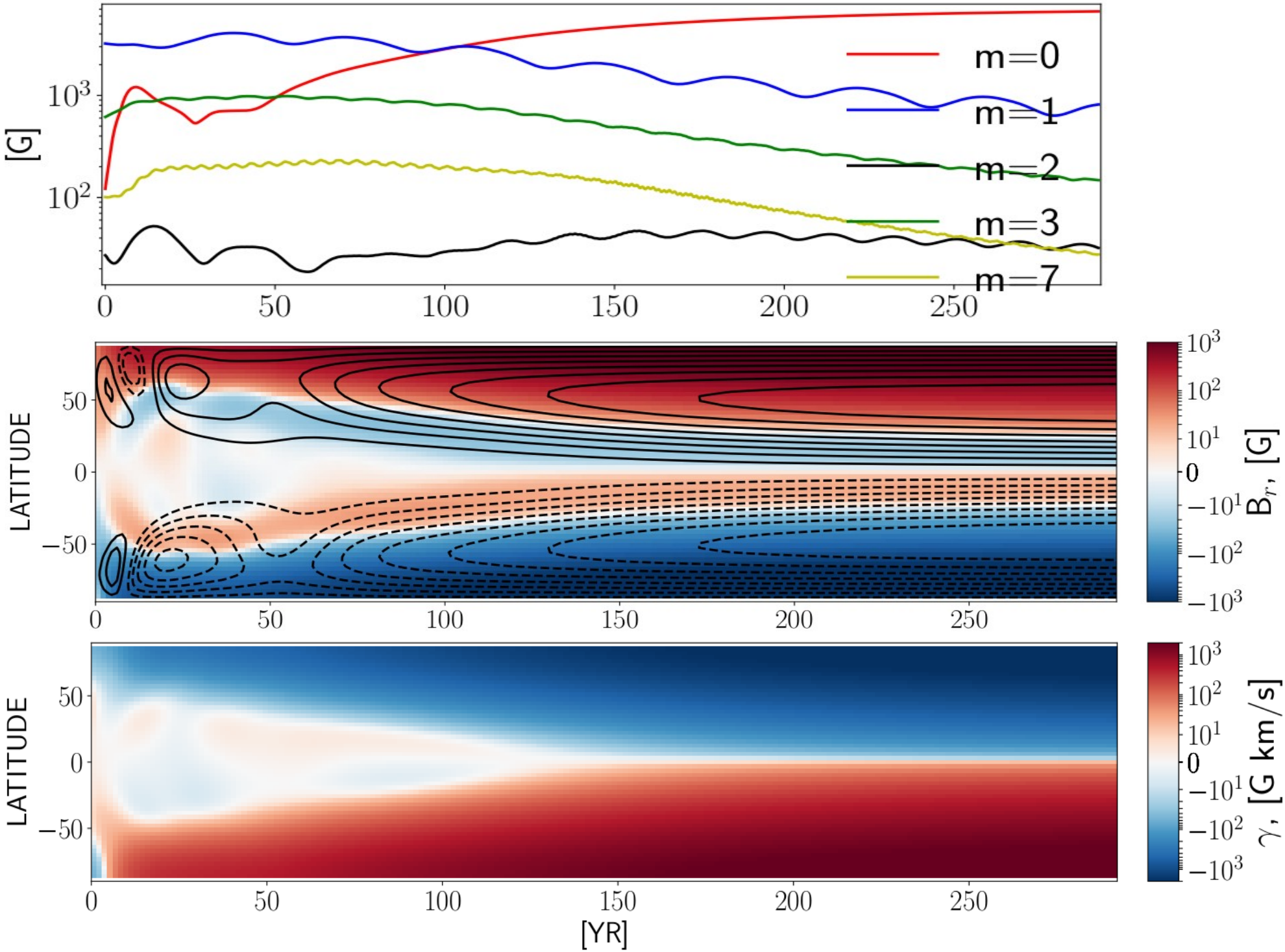
$$\boldsymbol{\mathcal{E}}^{\gamma} = C_{\gamma} \frac{\boldsymbol{\Omega}}{\Omega} \underbrace{\langle \gamma \psi_{\gamma} (\beta) \rangle} f_{\gamma} (\Omega^*)$$

γ^2

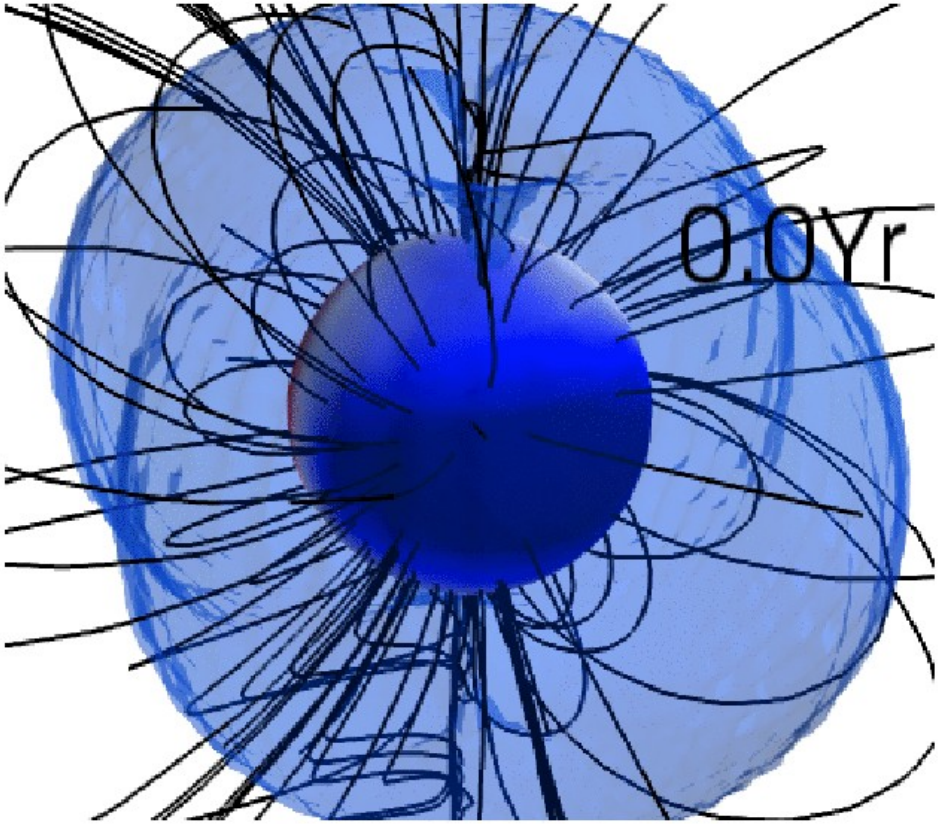


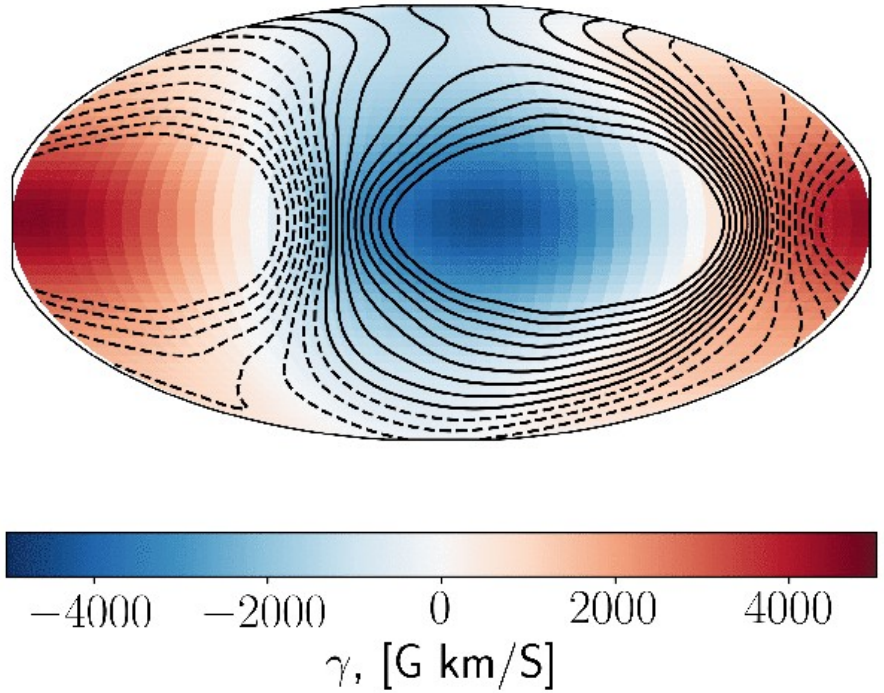


Nonlinear regime $\Delta^2\gamma^2$

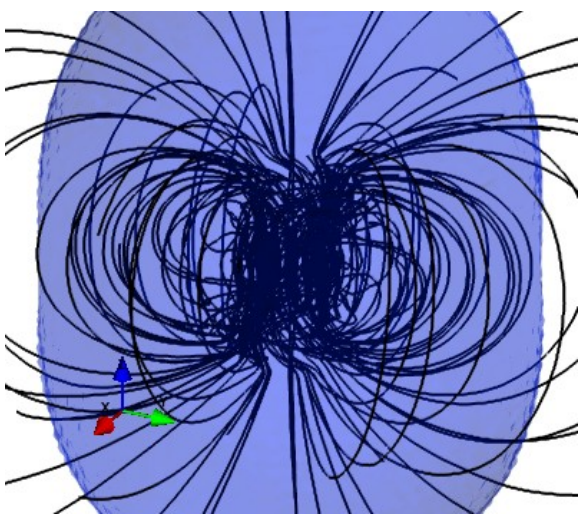


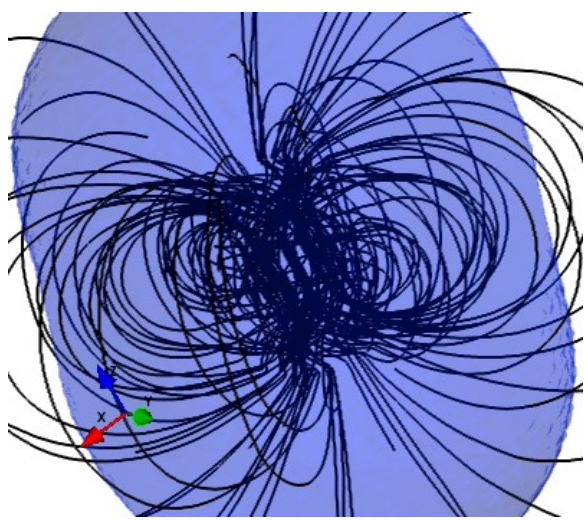
$\lambda^2\gamma^2$

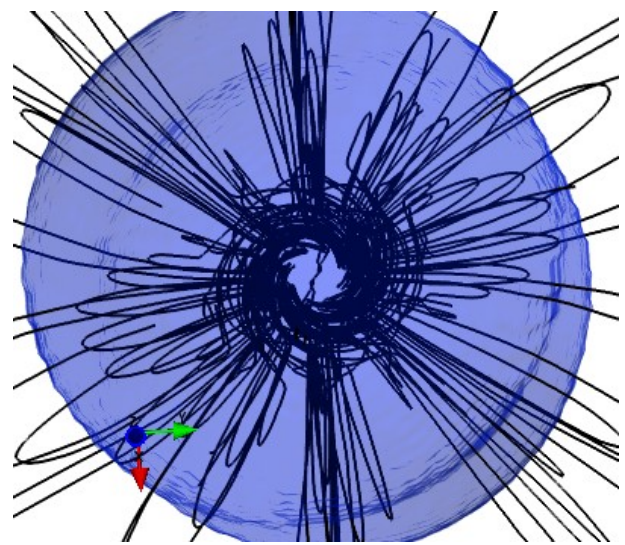




View from the pole







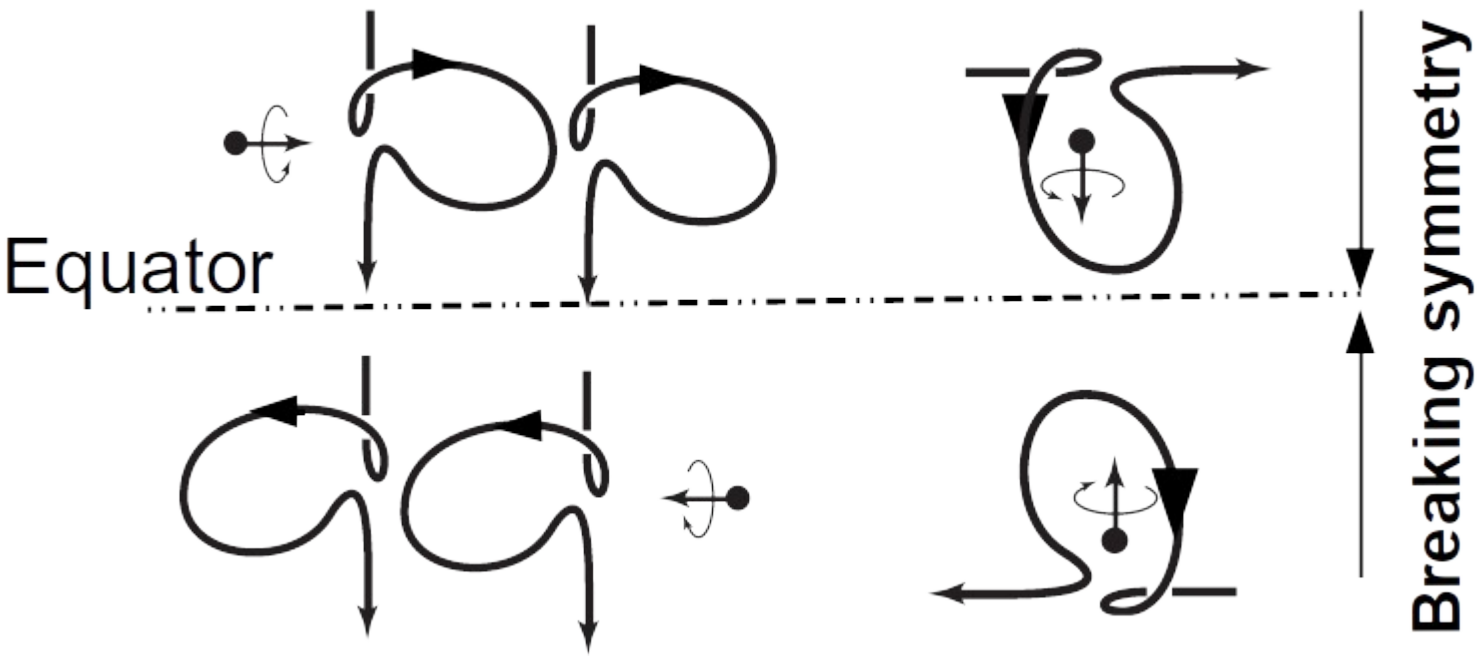
Discussion

For regime of the fast rotation there is no α effect for B-field parallel to the axis of rotation. This is because of anisotropy of convection motions.

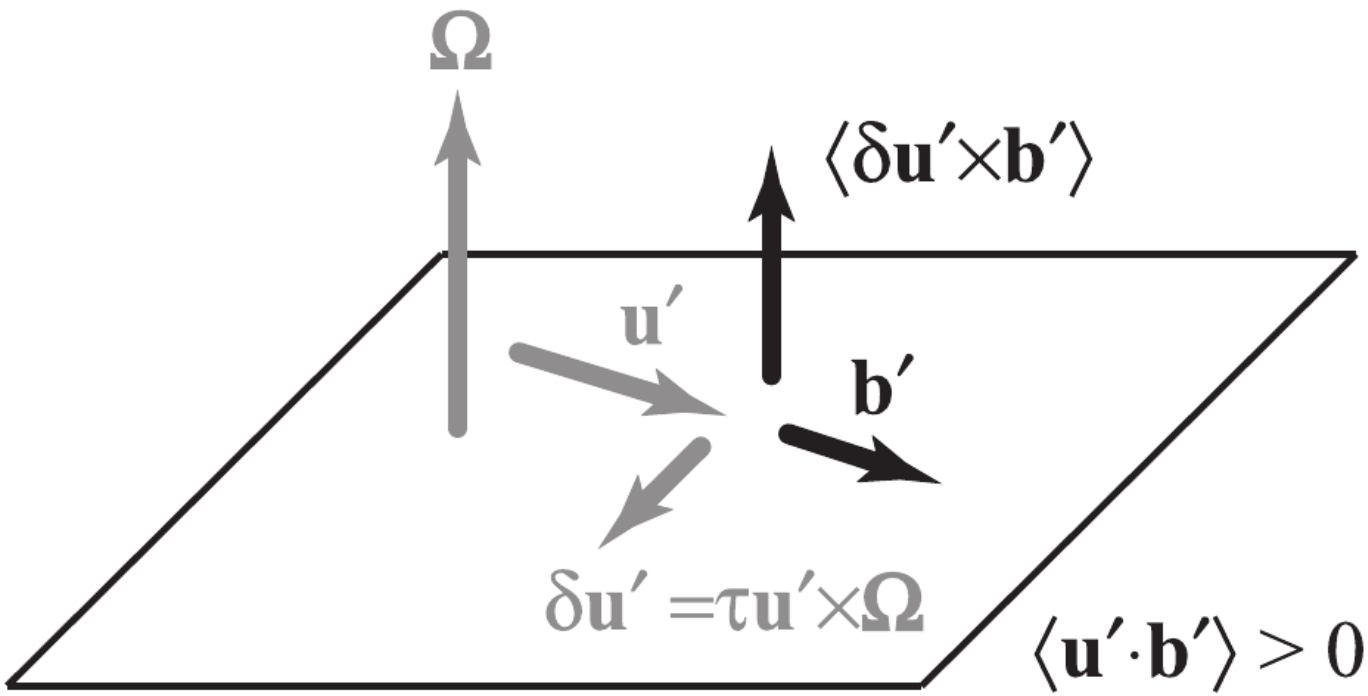
$$\alpha_{ij} \sim \alpha_0 \left(\delta_{ij} - \frac{\Omega_i \Omega_j}{\Omega^2} \right)$$

Ruediger&Kitchatinov, 1993;
Kitchatinov et al 2001

In this case only non-axisymmetric B-field can use the α effect for dynamo.



Generation by cross-helicity



It remains efficient even for the case of 2d state of turbulent flow in the fast rotating regime.

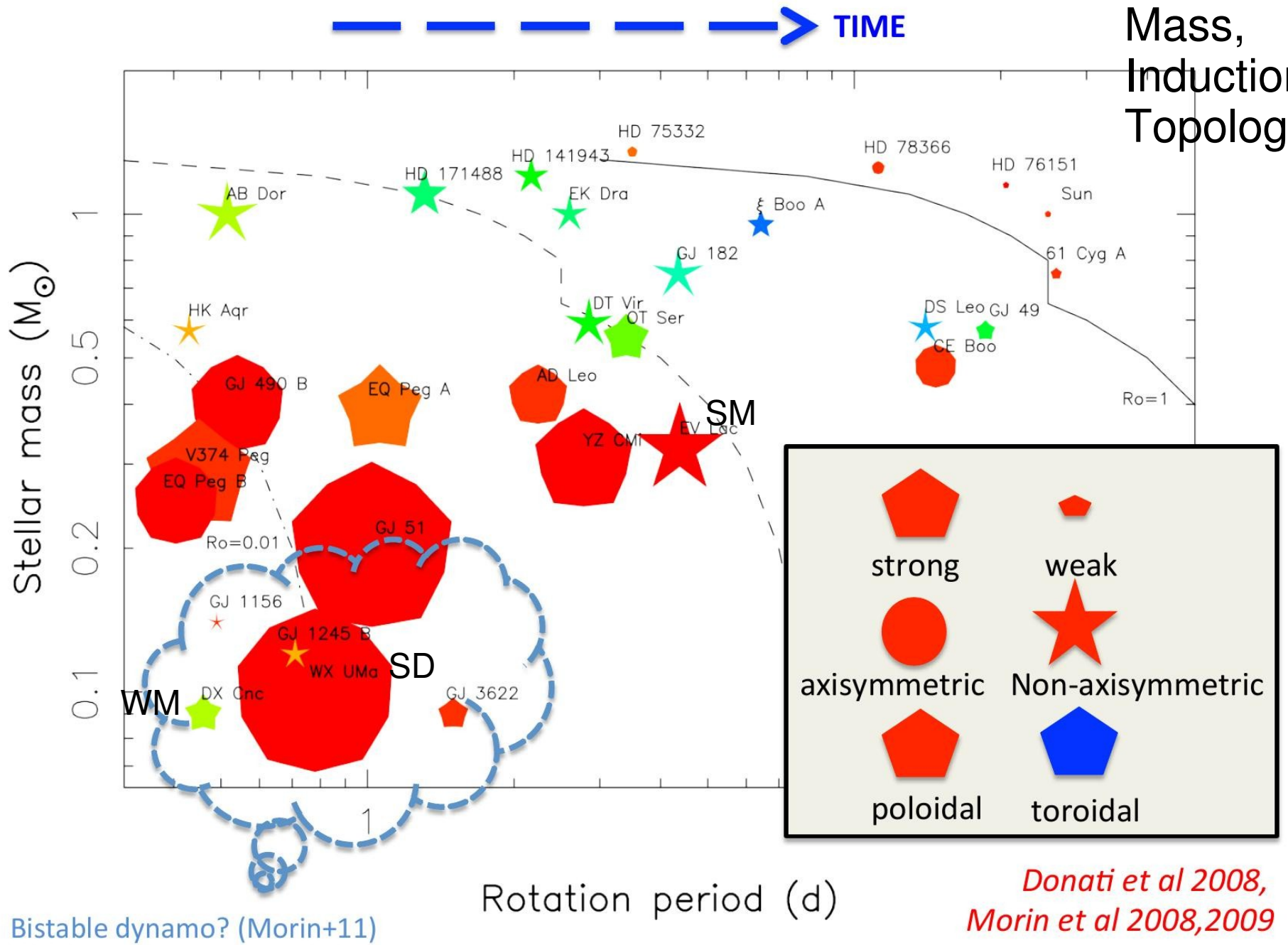
For overcritical Cγ the axisymmetric dynamo works via:

$$\mathbf{B}^{(P)} \to \langle \gamma \rangle \to \mathbf{B}^{(T)} \to \mathbf{B}^{(P)} \to \langle \gamma \rangle \ldots$$

Stellar magnetic topology

Jardine, 2015: 5D "Confusogram"

:Rotation, Mass, Induction, Topology(2)



Bistable dynamo? (Morin+11)

Donati et al 2008, Morin et al 2008,2009

Conclusions

- Cross-helicity effects provide several new scenarios for dynamo on the fully-convective stars
- For a particular case of the fast rotating stars with solid body rotation regime we show a possibility to sustain the strong dipolar B-field via $\alpha^2\gamma^2$ dynamo.
- Generation of axisimmetric toroidal magnetic field from non-axisymmetric cross-helicity with $\overline{\gamma}$ =0, potentially can be important in solar dynamo, in particullar for the young spotted Sun