INVERSION FOR MERIDIONAL FLOW IN THE SOLAR INTERIOR

S. P. Rajaguru and H. M. Antia

- Rajaguru & Antia (2015) used 4 years of data from HMI
- The travel-time difference between N–S and S–N along a ray path Γ_0 is given by

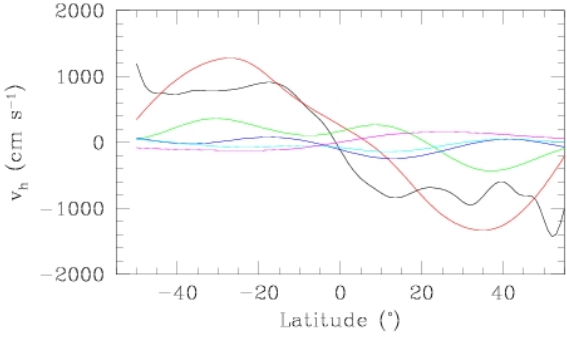
$$\delta\tau = -2\int_{\Gamma_0} \frac{\mathbf{u}\cdot\hat{\mathbf{n}}}{\mathbf{c^2}}\, ds$$

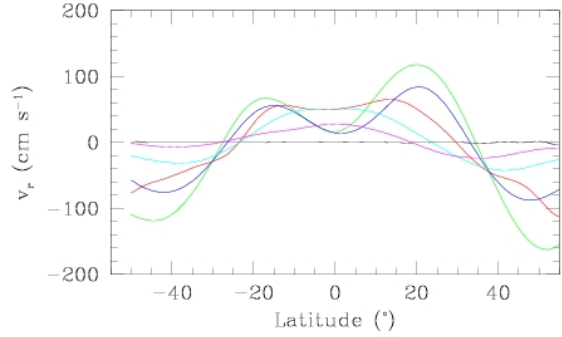
- Stream function was used to represent meridional velocity:

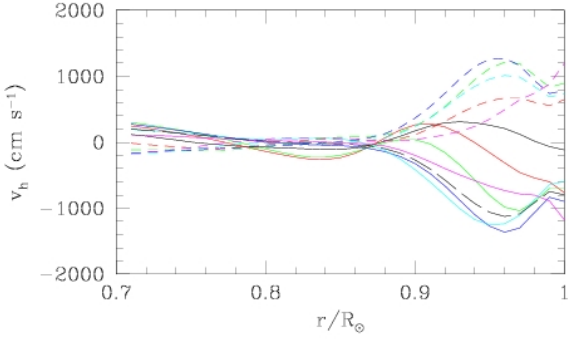
$$\rho u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta} + \frac{\cos\theta}{r\sin\theta}\psi$$
$$\rho u_\theta = -\frac{\partial\psi}{\partial r} - \frac{\psi}{r}$$

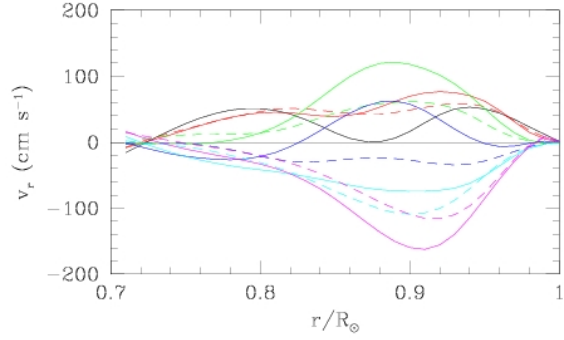
which ensures that the solution automatically satisfies the continuity equation and also enables u_r to be determined.

- Inversions are done using the Regularised Least Squares (RLS) technique.
- Boundary condition of $u_r = 0$ at outer boundary is used
- To get more realistic error estimates on inverted velocity we also perturb the smoothing used (apart from travel times) in inversion which gives some estimate of systematic errors in inversions.
- Results using travel times calculated for different frequencies using 6-year HMI data are as follows:

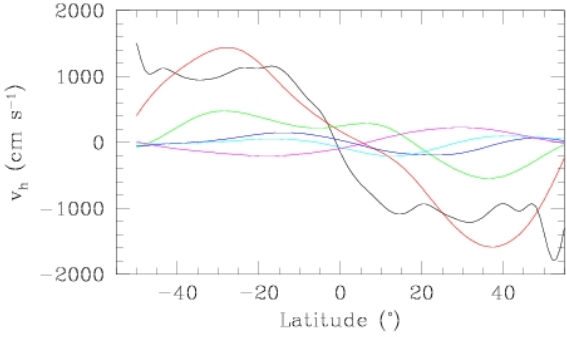

2000
1000
0
−1000
−2000
v_h (cm s^{-1})
−40
−20
0
20
40
Latitude (°)

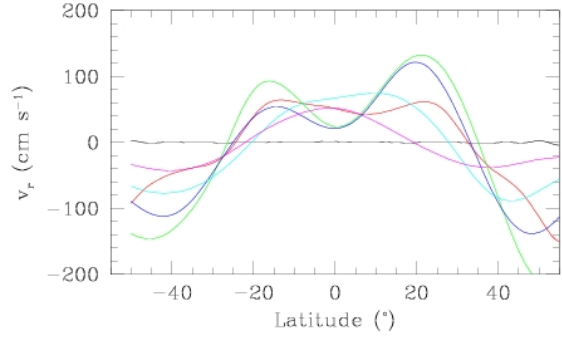

200
100
0
−100
−200
v_r (cm s^{-1})
−40
−20
0
20
40
Latitude (°)

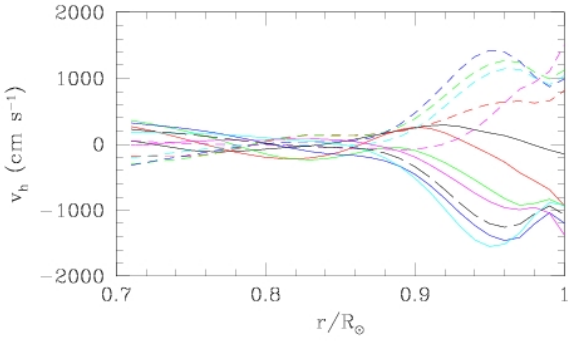

2000
1000
0
−1000
−2000
v_h (cm s^{-1})
0.7
0.8
0.9
1
$r/R_\odot$

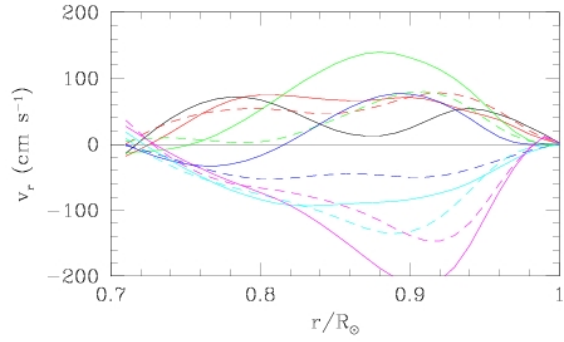

200
100
0
−100
−200
v_r (cm s^{-1})
0.7
0.8
0.9
1
$r/R_\odot$

$\nu = 2.7$ mHz

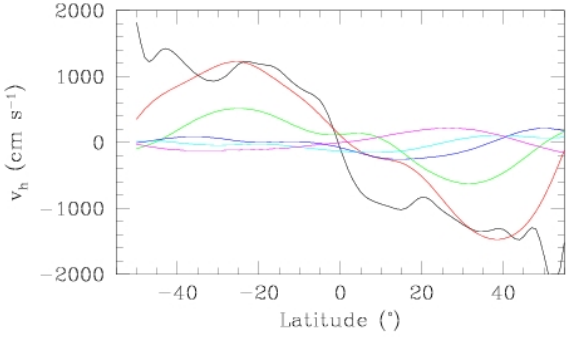

2000
1000
0
−1000
−2000
v_h (cm s^{-1})
−40
−20
0
20
40
Latitude (°)

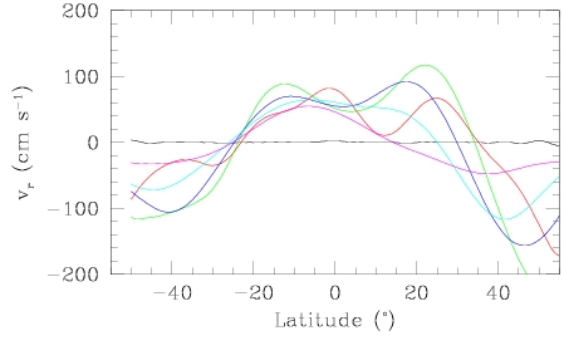

200
100
0
−100
−200
v_r (cm s^{-1})
−40
−20
0
20
40
Latitude (°)

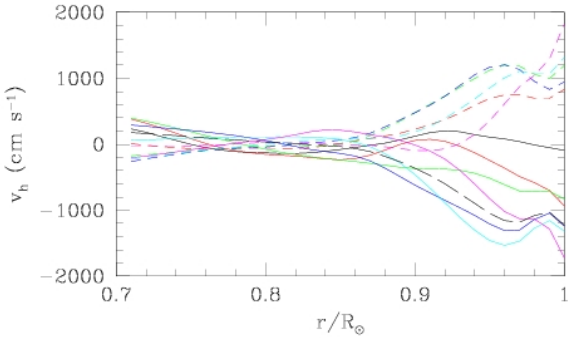

2000
1000
0
−1000
−2000
v_h (cm s^{-1})
0.7
0.8
0.9
1
$r/R_\odot$

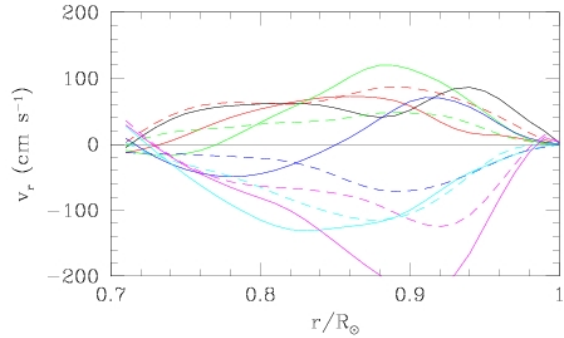

200
100
0
−100
−200
v_r (cm s^{-1})
0.7
0.8
0.9
1
$r/R_\odot$

$\nu = 3.0$ mHz

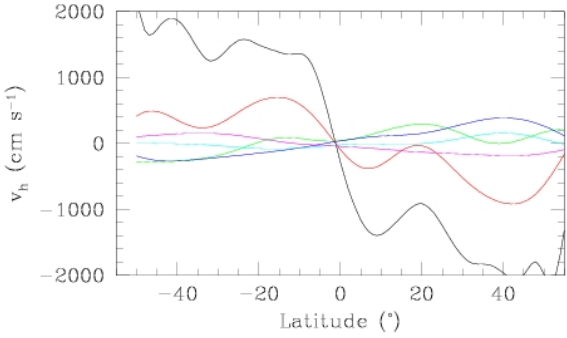

2000
1000
0
−1000
−2000
v_h (cm s^{-1})
−40
−20
0
20
40
Latitude (°)

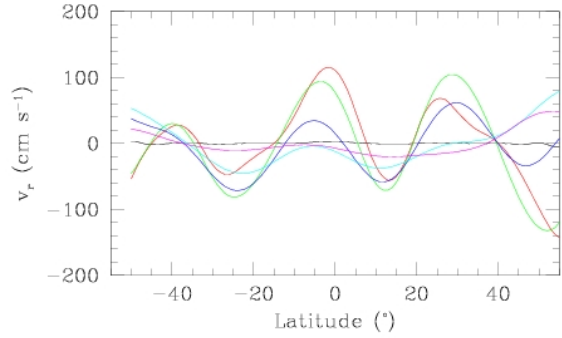

200
100
0
−100
−200
v_r (cm s^{-1})
−40
−20
0
20
40
Latitude (°)

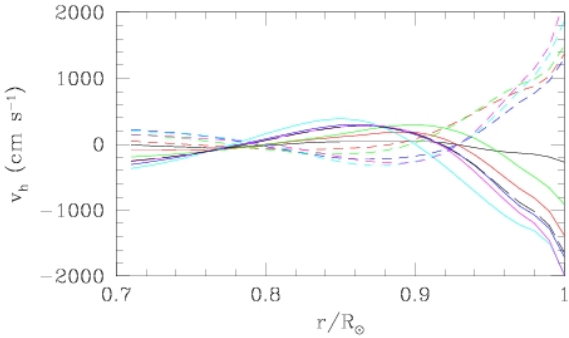

2000
1000
0
−1000
−2000
v_h (cm s^{-1})
0.7
0.8
0.9
1
$r/R_\odot$

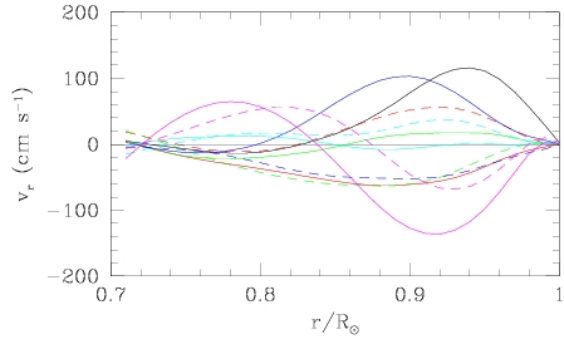

200
100
0
−100
−200
v_r (cm s^{-1})
0.7
0.8
0.9
1
$r/R_\odot$

$\nu = 3.3$ mHz

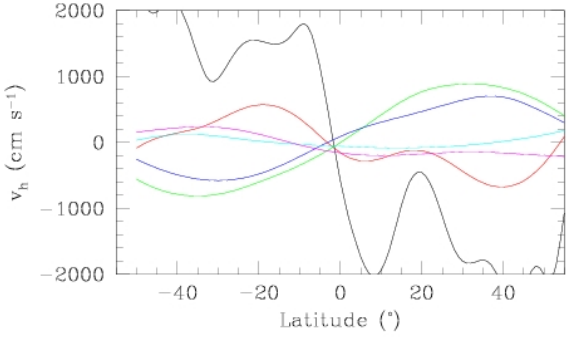

2000
1000
0
−1000
−2000
v_h (cm s^{-1})
−40
−20
0
20
40
Latitude (°)

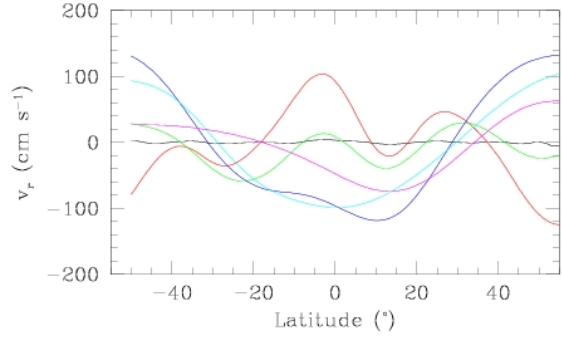

200
100
0
−100
−200
v_r (cm s^{-1})
−40
−20
0
20
40
Latitude (°)

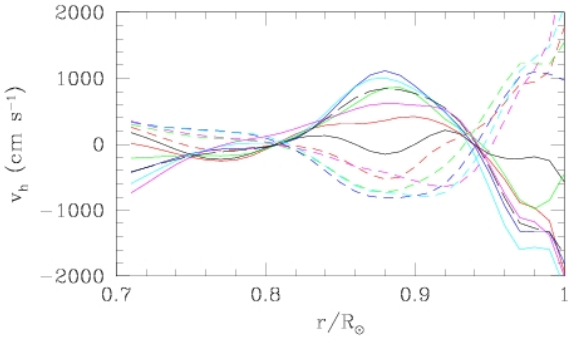

2000
1000
0
−1000
−2000
v_h (cm s^{-1})
0.7
0.8
0.9
1
$r/R_\odot$

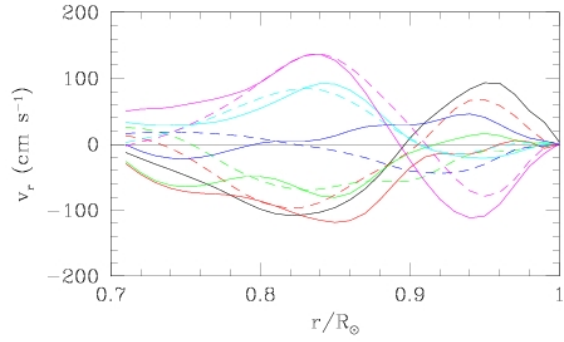

200
100
0
−100
−200
v_r (cm s^{-1})
0.7
0.8
0.9
1
$r/R_\odot$

$\nu = 4.0$ mHz

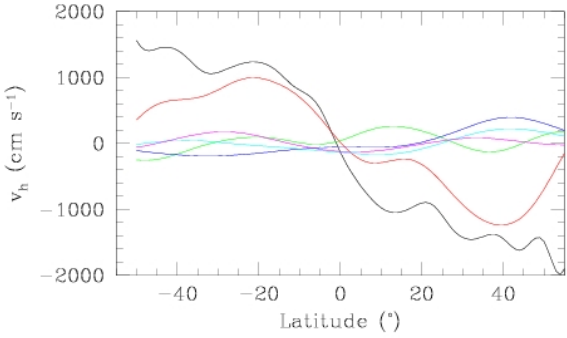

2000
1000
0
−1000
−2000
v_h (cm s^{-1})
−40
−20
0
20
40
Latitude (°)

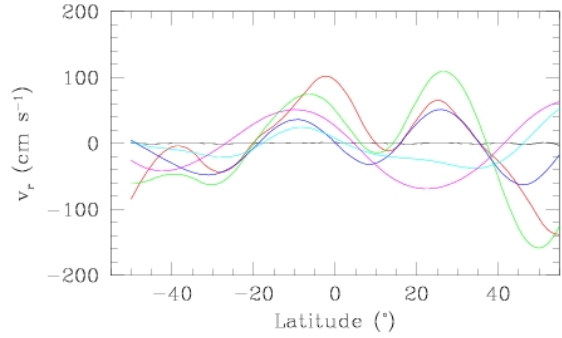

200
100
0
−100
−200
v_r (cm s^{-1})
−40
−20
0
20
40
Latitude (°)

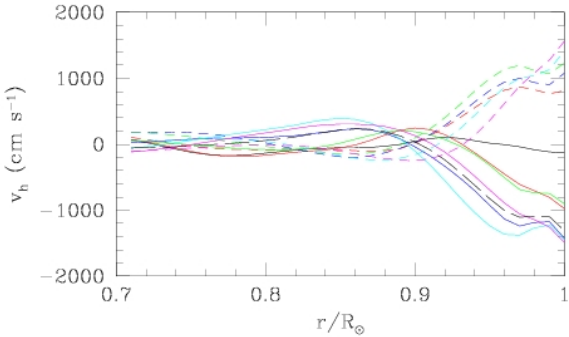

2000
1000
0
−1000
−2000
v_h (cm s^{-1})
0.7
0.8
0.9
1
$r/R_\odot$

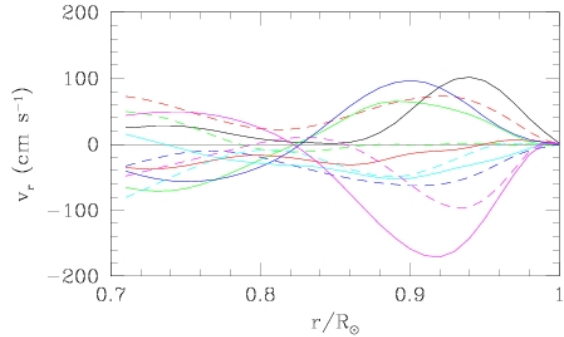

200
100
0
−100
−200
v_r (cm s^{-1})
0.7
0.8
0.9
1
$r/R_\odot$

$\nu = 4.5$ mHz









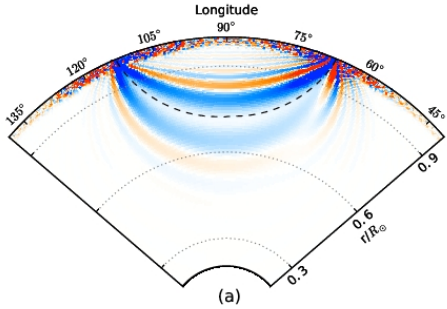
all frequencies combined

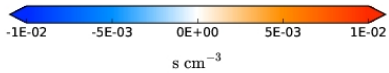
Sensitivity Kernels using Born Approximation

- Hanasoge and Mandal (2017) have calculated the sensitivity kernels using Born Approximation. The difference in travel times due to flows is defined by

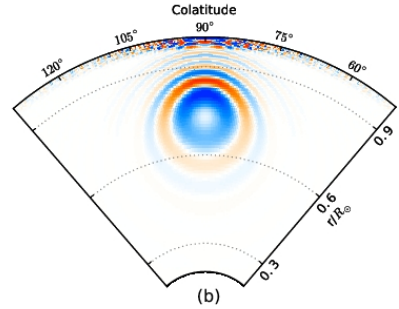
$$\delta\tau(\mathbf{x_r}, \mathbf{x_s}) = \int_{\odot} d\mathbf{x}\mathbf{K_v}(\mathbf{x_r}, \mathbf{x_s}, \mathbf{x}) \cdot \delta\mathbf{v}$$

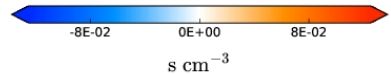
  These have been tested for the case of uniform rotation. For the v_ϕ component a sample kernel is


Longitude
90°
105°
75°
120°
60°
135°
45°
0.9
0.6
r/R⊙
0.3
(a)


-1E-02
-5E-03
0E+00
5E-03
1E-02

s cm^{-3}


Colatitude
90°
105°
75°
120°
60°
0.9
0.6
r/R⊙
0.3
(b)

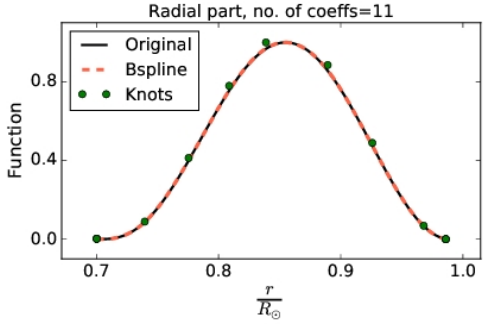

-8E-02
0E+00
8E-02

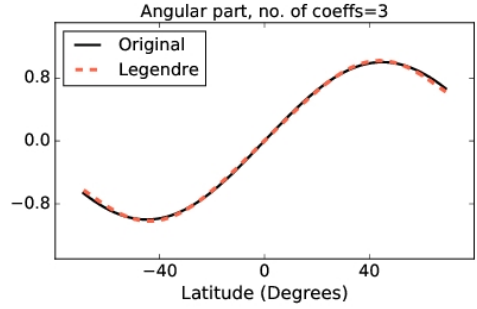
s cm^{-3}

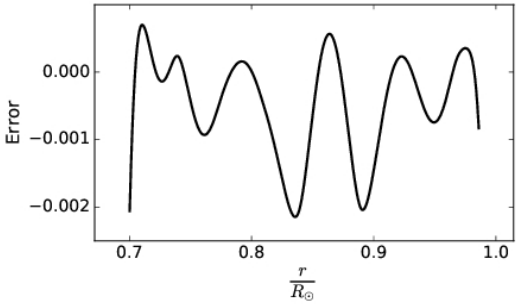
- For meridional flow they use the stream function χ. To invert for the meridional flow the stream function is expressed as

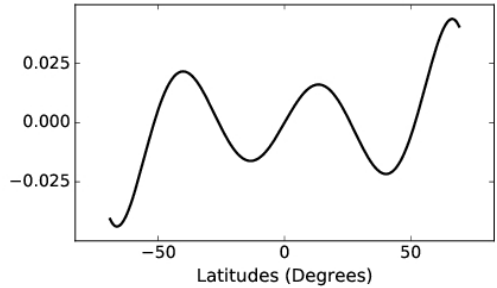
$$\chi(r,\theta) = \sum_{i,j} a_{ij} B_i(r) P_j(\cos\theta)$$

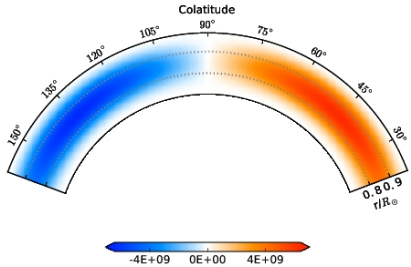
with 11 knots to define B-splines in r and 3 Legendre polynomials. Using a simple 1-cell and 2-cell meridional flow profile the travel times were calculated and these were inverted to test the process. No errors are added in these calculations.


Radial part, no. of coeffs=11
Original
Bspline
Knots
Function
0.0
0.4
0.8
0.7
0.8
0.9
1.0
$\frac{r}{R_\odot}$

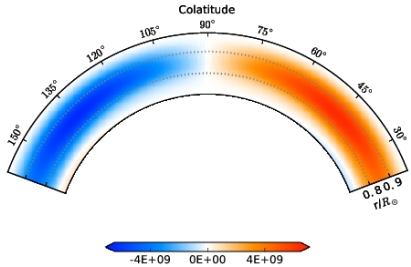

Angular part, no. of coeffs=3
Original
Legendre
0.8
0.0
−0.8
−40
0
40
Latitude (Degrees)


Error
0.000
−0.001
−0.002
0.7
0.8
0.9
1.0
$\frac{r}{R_\odot}$

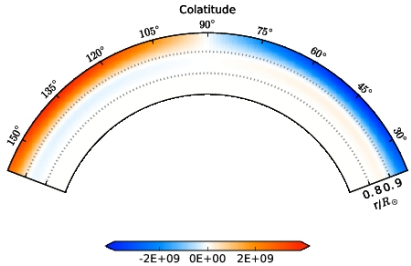

0.025
0.000
−0.025
−50
0
50
Latitudes (Degrees)



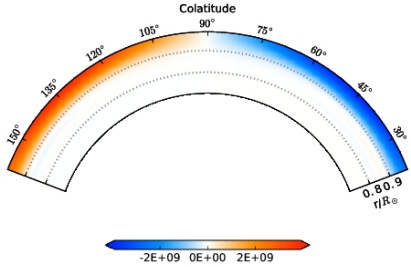
Input profile



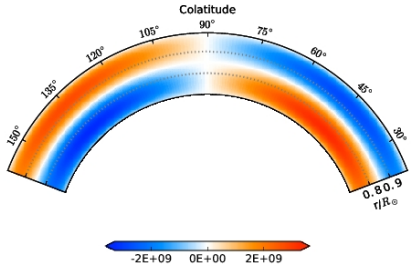
Inverted profile



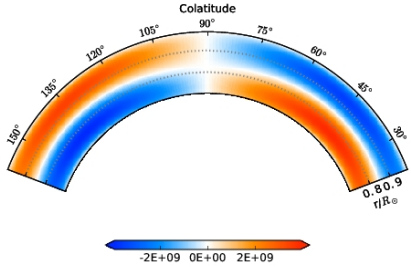
Input profile



Inverted profile



Input profile



Inverted profile