The Envelope Spectrum

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Abstract

Solar-like oscillations exhibit a regular pattern of frequencies. This pattern is dominated by the small and large frequency separations between modes. The accurate determination of these parameters is of great interest, because they give information about e.g. the evolutionary state and the mass of a star. Here, we present a robust method to determine the large and small frequency separations for time series with low signal-to-noise ratio. For this purpose, we analyse a time series of the star KIC 5184732 from the NASA Kepler satellite by employing a combination of Fourier and Hilbert transform. We use the analytic signal of the filtered time series to compute the signal envelope. Spectral analysis of the signal envelope then reveals frequency differences of dominant modes in the periodogram of the stellar time series. With this method, the large frequency separation $\Delta\nu$ can be extracted from the envelope spectrum even for data of poor signal-to-noise ratio. A modification of the method allows for an overview of regularities in a periodogram.

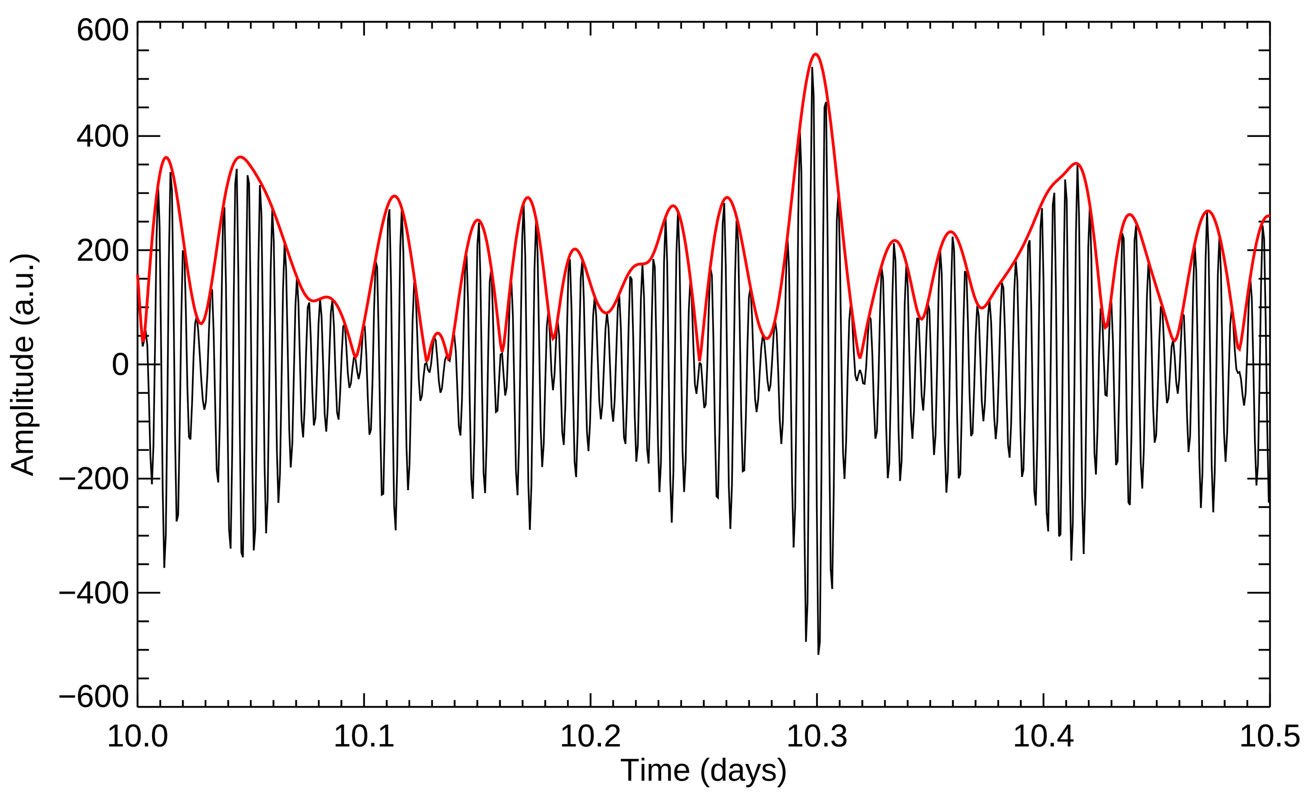
Beatings

The square root of a periodogram or a filtered range of interest is transformed back to the temporal domain by an inverse Fourier transform, thereby resulting in a new time series $x(t)$. The time series $x(t)$ is a superposition of all the modes in the inversely transformed range of the periodogram. The analytic signal $x_a(t)$ of a real valued signal $x(t)$ is defined by

$$x_a(t) = x(t) + i\,\mathrm{H}[x](t),$$

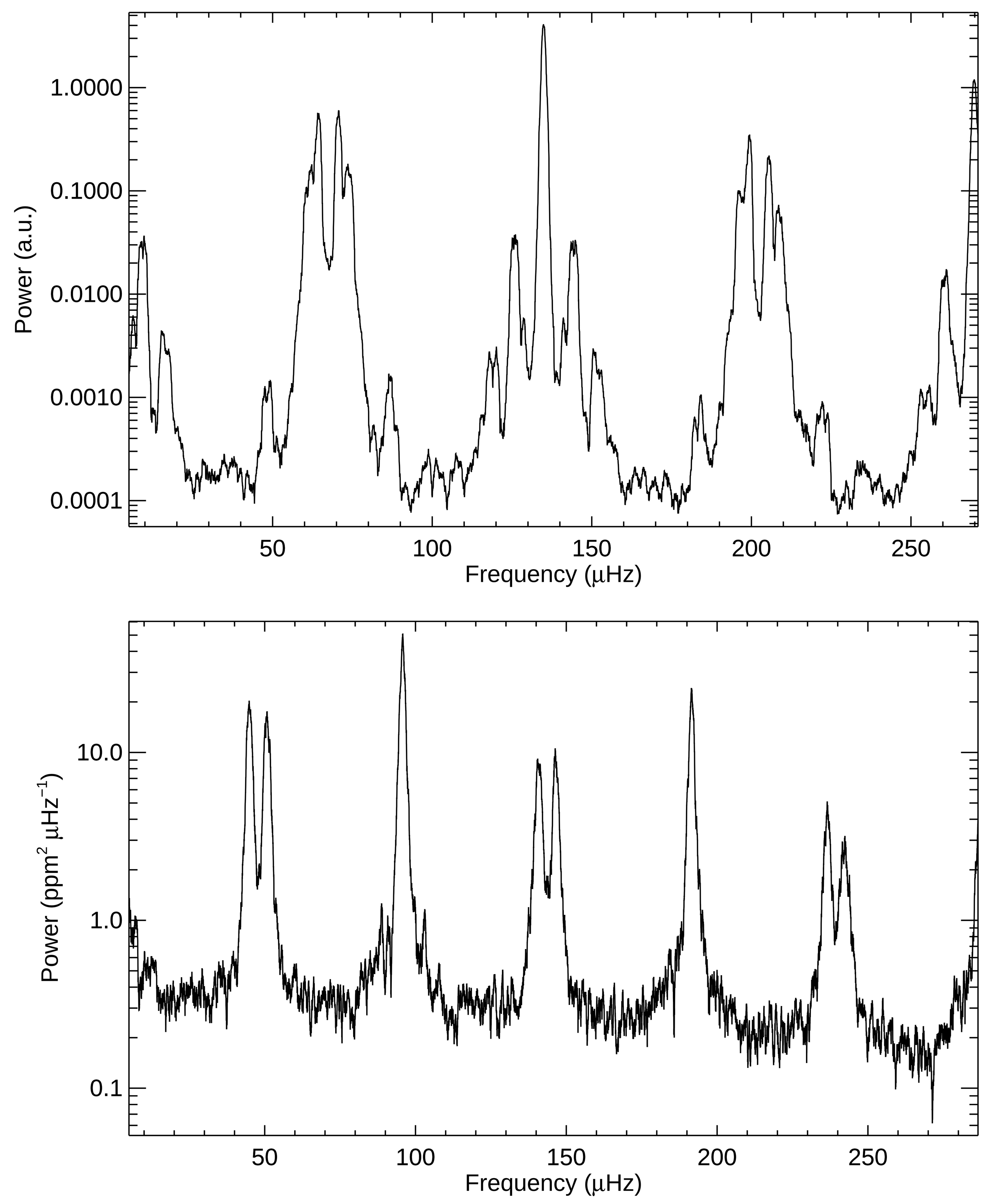
$$\mathrm{H}[x](t) = x(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}\,\mathrm{d}\tau,$$

where $\mathrm{H}[x](t)$ is the Hilbert transform. The $*$ operator indicates convolution. By taking the modulus of $x_a(t)$, the envelope of $x(t)$ is computed. In the spectrum of abs($x_a(t)$), henceforth called the envelope spectrum, the frequency differences of modes which are regularly spaced in the periodogram of the original time series appear as strong peaks.



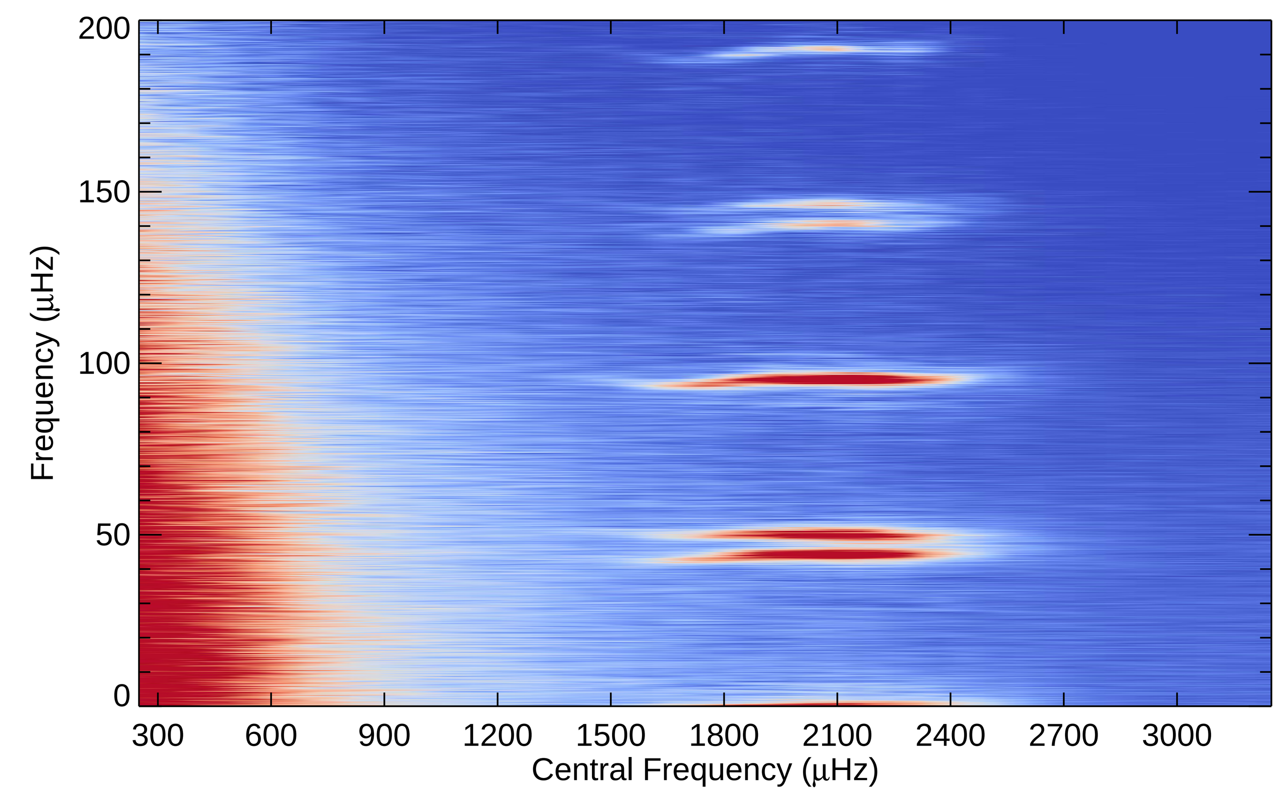
This figure shows a segment of the time series $x(t)$ for the *Kepler* star KIC 5184732 (black) and the corresponding signal envelope (red). The frequency range in which [1] reported p mode oscillations was chosen for the frequency filter $(1.4-2.7\,\mathrm{mHz})$. The modes present in $x(t)$ all have the same phase lag $\phi_0 = 0$. Because of this, modes with a regular frequency spacing superpose to produce beatings in $x(t)$.

The Envelope Spectrum



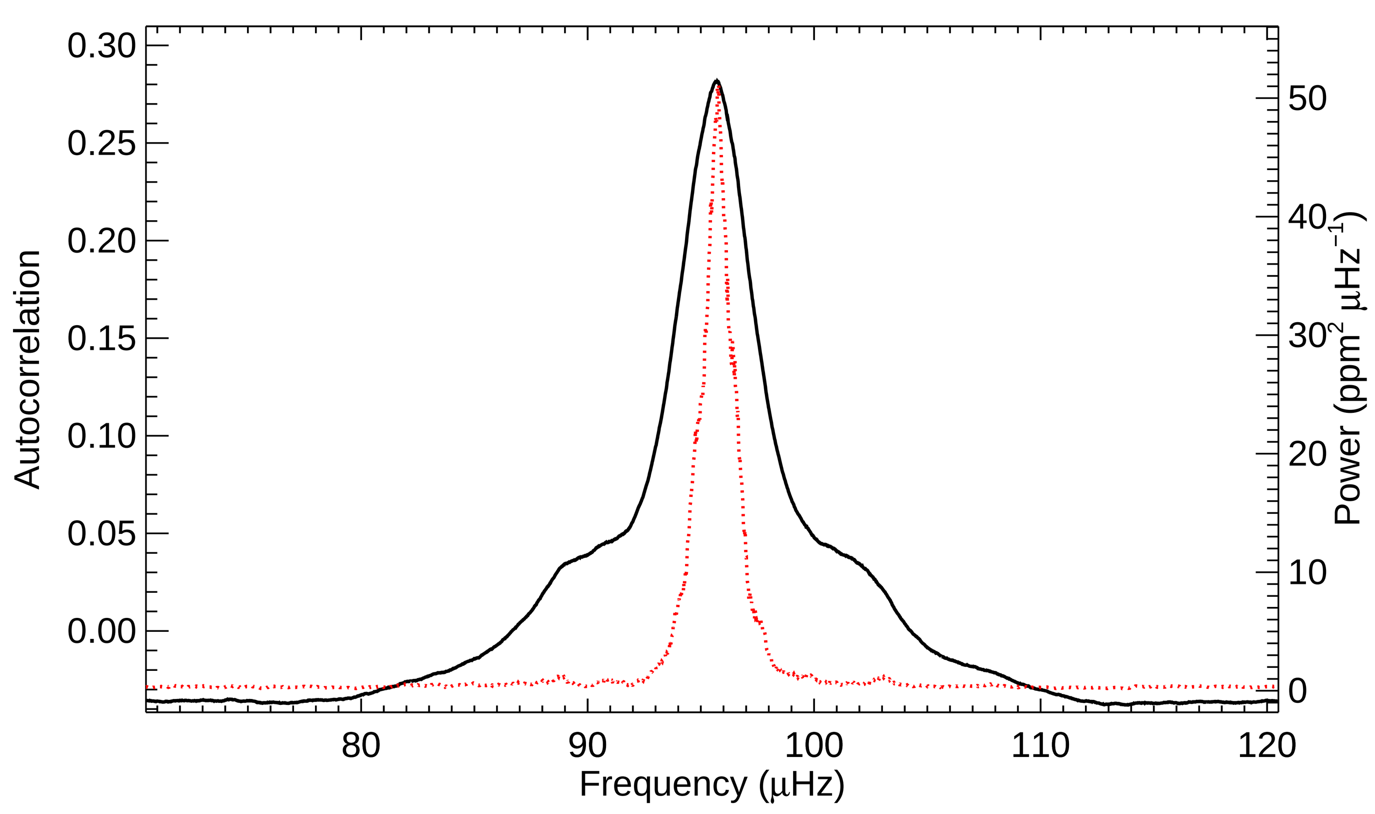
Top panel: The solar envelope spectrum computed from GOLF data [2] on a semi logarithmic scale (boxcar smoothed over $0.5\,\mu\mathrm{Hz}$). The periodogram was filtered for the frequency range 1.7-3.5 mHz. Many frequency separation can be identified, e.g. the large separation $\Delta\nu$ at $\sim 135\,\mu\mathrm{Hz}$ or $\Delta\nu - \delta\nu$ at $\sim 135 \pm 9\,\mu\mathrm{Hz}$.
Bottom panel: Envelope spectrum of KIC 5184732 computed from the frequency range mentioned above. Peaks from dominant beat frequencies stand out against the background. Boxcar smoothed over $0.5\,\mu\mathrm{Hz}$. The large separation $\Delta\nu$ can be identified at $\sim 95\,\mu\mathrm{Hz}$.

Where are regularites?



The envelope spectrum can be used to locate the spectral range of regularities in a periodogram. This is done by incrementally shifting a narrow spectral filter through the periodogram. The figure above shows a map of the envelope spectrum of KIC 5184732 as a function of central frequency of the used spectral filter (filter width 500 μHz, shift increment 10 μHz, boxcar smoothed in y direction over 0.1 μHz). A vertical cut at any central frequency value through this diagram is the respective envelope spectrum. Red colour represents a large amplitude in the envelope spectrum, blue a small amplitude. This map is produced without any previous knowledge of the frequency separations in the periodogram. Therefore, its calculation is less dependent on critical parameters as the common echelle diagram, which needs an accurate estimate of $\Delta\nu$ to be of diagnostic value. The large amplitudes at low frequencies are caused by noise in the time series. Regularities are present in the $\sim 1.4-2.7\,\mathrm{mHz}$ range, hence the large amplitudes in the envelope spectrum. Also, $\Delta\nu$ can be identified at $\sim 95\,\mu\mathrm{Hz}$.

Comparison to the ACF



Comparison of the ranges around the value of $\Delta\nu$ of KIC 5184732 of the autocorrelation of the periodogram (solid black line) and the corresponding section of the envelope spectrum (dashed red line). All data is boxcar smoothed over $0.5\,\mu\mathrm{Hz}$. The peak in the envelope spectrum is a factor ~2 narrower than in the ACF. Therefore, $\Delta\nu$ can be measured somewhat more precise from the envelope spectrum than from the ACF.

Summary

The envelope spectrum allows a reliable detection of regularities in the periodograms of solar and stellar oscillation time series. For solar-like oscillators the mean small and large frequency separations are the prime candidates for these kinds of regularities in the periodogram.
We showed that by incrementally shifting the frequency filter through the periodogram, regularities in the periodogram can be visualised. This map of regularities and their frequency dependence can be created without the choice of a critical parameter such as the large frequency separation for echelle diagrams. It is particularly useful for a first visual estimation of the frequency range of p modes and of $\Delta\nu$.
In comparison to the autocorrelation of the periodogram, we find that the error on the large frequency separation extracted from the envelope spectrum is smaller or equal to the corresponding value extracted from the ACF. This is an indication that the envelope spectrum performs somewhat better than the ACF in the extraction of the large frequency separation.
The results of this work are published in [3].

References

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[3] R. Kiefer, A. Schad, W. Herzberg, and M. Roth. Determination of fundamental asteroseismic parameters using the Hilbert transform. *A&A*, 578:A56, June 2015. doi: 10.1051/0004-6361/201425474.

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