

# The Envelope Spectrum

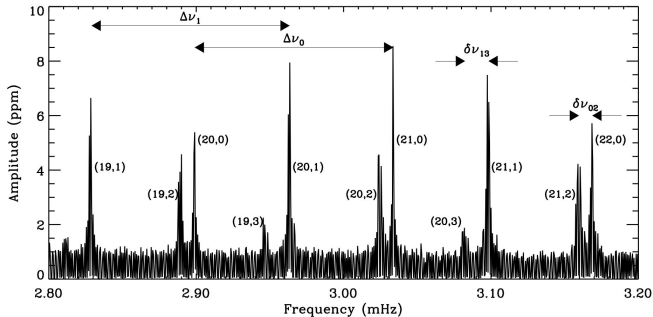
A novel approach to determine frequency separations

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From: Bedding, T. R.; Kjeldsen, H.: Solar-like Oscillations. In: Publications of the Astronomical Society of Australia 20 (2003), p. 203–212.

# The Envelope Spectrum

The superposition of two oscillating signals:

$$\cos(\omega_1 t) + \cos(\omega_2 t) = A(t) \cos\left(\frac{1}{2}(\omega_1 + \omega_2)t\right),$$

$$A(t) = 2 \cos\left(\frac{1}{2}(\omega_1 - \omega_2)t\right),$$

where  $A(t)$  is the slowly varying amplitude of the signal which oscillates with frequency

$$\Omega = \frac{1}{2}(\omega_1 - \omega_2).$$

# The Envelope Spectrum

Compute the periodogram (FFT or LSP)



Take absolute value, set all phases to zero

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Filter for frequency range of interest (Tukey)



Apply inverse FFT on periodogram

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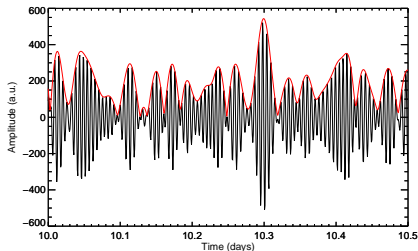


Apply inverse FFT on periodogram



Compute the analytic signal and the signal envelope

# The Envelope Spectrum



Analytic signal:

$$x_a(t) = x(t) + i H[x](t)$$

Hilbert transform:

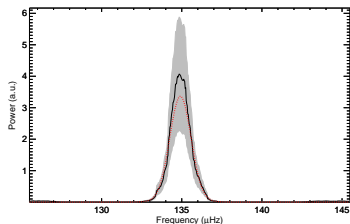
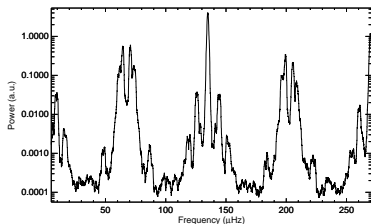
$$H[x](t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau$$

Signal envelope:

$$\text{env} = \text{abs}(x_a(t))$$

# Solar envelope spectrum

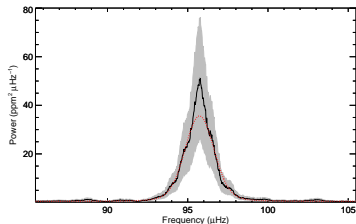
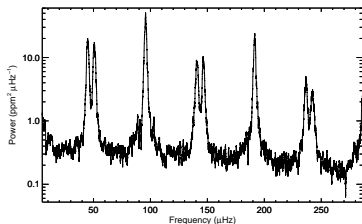
- GOLF time series: July 2007 – July 2008
- Filtering as described above, Tukey with  $\alpha = 0.9$
- Frequency range: 1.7–3.5 mHz
- Envelope spectrum 1  $\mu\text{Hz}$  boxcar smoothed
- $\Delta\nu_{\odot} = 134.92 \pm 0.06 \mu\text{Hz}$



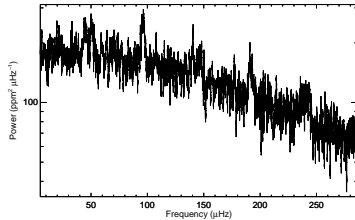
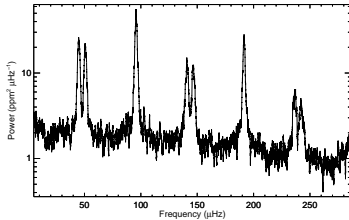
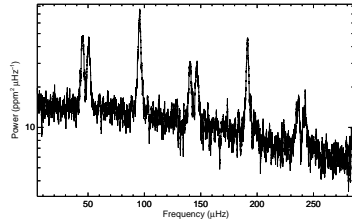
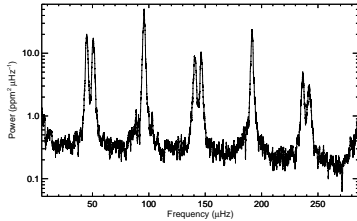


# Stellar envelope spectrum

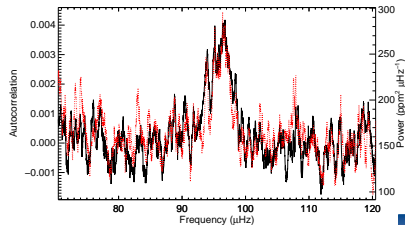
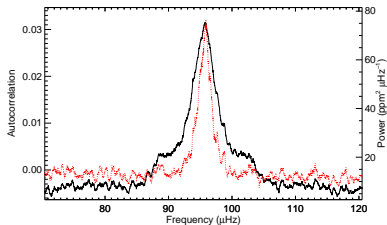
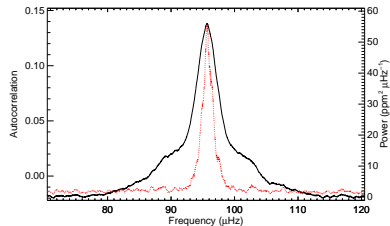
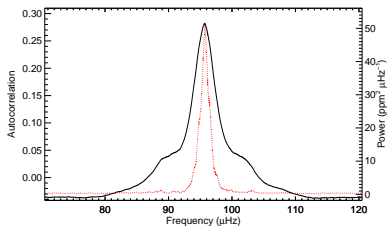
- Kepler time series: Q7-Q15
- Filtering as described above, Tukey with  $\alpha = 0.9$
- Frequency range: 1.4–2.7 mHz,
- Envelope spectrum 0.5  $\mu\text{Hz}$  boxcar smoothed
- $\Delta\nu = 95.7 \pm 0.2 \mu\text{Hz}$



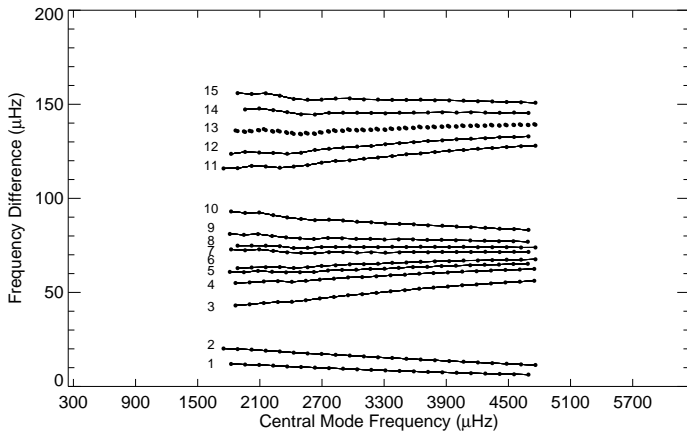
# Robustness to noise



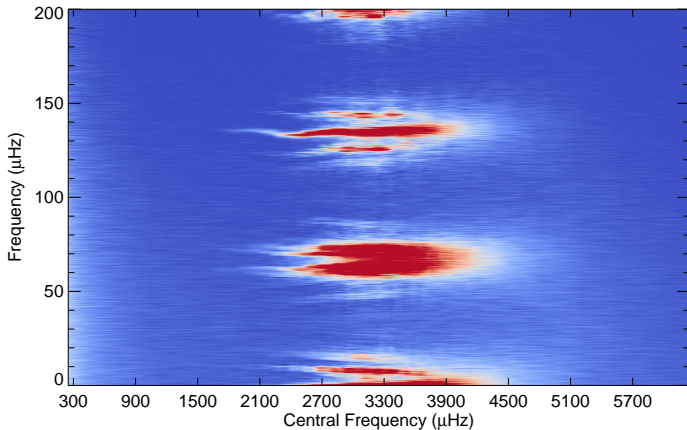
# Comparison to ACF



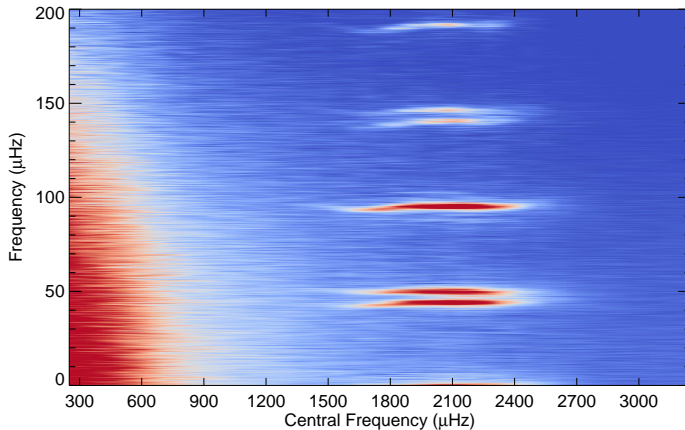
# Theoretical frequency differences



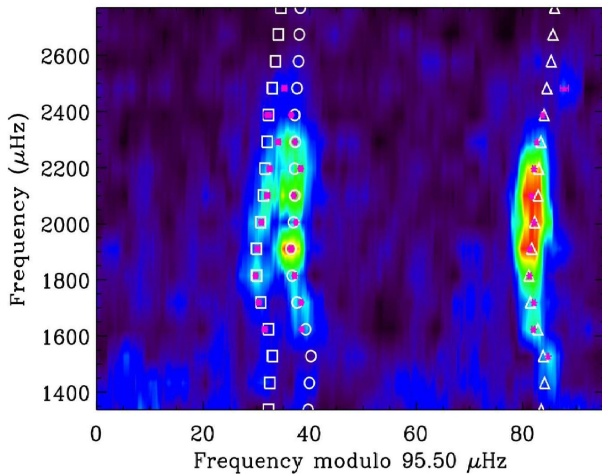
# Map of regularities in GOLF periodogram



# Map of regularities of KIC 5184732



# Echelle diagram of KIC 5284732



From Mathur et al. (2012)

# Summary

- Reliable detection of regularities in periodogram
- Alternative way for first estimation of  $\Delta\nu$
- Robust to low S/N data
- Paper submitted to A&A:

## Determination of fundamental asteroseismic parameters using the Hilbert transform

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### ABSTRACT

*Context.* Solar-like oscillations exhibit a regular pattern of frequencies. This pattern is dominated by the small and large frequency separations between modes. The accurate determination of these parameters is of great interest, because they give information about e.g. the evolutionary state and the mass of a star.

*Aims.* We want to develop a robust method to determine the large and small frequency separations for time series with low signal-to-noise ratio. For this we analyse time series from the GOLF instrument aboard SOHO and the NASA *Kepler* satellite by employing a combination of the *Redundant* and the *Hilbert* transform.



Thank you for your attention.

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