Two-scale analysis of solar magnetic helicity

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Important for understanding the dynamo:
→ Precursers of ARs and CMEs
→ Opposite helicity at large/small scales?
→ Inside/outside dynamo region (solar wind)

Coronal mass ejections

→ important carrier of magnetic helicity

Solar magnetic helicity

- Important indicator for CMEs
  - Topological invariant
- Need field extrapolation into a box
  - Or is the field turbulent?
- Large-scale vs small-scale mag helicity
  - Is there an α effect in the Sun

Valori et al. (2016)

Finite volume (FV)

$$\mathscr{H}_\mathcal{V} = \int_\mathcal{V} (\mathbf{A} + \mathbf{A}_\mathrm{p}) \cdot (\mathbf{B} - \mathbf{B}_\mathrm{p})\, \mathrm{d}\mathcal{V}$$

see Eq. (3)

- Requires $\mathbf{B}$ in $\mathcal{V}$ *e.g.*, from MHD simulations or NLFFF
- Compute $\mathscr{H}_\mathcal{V}$ at one time
- May employ different gauges (see Table 2)

Helicity-flux integration (FI)

$$\frac{\mathrm{d}\mathscr{H}_\mathcal{V}}{\mathrm{d}t} = 2 \int_{\partial\mathcal{V}} \left[(\mathbf{A}_\mathrm{p} \cdot \mathbf{B}) v_n - \left(\mathbf{A}_\mathrm{p} \cdot \mathbf{v}_\mathrm{t} \right) B_n \right] \mathrm{d}S$$

- Requires time evolution of vector field on $\partial\mathcal{V}$
- Requires knowledge or model of flows on $\partial\mathcal{V}$
- Valid for a specific set of gauge and assumptions, see Pariat et al. (2017)

Discrete flux-tubes (DT)

$$\mathscr{H} \simeq \sum_{i=1}^{M} \mathcal{T}_i \Phi_i^2 + \sum_{i=1}^{M} \sum_{j=1, j\neq i}^{M} \mathcal{L}_{i,j} \Phi_i \Phi_j\,,$$

see Eq. (31)

Twist-number (TN)

$$\mathscr{H} \simeq \mathcal{T} \Phi^2$$

see Eq. (32)

- Estimation of the twist contribution to $\mathscr{H}$
- Requires $\mathbf{B}$ in $\mathcal{V}$
- Requires a flux-rope-like structure for computing the twist $\mathcal{T}$

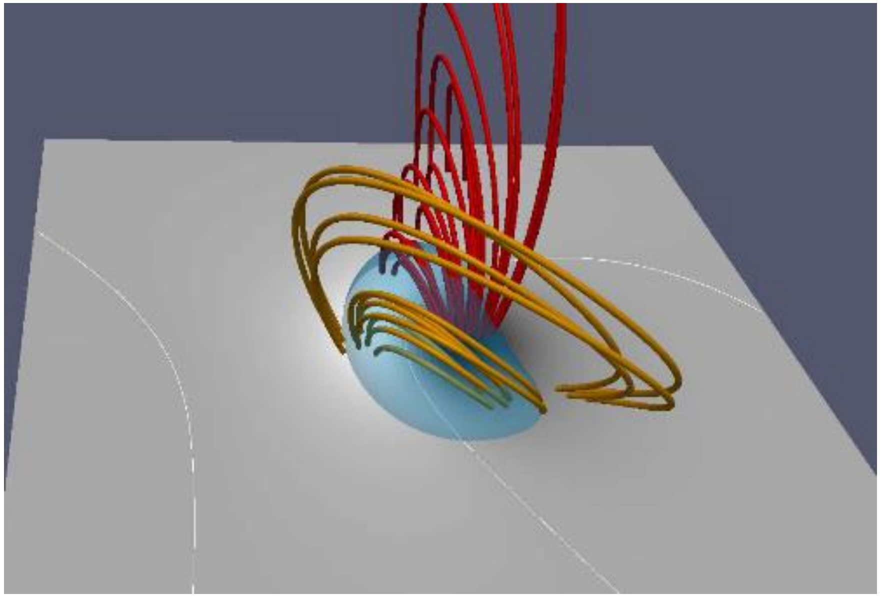
Connectivity-based (CB)

$$\mathscr{H} = A \sum_{i=1}^{M} \alpha_i \Phi_i^{2\delta} + \sum_{l,m=1}^{M} \mathcal{L}_{lm} \Phi_l \Phi_m$$

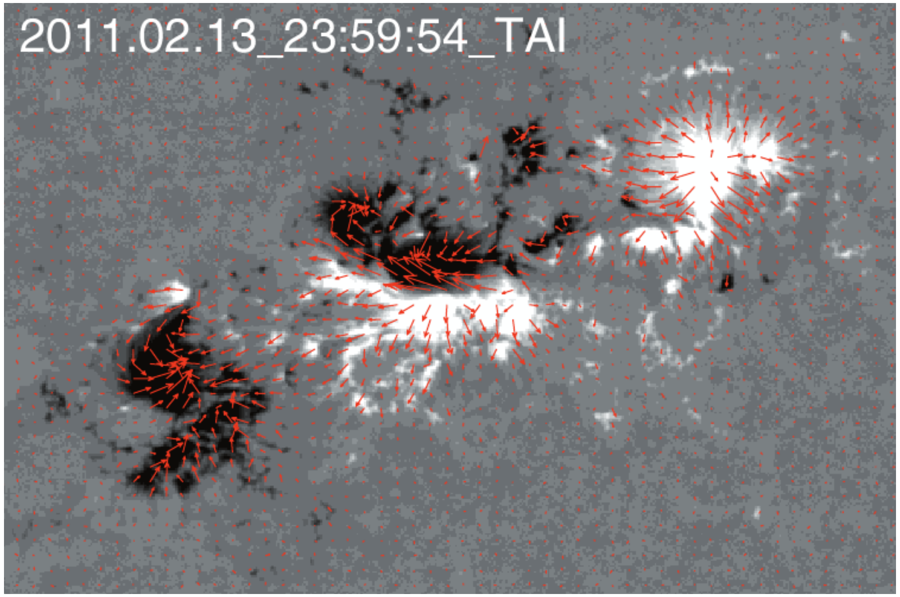
see Eq. (35)

- Requires the vector field on photosphere at one time
- Models the corona connectivity as a collection of M force-free flux tubes
- Minimal connection length principle

Approaches to solar magnetic helicity



Review by Valori et al. (2016)



Zhang, Brandenburg, & Sokoloff (2016)

$$\mathscr{H}_{\mathcal{V}} \equiv \int_{\mathcal{V}} \left(\mathbf{A} + \mathbf{A}_{\mathrm{p}}\right) \cdot \left(\mathbf{B} - \mathbf{B}_{\mathrm{p}}\right) \, \mathrm{d}\mathcal{V}$$

Berger & Field (1984)

→ No length scale information

$$H_{\mathrm{M}}(k) = \int_{2\pi} \hat{\mathbf{A}}(\mathbf{k}) \cdot \hat{\mathbf{B}}^{*}(\mathbf{k}) \, k d\phi + \mathrm{c.c.}$$

$$kH_{\mathrm{M}}(k) = \mathrm{Im}\, \varepsilon_{ijk} \int_{2\pi} k_i \hat{B}_j(\mathbf{k}) \hat{B}_k^{*}(\mathbf{k}) \, k d\phi$$

$$\left|kH_{\mathrm{M}}(k)\right| \le 2E_{\mathrm{M}}(k)$$

gauge-invariant
(periodic → FFT)

Background of our approach

- Application to turbulent fields
- Density of linkages (gauge-invariant)

Subramanian & Brandenburg (2006)

- Mean helicity density of mean/fluct fields

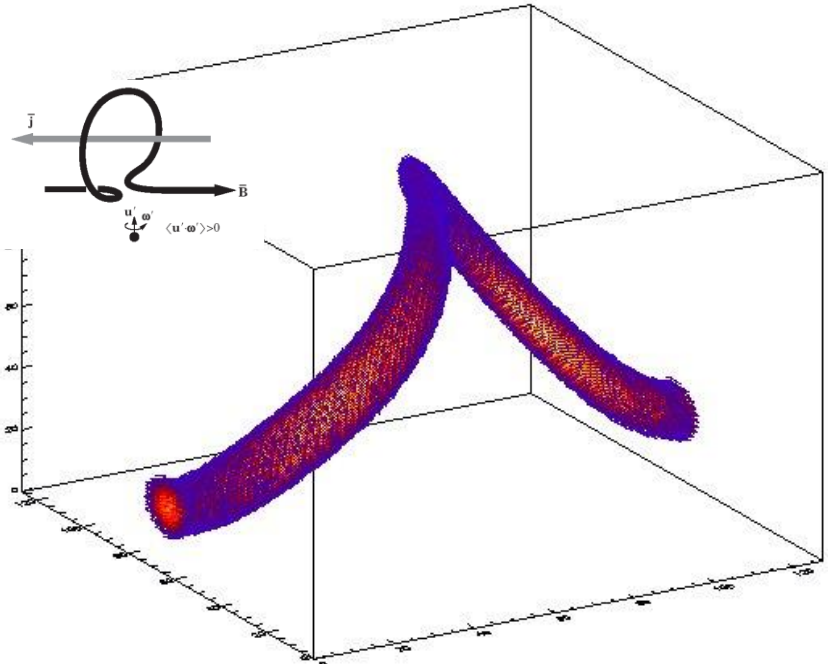
$$\frac{\mathrm{d}}{\mathrm{d}t}\overline{\mathbf{A}}\cdot\overline{\mathbf{B}} = +2\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}} - 2\eta\overline{\mathbf{J}}\cdot\overline{\mathbf{B}} - \nabla\cdot\left(\overline{\mathbf{E}}\times\overline{\mathbf{A}}\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\overline{\mathbf{a}\cdot\mathbf{b}} = -2\overline{\boldsymbol{\mathcal{E}}}\cdot\overline{\mathbf{B}} - 2\eta\overline{\mathbf{j}\cdot\mathbf{b}} - \nabla\cdot\overline{\mathbf{e}\times\mathbf{a}}$$

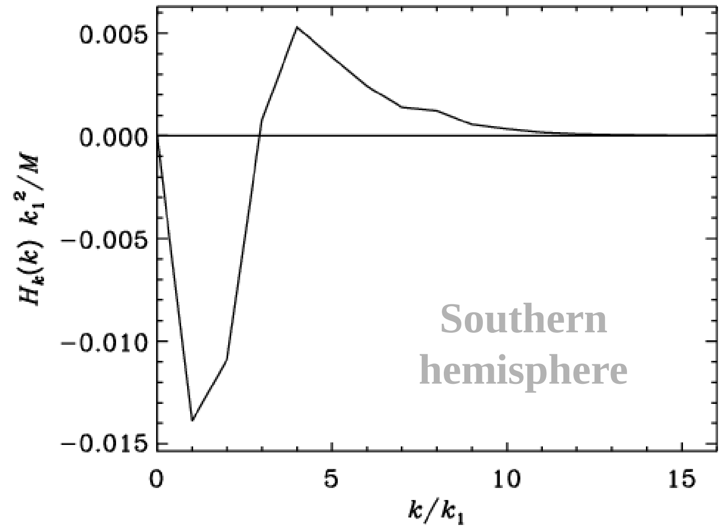
$\overline{\mathbf{e}\times\mathbf{a}}$ → advective & turbulent diffusive flux contributions

Dynamos produce bi-helical fields

***zero* net helicity**



Magnetic helicity spectrum



$$\int H(k)dk = \langle \mathbf{A}\cdot\mathbf{B} \rangle$$

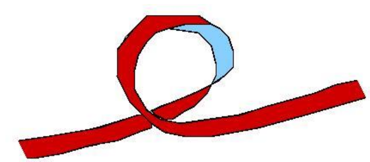
$$= \langle \overline{\mathbf{A}}\cdot\overline{\mathbf{B}} \rangle + \langle \mathbf{a}\cdot\mathbf{b} \rangle$$

$$\approx k_{\rm f}^2 \langle \mathbf{a}\cdot\mathbf{b} \rangle$$

g.Ω → u.ω → a.b

Pouquet, Frisch, & Leorat (1976)

$$\alpha = -\tfrac{1}{3}\tau\left(\overline{\boldsymbol{\omega}\cdot\mathbf{u}} - \overline{\mathbf{j}\cdot\mathbf{b}}/\rho_0\right)$$



Local technique: 2-point correlation tensor

Inspired by earlier 1-D work:
Matthaeus et al. (1982)

Real space

$$M_{ij}(\mathbf{r}) = \langle B_i(\mathbf{x}) B_j(\mathbf{x}+\mathbf{r}) \rangle$$

Fourier space

$$\left\langle \hat{B}_i(\boldsymbol{k},t)\hat{B}_j^*(\boldsymbol{k}',t)\right\rangle = \Gamma_{ij}(\boldsymbol{k},t)\delta^2(\boldsymbol{k}-\boldsymbol{k}'),$$

$$\Gamma_{ij}(\boldsymbol{k},t) = \frac{2E_M(k,t)}{4\pi k}(\delta_{ij}-\hat{k}_i\hat{k}_j) + \frac{iH_M(k,t)}{4\pi k}\varepsilon_{ijk}k_k,$$

$$= \begin{pmatrix} (1-\cos^2\phi_k)2E_M & -\sin 2\phi_k E_M & -ik\sin\phi_k H_M \\ -\sin 2\phi_k E_M & (1-\sin^2\phi_k)2E_M & ik\cos\phi_k H_M \\ ik\sin\phi_k H_M & -ik\cos\phi_k H_M & 2E_M \end{pmatrix}$$

$k_x = k\cos\phi_k$
$k_y = k\sin\phi_k$
no z derivatives

→ **compute**

$$2E_M(k) = 2\pi k\,\mathrm{Re}\,\langle \Gamma_{xx} + \Gamma_{yy} + \Gamma_{zz}\rangle_{\phi_k},$$

$$kH_M(k) = 4\pi k\,\mathrm{Im}\,\langle \cos\phi_k \Gamma_{yz} - \sin\phi_k \Gamma_{xz}\rangle_{\phi_k},$$

Zhang, Brandenburg, & Sokoloff (2014, ApJ 784, L45)

NOAA 11158

Helicity injection	NFFF extrapol
Vemareddy+12	Jing+12
Liu+Schuck12	Tziotziou+13

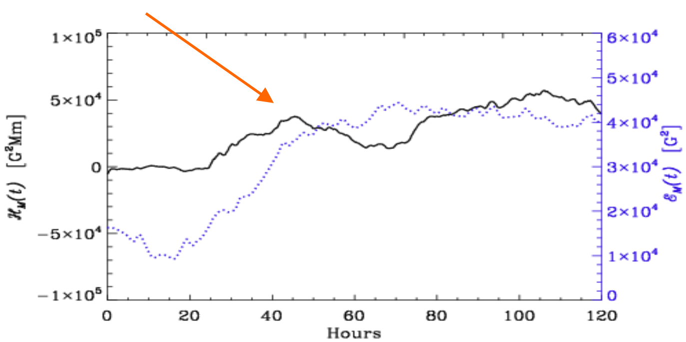
$$\langle \mathbf{A}\cdot\mathbf{B}\rangle = \int H_{\mathrm{M}}(k)\,dk$$

$$\mathcal{H}_{V} = 10^{43}\ \mathrm{Mx}^2$$

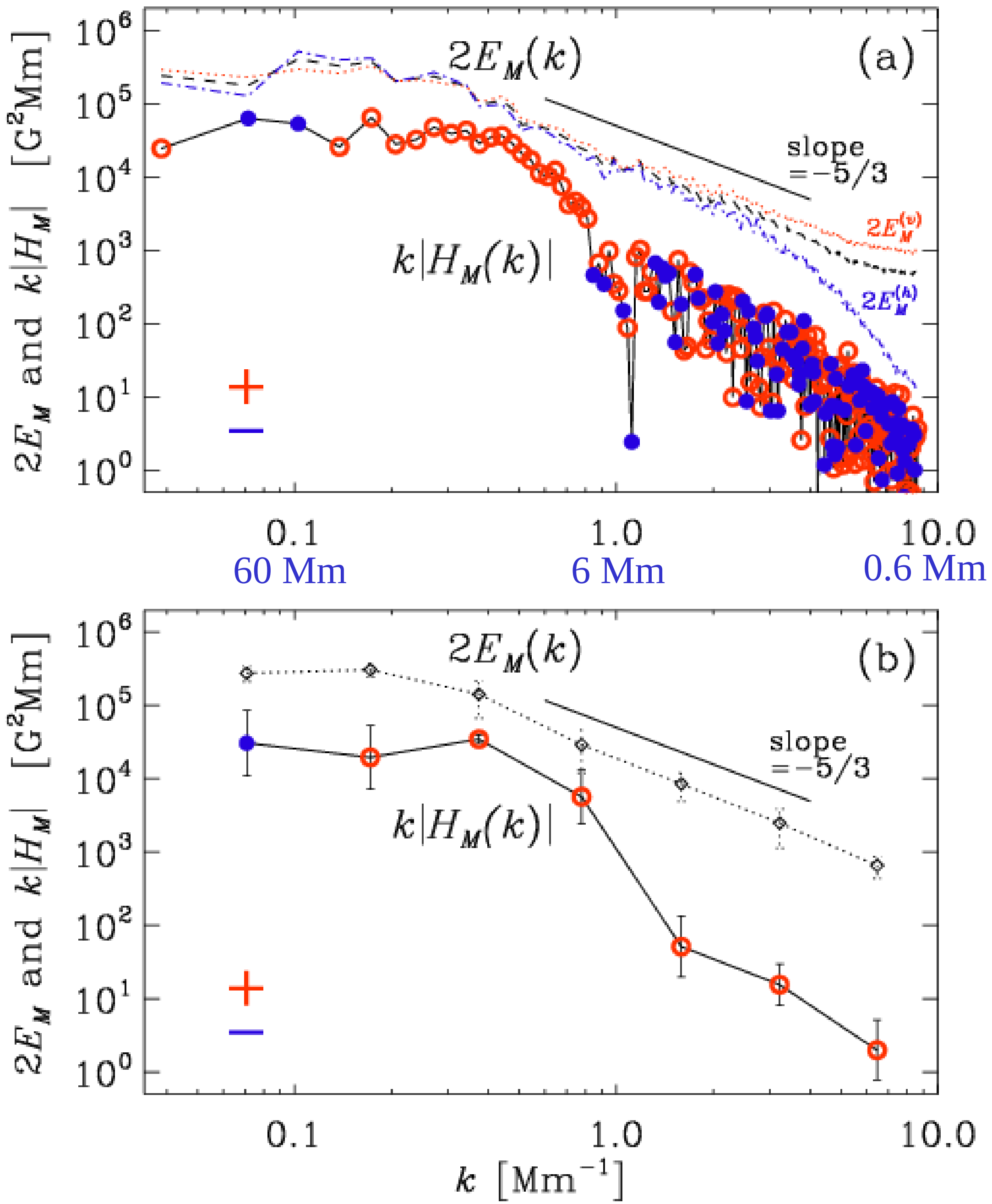
Compare:

$$3\times10^4\,\mathrm{G}^2\mathrm{Mm}\times(186\mathrm{Mm})^2\times100\mathrm{Mm}$$

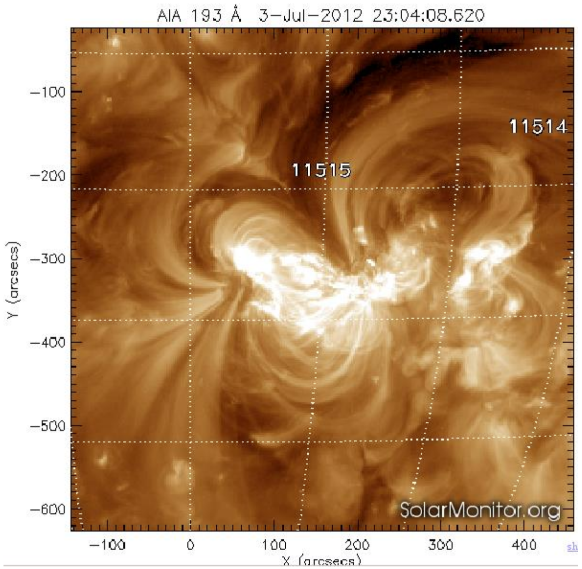
$$=10^{43}\mathrm{Mx}^2$$

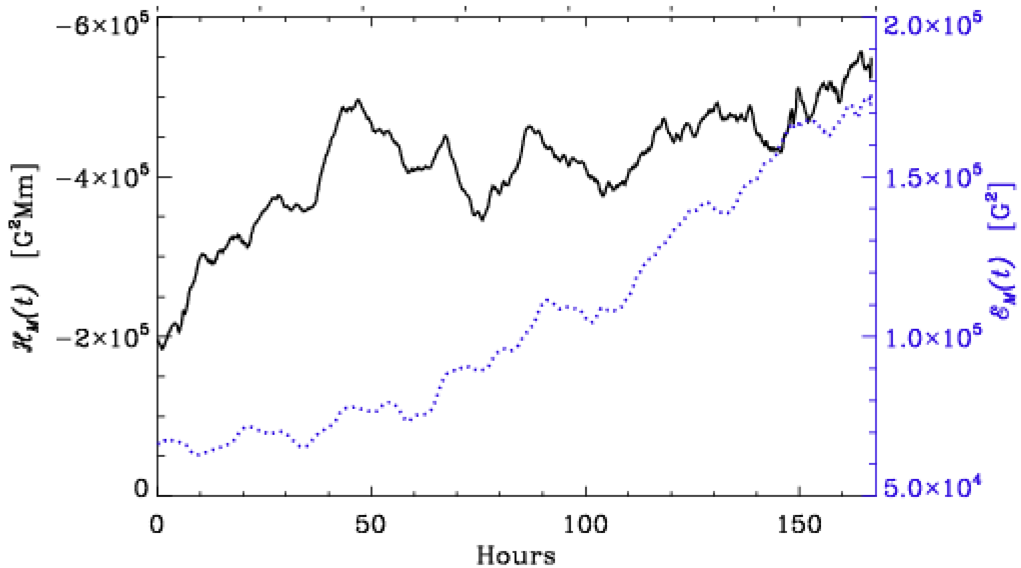


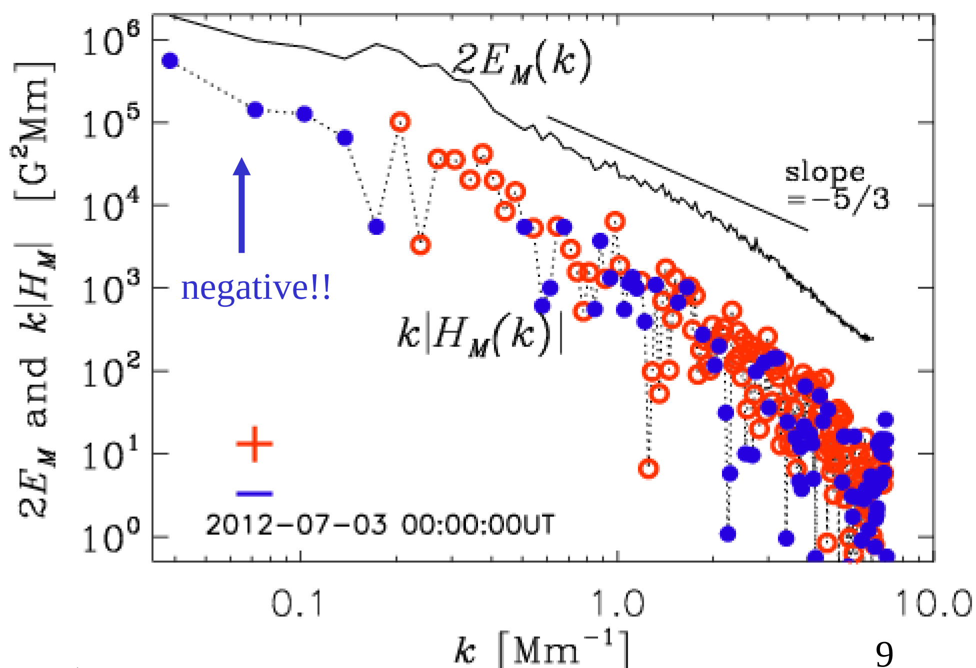
cf. Manolis' question



NOAA 11515: wrong sign?







- 18 degr south
- but negative hel.
- dominance of LS field

Zhang, Brandenburg, & Sokoloff (2016, ApJ)

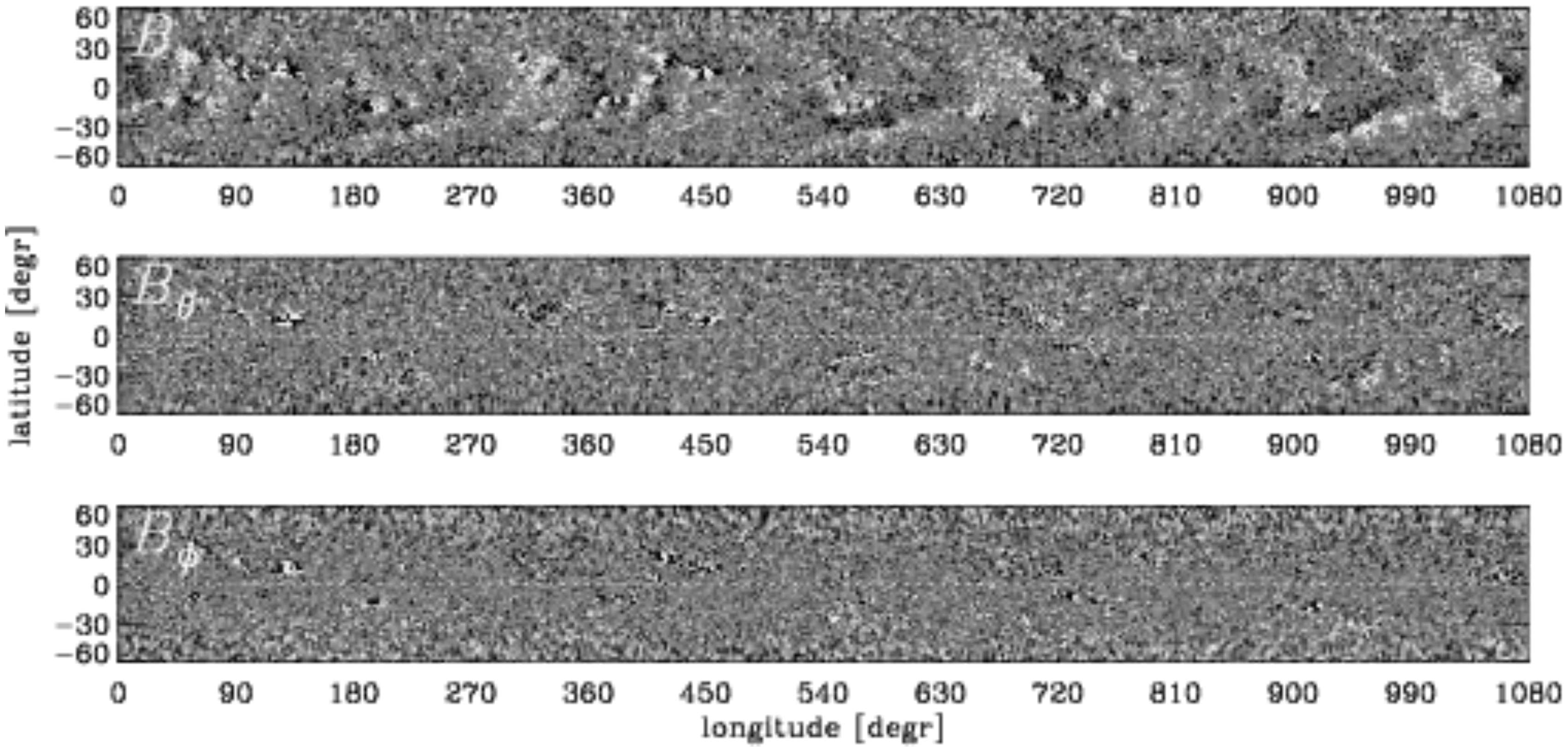
Global vector magnetograms

- First constructed from SOLIS/VSM (Gosain+13)
- Same pipeline now used for HMI (Hughes+16)
- Here earlier independent version (Yang Liu)
- 3 x 3600 x 1440 pixels (for 3 CRs)

Brandenburg, Petrie, & Singh (2017, ApJ 836, 21)

Toward global analysis

CR2161-2163 patched together (Feb 28-May 20, 2015)



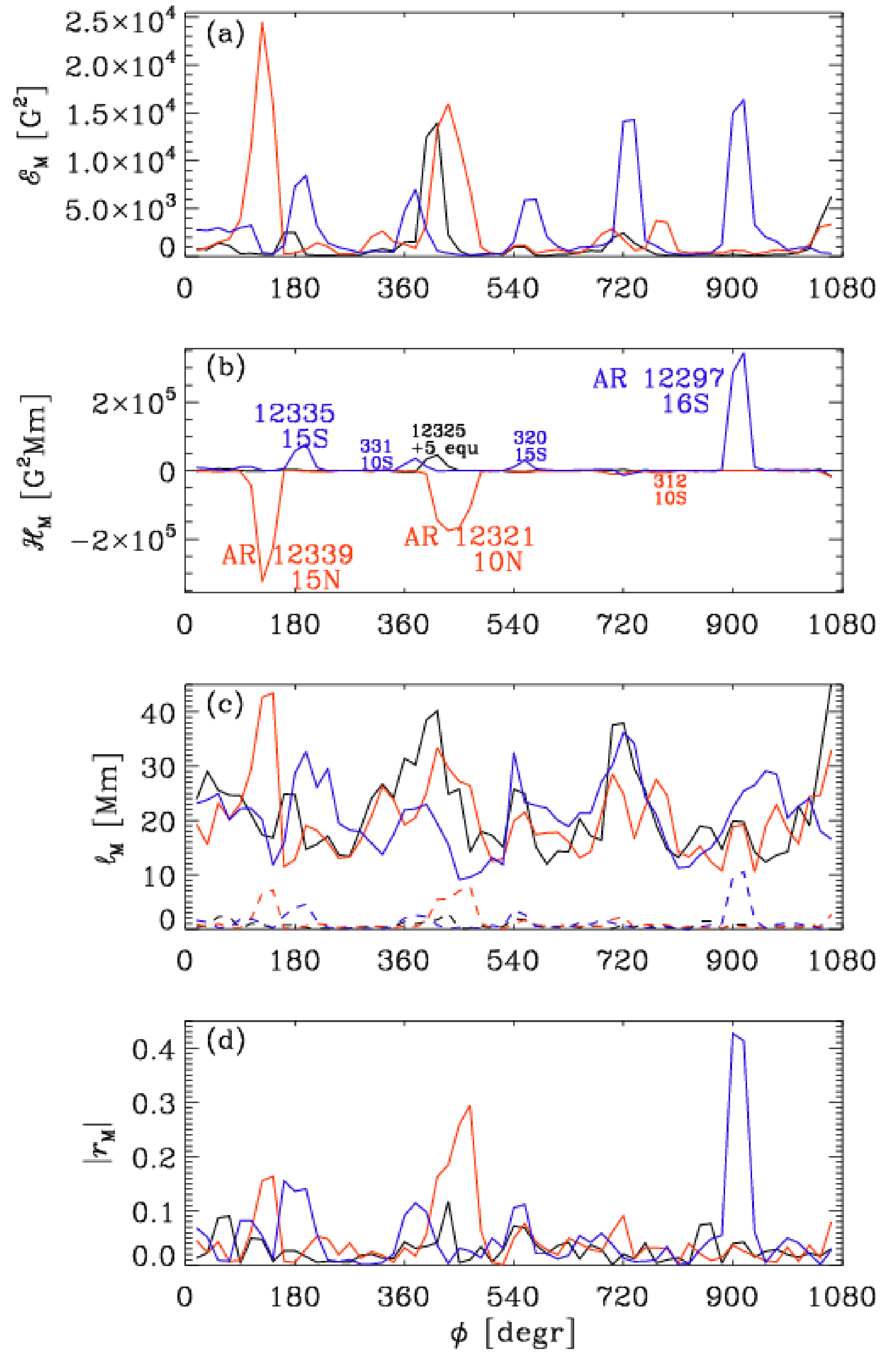
HMI, downsampled to 0.1 degr x 0.1 degr

Two approaches:

- Rolling patches in lat. strips, 2-D standard FFT
- Double Fourier transform (Roberts & Soward 1975)

(i) Rolling patches

- Helicity emerges intermittently!
  - Lots of variability
  - Not all are helical
  - Hemispheric sign dep.
- Spectral peak 20 Mm
  - Fractional helicity 10%-40%



(ii) Double Fourier transform

"Sliding" 2-point Correlation function

$$M_{ij}(\mathbf{x}) = \langle B_i(\mathbf{X}) B_j(\mathbf{X}+\mathbf{x}) \rangle$$

$$M_{ij}(\mathbf{X},\mathbf{x}) = \langle B_i(\mathbf{X}+\tfrac{1}{2}\mathbf{x}) B_j(\mathbf{X}-\tfrac{1}{2}\mathbf{x}) \rangle$$

First Fourier transform

$$\hat{M}_{ij}(\mathbf{X},\mathbf{k}) = \int M_{ij}(\mathbf{X},\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}} d^2x/(2\pi)^2$$

Second Fourier transform (now over ***X***)

$$\tilde{M}_{ij}(K,k) = \langle \hat{B}_i(\mathbf{k}+\tfrac{1}{2}\mathbf{K}) \hat{B}_j^*(\mathbf{k}-\tfrac{1}{2}\mathbf{K}) \rangle$$

Roberts & Soward (1975)

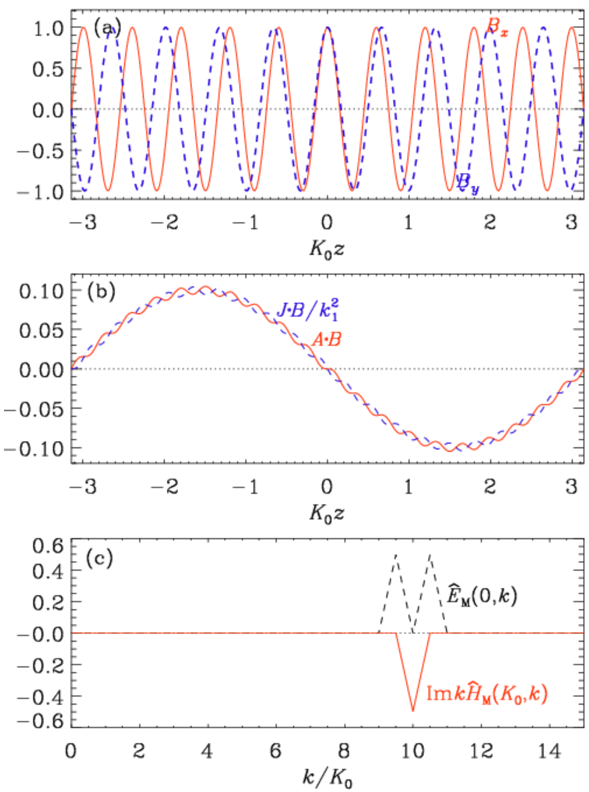
Validation of two-scale technique

1-D example:

$$\widetilde{M}_{ij}(K,k) = \left\langle \hat{B}_i(\mathbf{k}+\tfrac{1}{2}\mathbf{K})\hat{B}_j(\mathbf{k}-\tfrac{1}{2}\mathbf{K}) \right\rangle$$

$$B = \begin{pmatrix} \sin 10z \\ \cos 10z \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} \cos 10.5z \\ \cos 9.5z \\ 0 \end{pmatrix}$$

Is a helical field with modulation

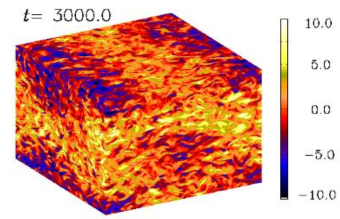


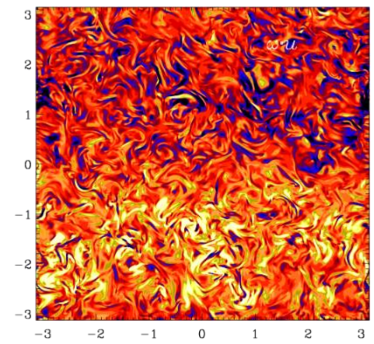
Yes, recover large-scale modulation!

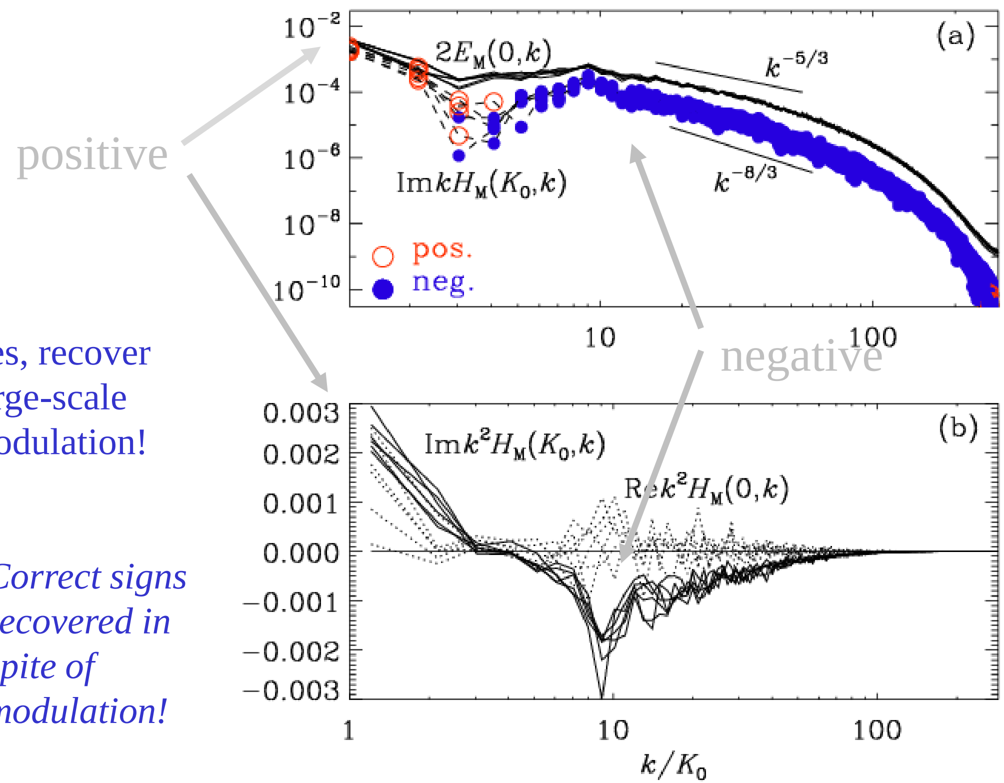
Correct signs recovered in spite of modulation!

3-D turbulence

w/ dynamo: bi-helical

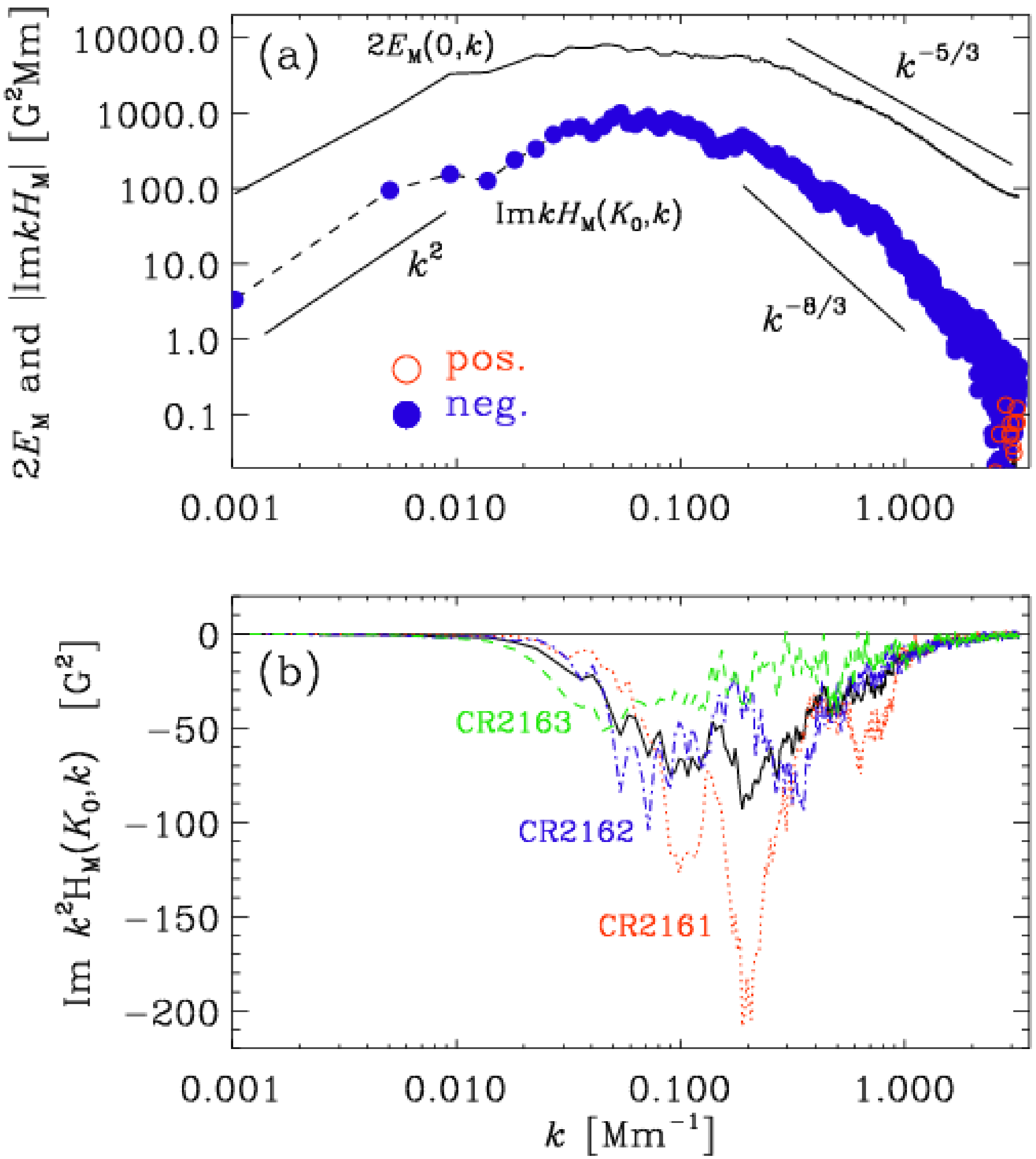






Result of two-scale approach

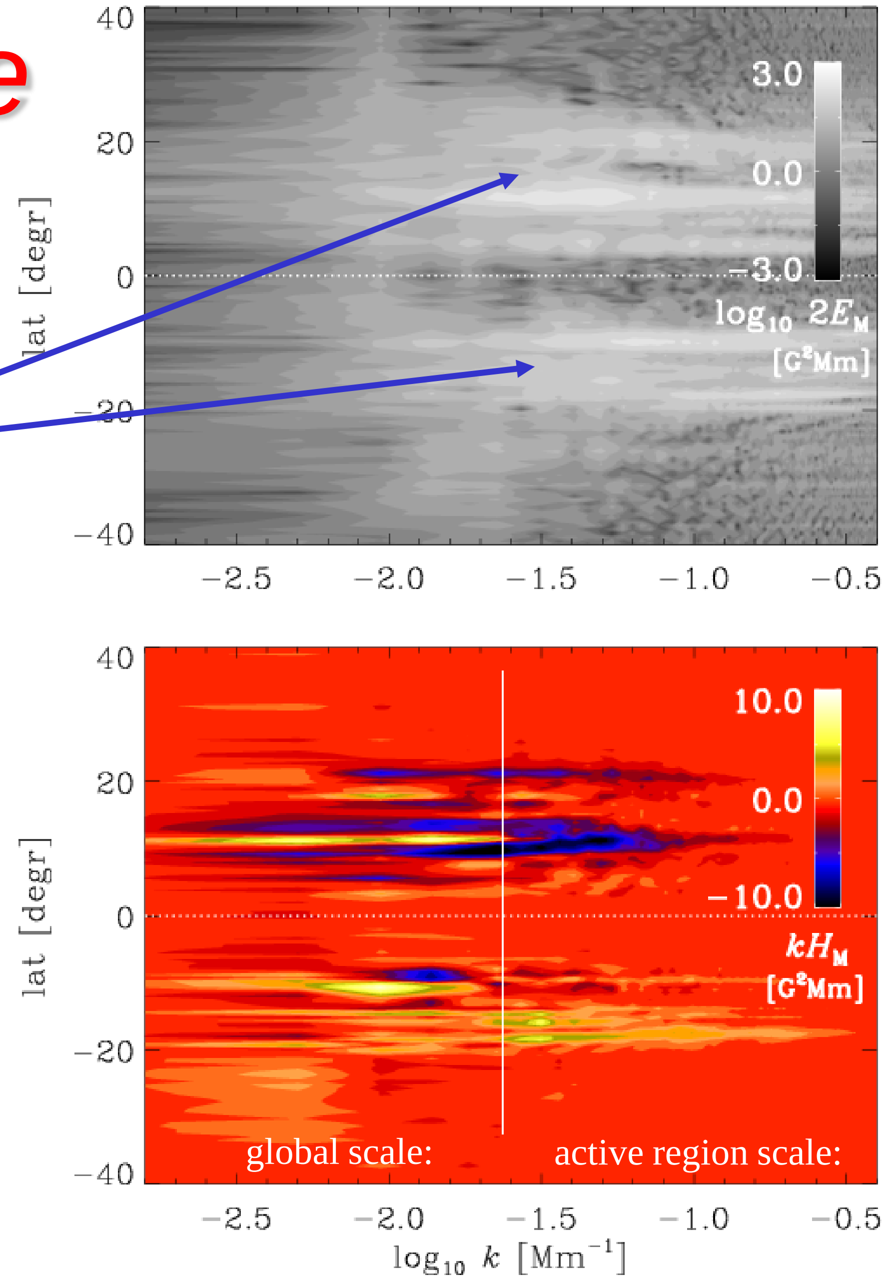
- Sign rule well obeyed
- Fluctuations small over 1-3 month
- No change of sign at small *k*



Back to X space

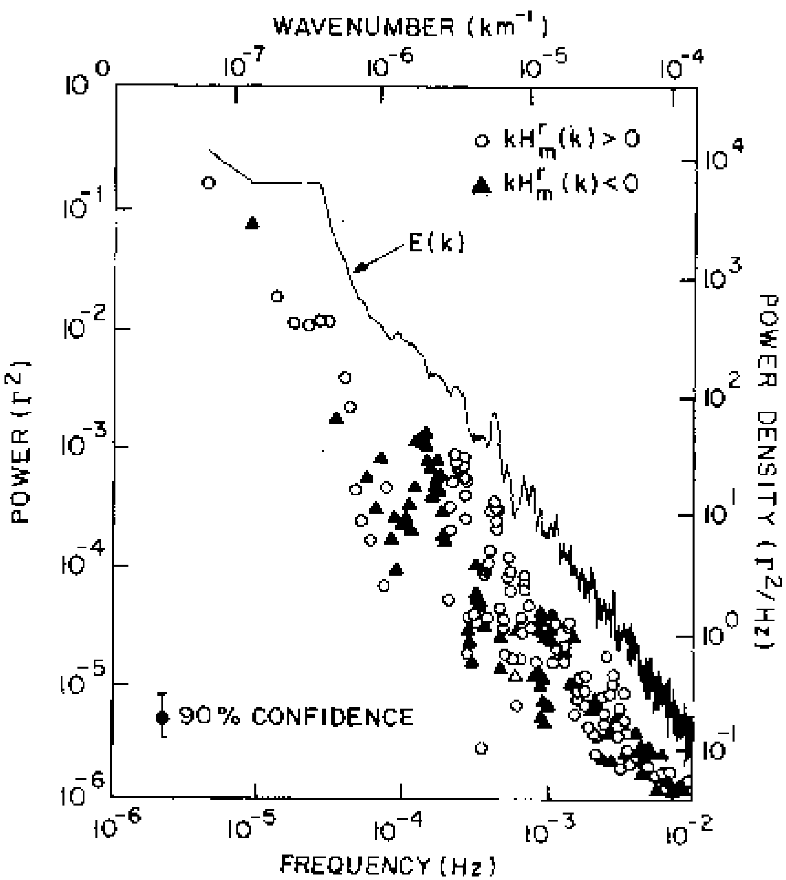
$$\hat{M}_{ij}(\mathbf{X},\mathbf{k}) = \int \widetilde{M}_{ij}(\mathbf{K},\mathbf{k}) e^{i\mathbf{K}\cdot\mathbf{X}} d^2K$$

- Magn. energy well reconstructed in latitude
  - Concentrated to +/- 15 degr
- At active region scale: negative in north, pos in south
  - But no clear evidence of reversal at global scales



Comparison with solar wind data

Matthaeus et al. (1982)



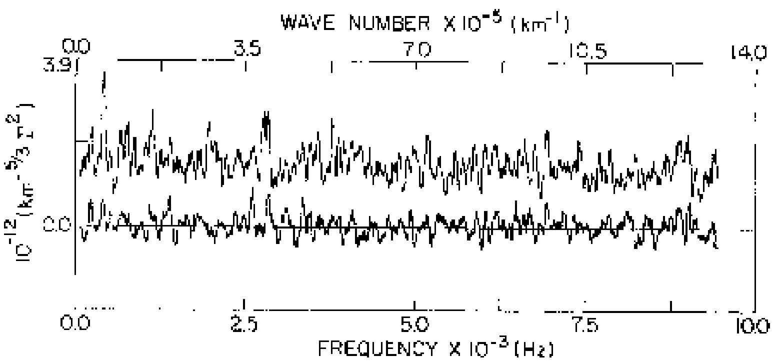
Measure correlation function

$$M_{ij}(\mathbf{r}) = \langle B_i(\mathbf{x}) B_j(\mathbf{x}+\mathbf{r}) \rangle$$

In Fourier space, calculate

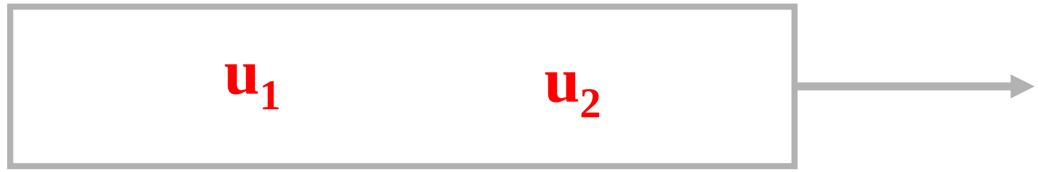
magnetic energy and helicity spectra

$$M_{ij}(k) = (\delta_{ij} - k_i k_j) E(k) - i\varepsilon_{ijk} k_k H(k)$$

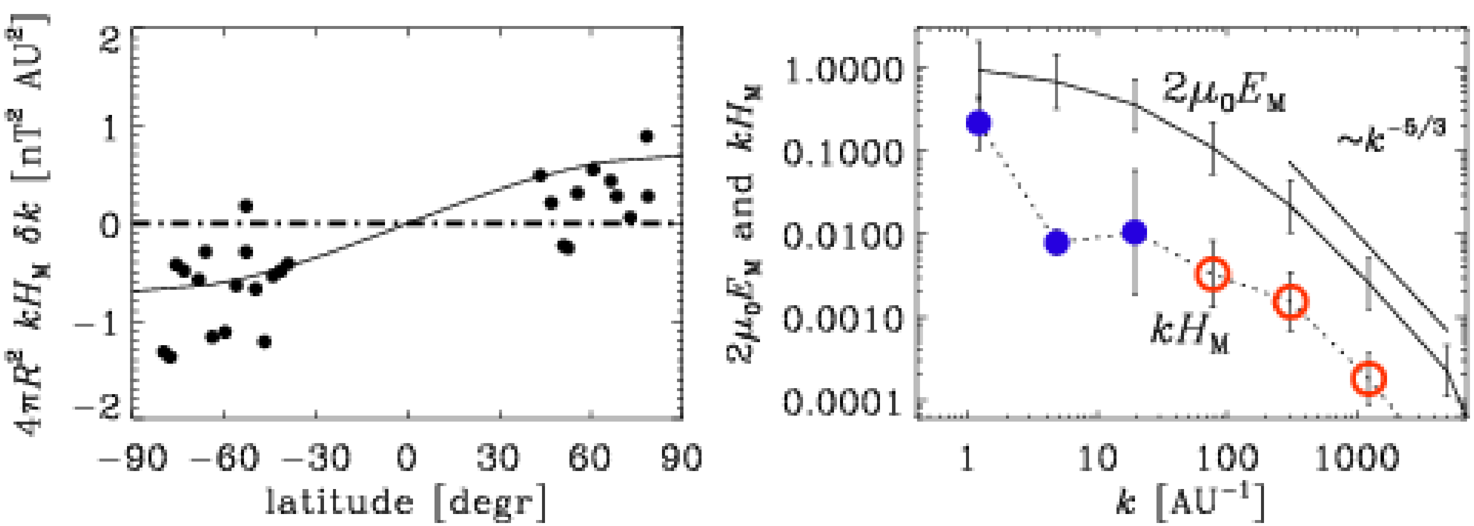


→ Should be done with Ulysses data away from equatorial plane

Measure 2-point correlation tensor

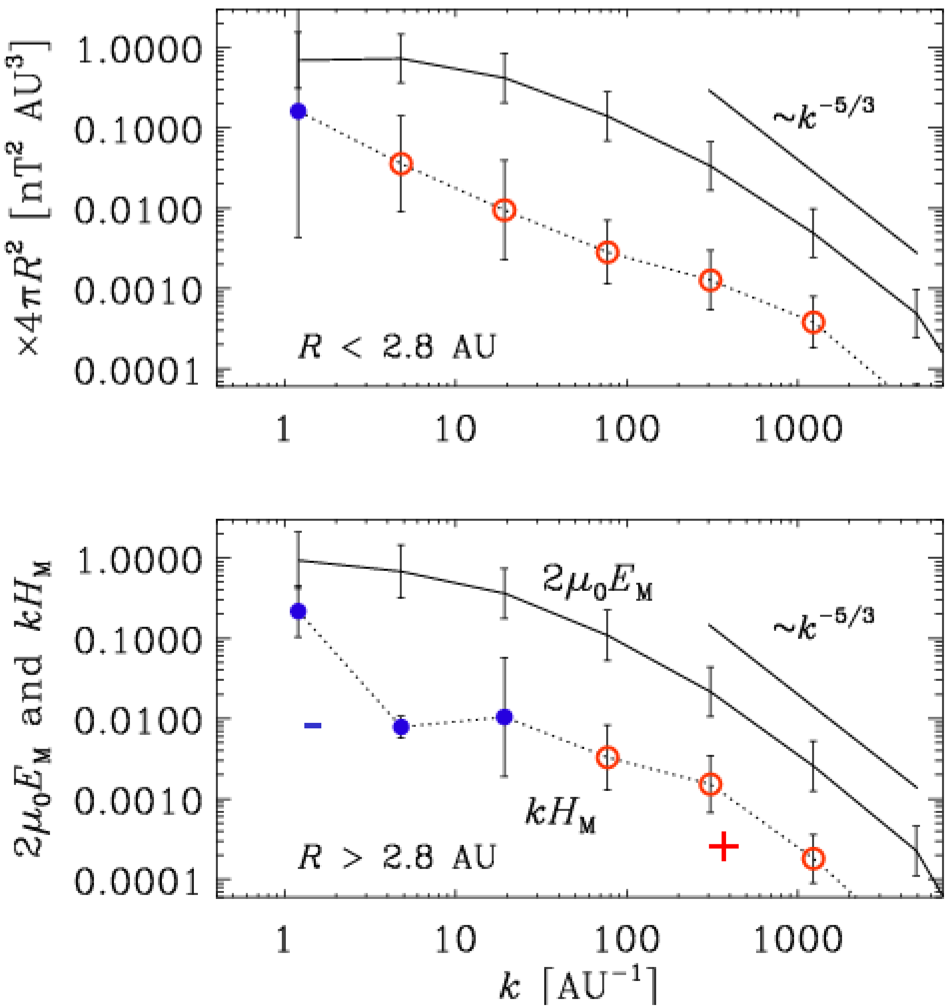


Taylor hypothesis: $$R = R_0 - u_R t$$



Change of sign: (i) in latitude, (ii) in scale

Bi-helical fields from Ulysses



Taylor hypothesis

Broad *k* bins

Southern latitude with opposite sign

Small/large distances

Positive *H* at large *k*

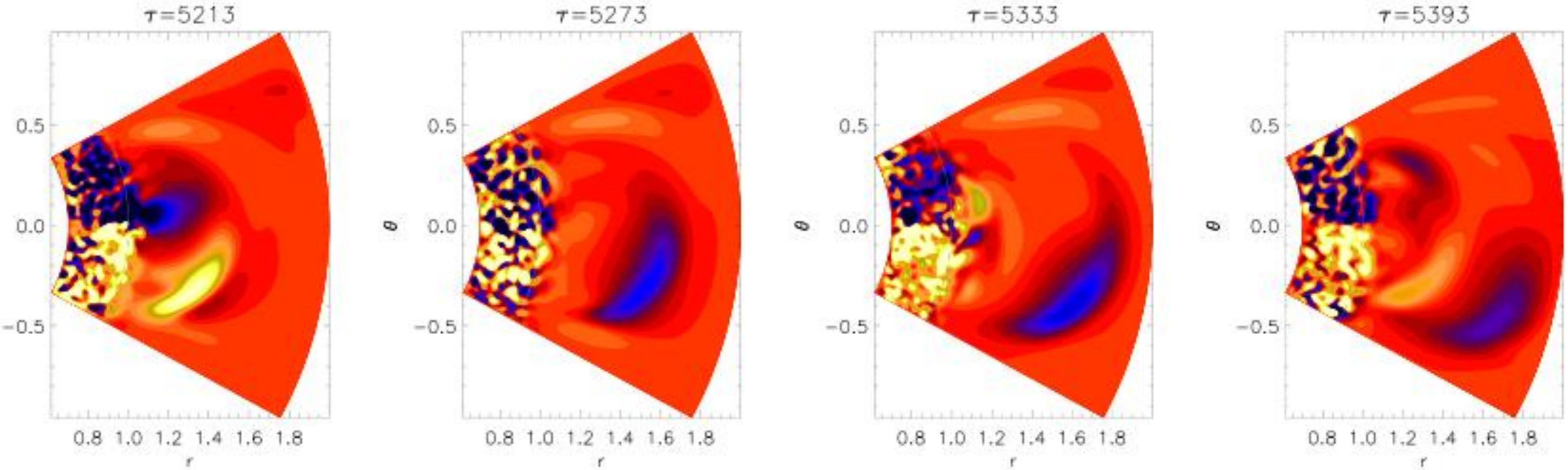
Break point with distance to larger *k*

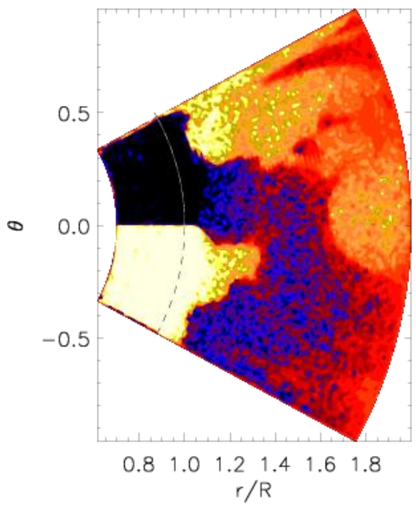
1500 AU^{-1} ” 0.01 Mm^{-1}

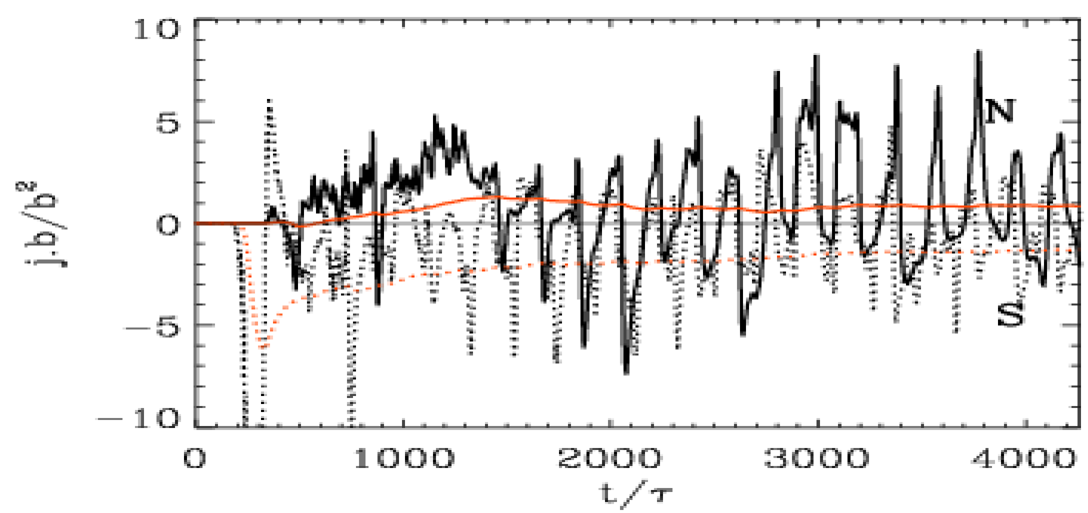
1 AU^{-1} ” 1 μHz

Shell dynamos with ~CMEs

Warnecke, Brandenburg, Mitra (2011, A&A, 534, A11)







Strong fluctuations, but positive in north

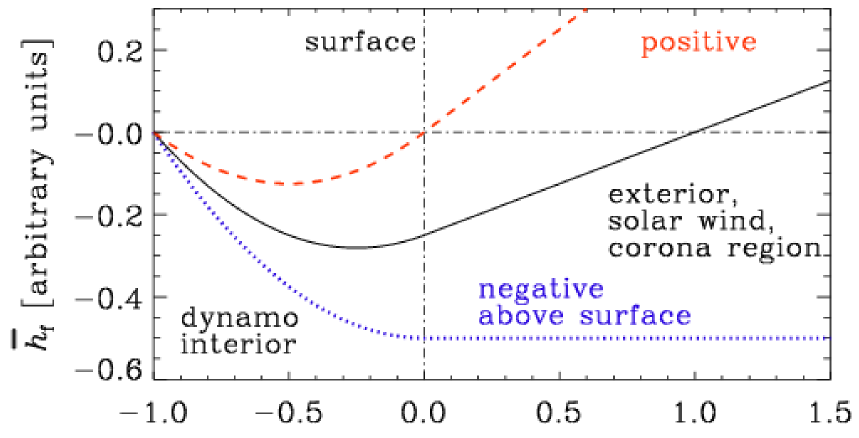
To carry negative flux: need positive gradient

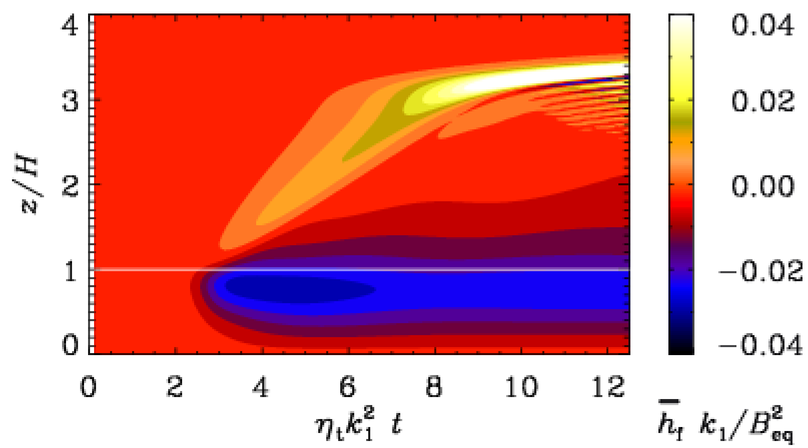
$$\overline{\mathbf{F}}_{\rm f} = -\kappa_h \nabla \overline{h}_{\rm f}$$

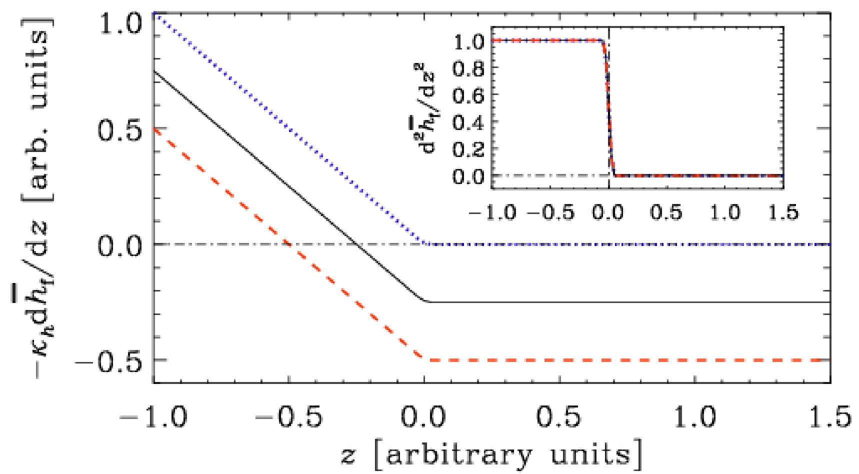
Brandenburg, Candelaresi, Chatterjee
(2009, MNRAS 398, 1414)

$$\frac{{\rm d}\overline{h}_{\rm m}}{{\rm d}t} = +2\alpha\overline{\mathbf{B}}^2 - 2\eta_t\overline{\mathbf{J}}\cdot\overline{\mathbf{B}} - \nabla\cdot\overline{\mathbf{F}}_{\rm m}$$

$$\frac{{\rm d}\overline{h}_{\rm f}}{{\rm d}t} = -2\alpha\overline{\mathbf{B}}^2 + 2\eta_t\overline{\mathbf{J}}\cdot\overline{\mathbf{B}} - \nabla\cdot\overline{\mathbf{F}}_{\rm f}$$

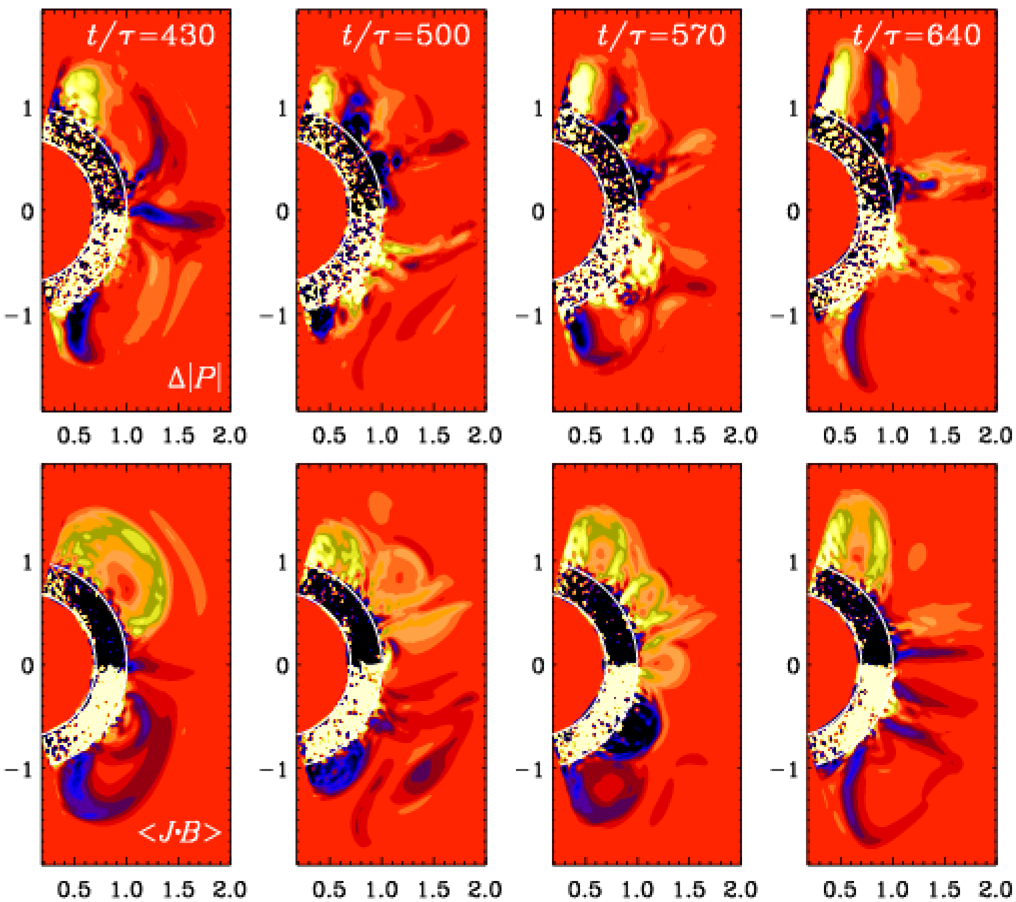


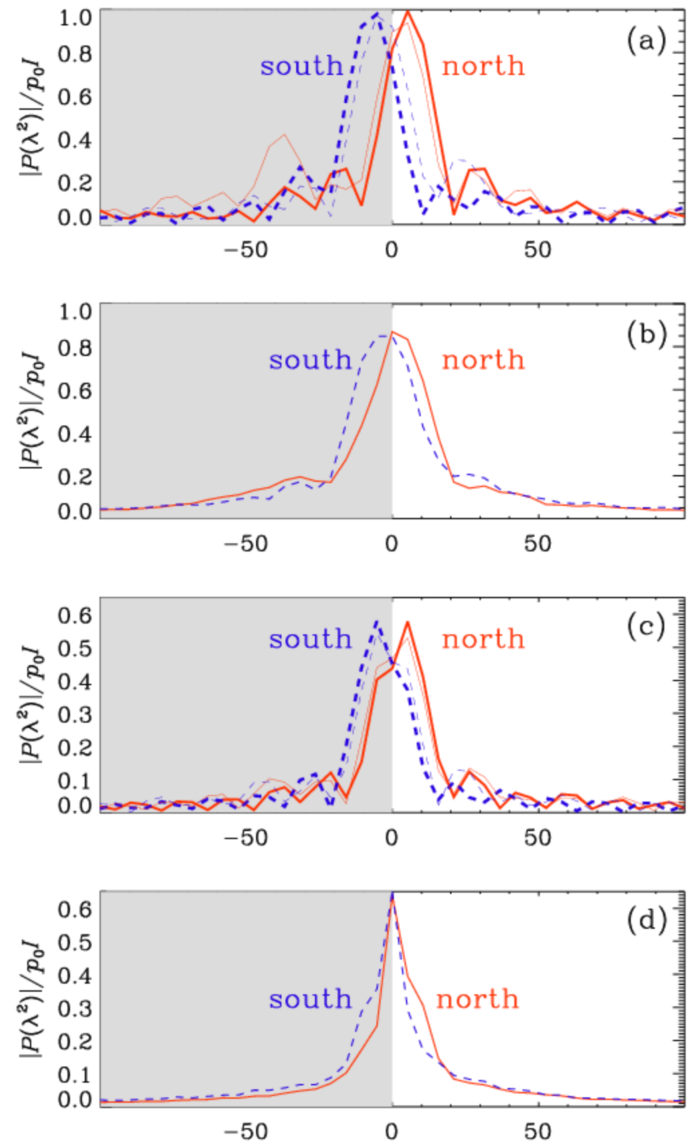




Sign reversal makes sense!

To carry negative flux: need positive gradient





Brandenburg, Ashurova, & Jabbari
(2017, ApJL 845, L15)

Conclusions I

- New technique: spectrum (scale dependence)
  - from correlation tensor: no ***A***, no gauge dep
- Isotropy ok, but inaccurate for k > 3/Mm (l < 2Mm)
  - to check with Daniel K Inouye Solar Telescope (DKIST)!
- North-south sign dependence obeyed
- For NOAA 11515: wrong sign explained (?)
  - Signature of tilt from α-effect
- Global analysis: Double Fourier transform

Conclusions II

- Need to generalize to spherical harmonics
- Is lack of large-scale helicity a boundary effect?
  - Recall: helicity reversal in solar wind (Brandenburg+11)
- Polar field underestimated?
  - Pipin & Pevtsov (2014)