The Rossby-number Dependence of Large-scale Dynamo in Solar-like Strongly-stratified Convection

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Youhei MASADA
(Aichi University of Education, JAPAN)

Collaborator:
Takayoshi SANO (ILE, Osaka University)

Outline

- A Key Parameter : <u>Rossby number</u> -

1. Current Status of Observation of Magnetic Activities of Low-Mass Stars

2. The Ro-dependence of Convective MHD Dynamo in a Simplified Semi-global Model

3. Mean-field Model Coupled with the DNS:
   Why the Ro is the key for the Success and Failure of Dynamo ?

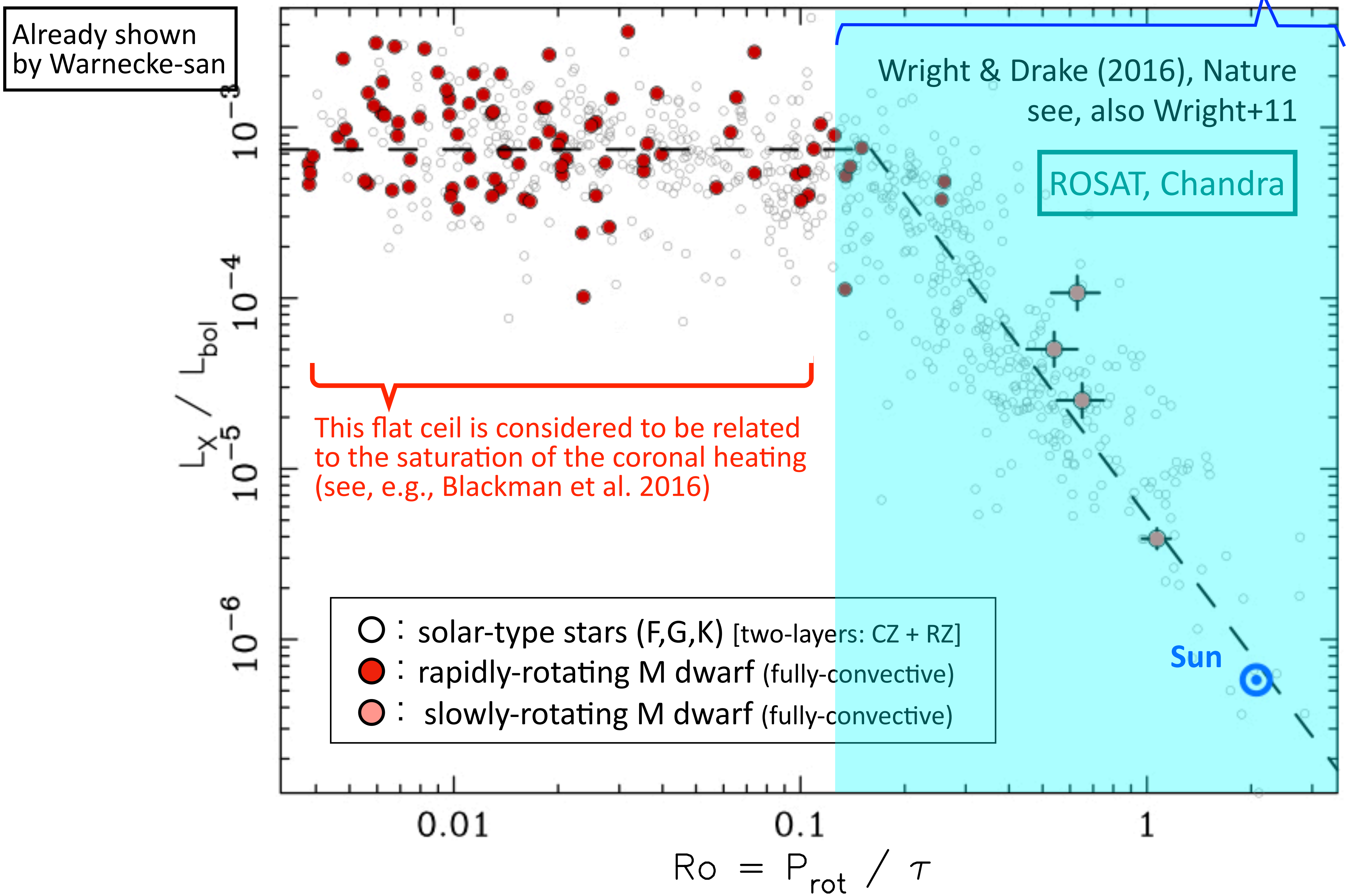
1. Current Status of Observation of Magnetic Activities of Low-Mass Stars

~ Why does we focus on the Rossby number ? ~

Stellar X-ray luminosity v.s., Rossby number

Since L_x reflects T_{corona}, which is determined by magnetic activity, we believe that it should be an indicator of stellar magnetic activity

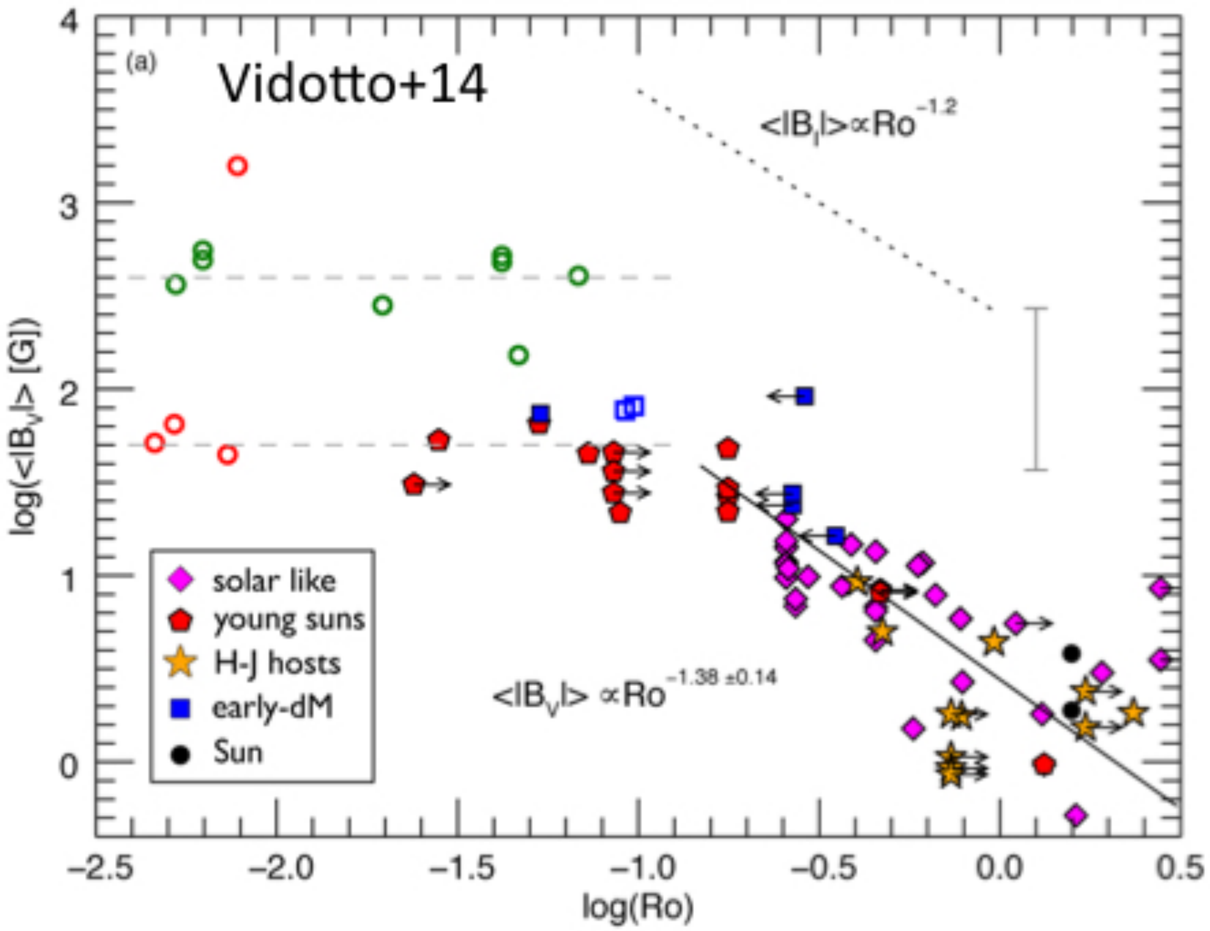
Focus on this regime
(The magnetic activity is directly reflected in the X-ray luminosity)



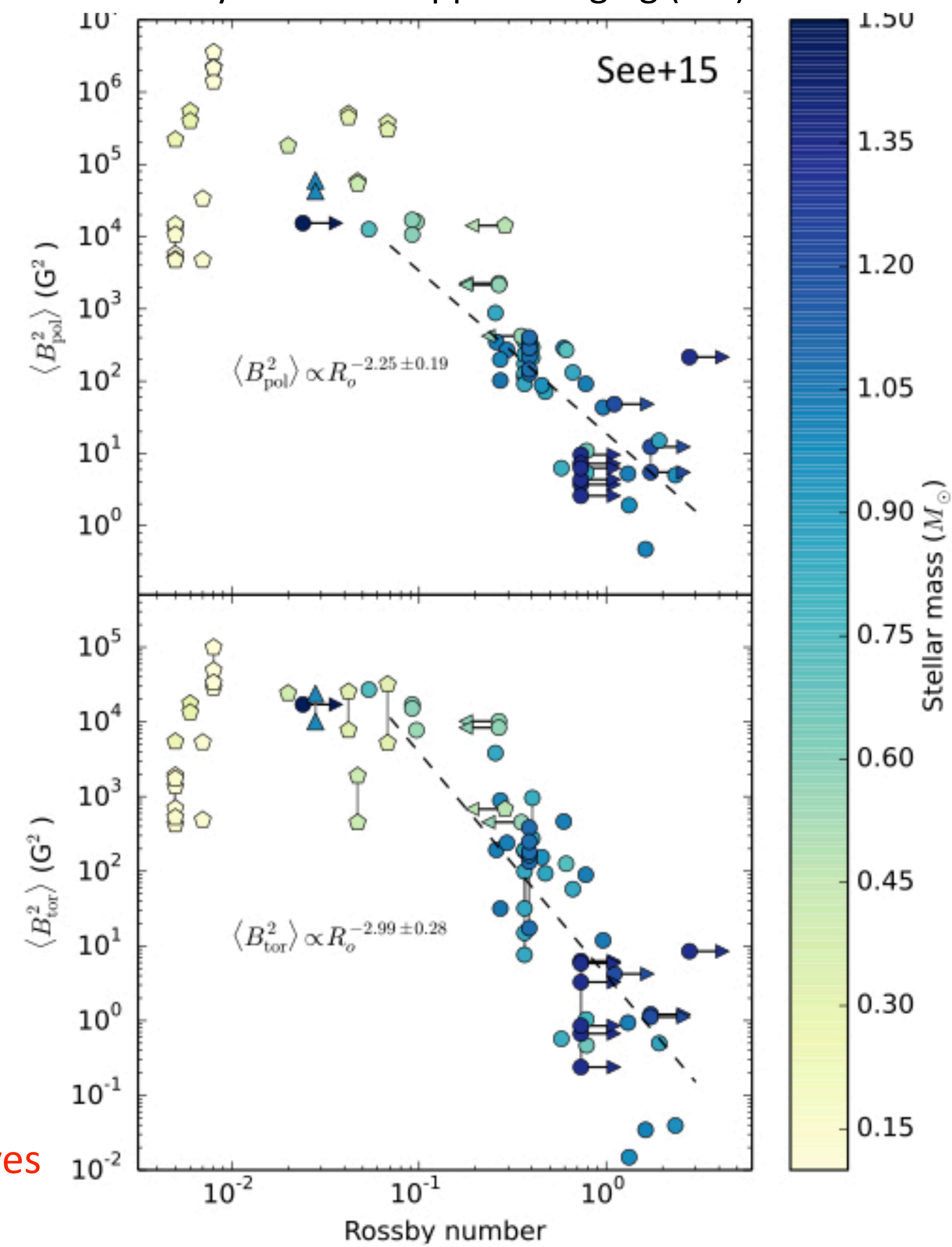
- Stellar X-ray luminosity is strongly dependent on the Ro with focusing on the regime Ro $\gtrsim$ 0.1
- The low mass star (F, G, K, and M-type here) has a same or similar Lx - Ro relationship
- The stellar magnetic activity is a function of Ro, not solely the stellar mass, luminosity, & structure

Magnetic field of Low Mass Stars v.s., Rossby number

In addition to the Lx, the information of the B-field is also obtained:



by Zeeman Doppler Imaging (ZDI)



- ZDI Observation of B-field of Stars：
  - Strength of mean-field — $B_{ave} \propto Ro^{-1.4}$ (Vidotto+14)
  - Energy of B_p-component — $B_p^2 \propto Ro^{-2.3}$ (See+15)
  - Energy of B_ϕ-component — $B_\phi^2 \propto Ro^{-3.0}$ (See+15)

・All the B-field components become weaker with the increase of the Ro

・Ro-dependence has been long known (e.g., Noyes +84; Brandenburg +98) but is not fully-resolved

2. The Ro-dependence of Convective MHD Dynamo in a Simplified Semi-global Simulation

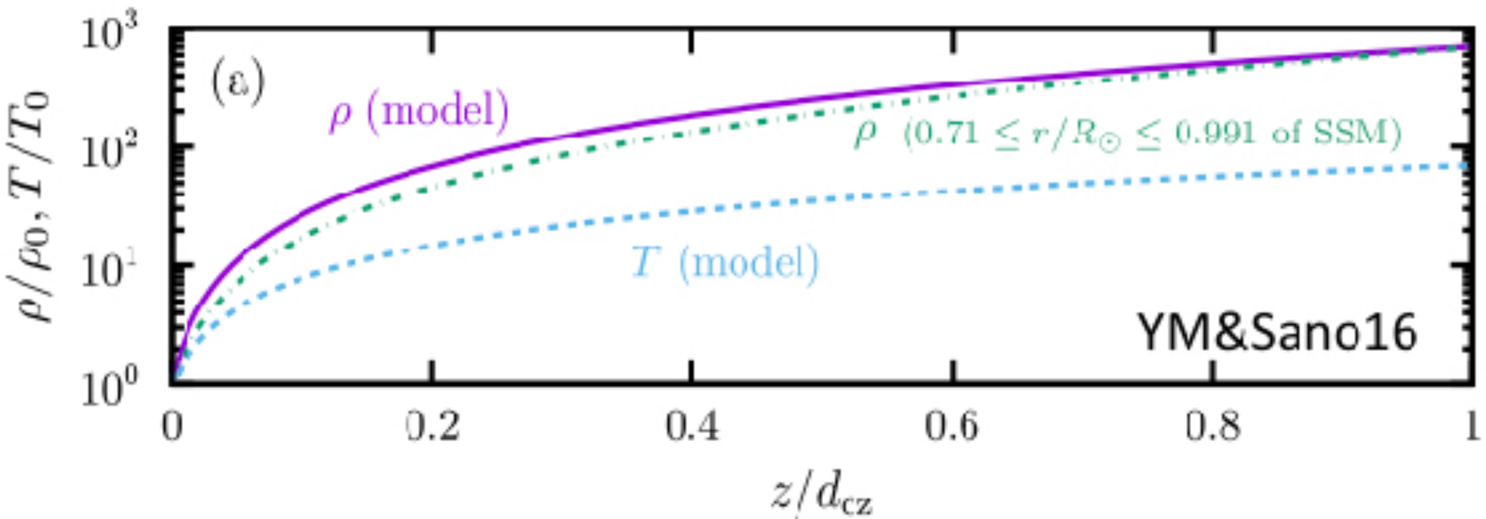
Our tool for capturing the essence of the physics
of the Ro-dependence of the convective dynamo

Semi-global Dynamo Model (see YM & Sano 2016, similar to Bekki-san's model)

We solve global structure in the depth direction but assume periodicity (local) in the horizontal direction

①Numerical Setting : strongly-stratified atmosphere modeling the solar CZ

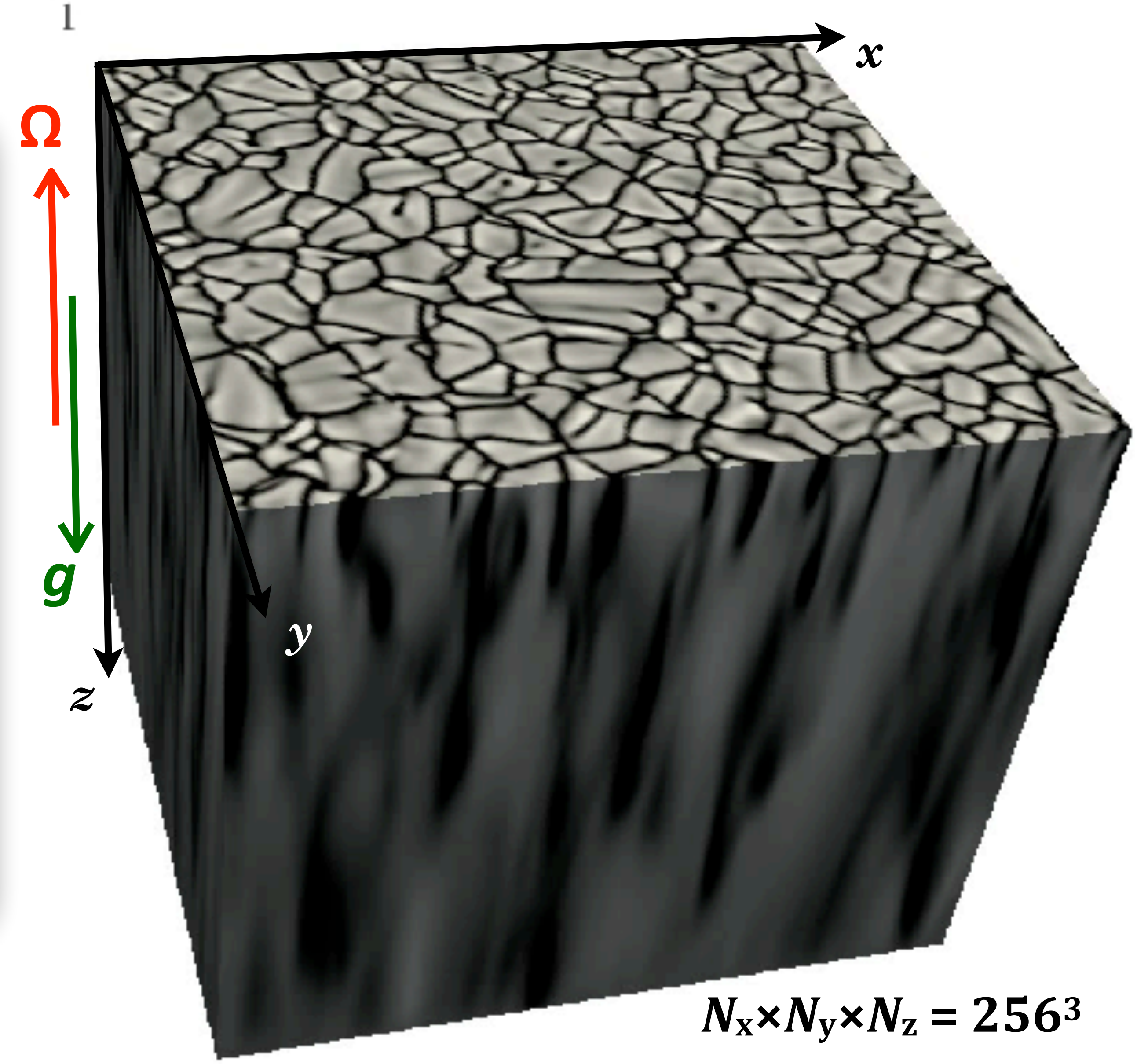
② Control Parameter : angular velocity （Ω）



- density contrast = 700
- covering over the layer of $0.71R_{sun} < r < 0.99R_{sun}$
- no mean-flow and thus no Ω-effect

By changing Ω, Ro-dependence is studied

- **Basic eq：Compressible MHD【rotating coordinate】**
- **1-layer polytrope 【convection zone only】**
  aspect ratio*：$L_x/L_z = L_y/L_z = 4$, *Ω* is antiparallel to *g
- **non-D parameter : *Pr* = 12, *Pm* = 2, *Ra* = 3.6×10⁷**
- **polytropic index：1.49 (super-adiabaticity $\delta=10^{-3}$)**
- **Boundary Condition （horizontally periodic） ：**
  - B-field ・・CZ surface：Open Boundary
    CZ bottom：Perfect Conductor
  - u-field・・ stress-free at CZ surface and bottom
  - constant dε/dz at the bottom → driving convection

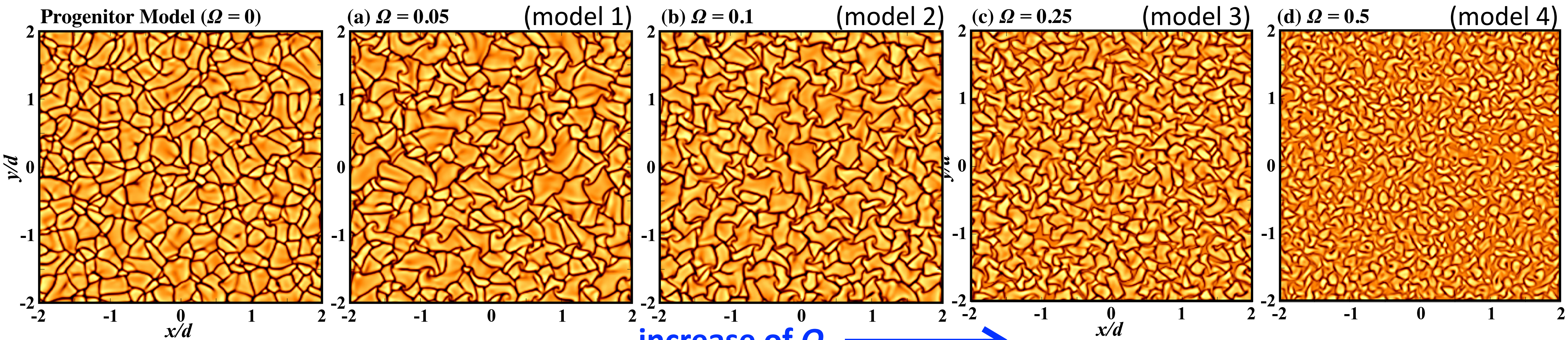


dynamo activity in the strongly-stratified convection

Response of Turbulent Convection to the Change of Ω

YM&Sano18 in prep.

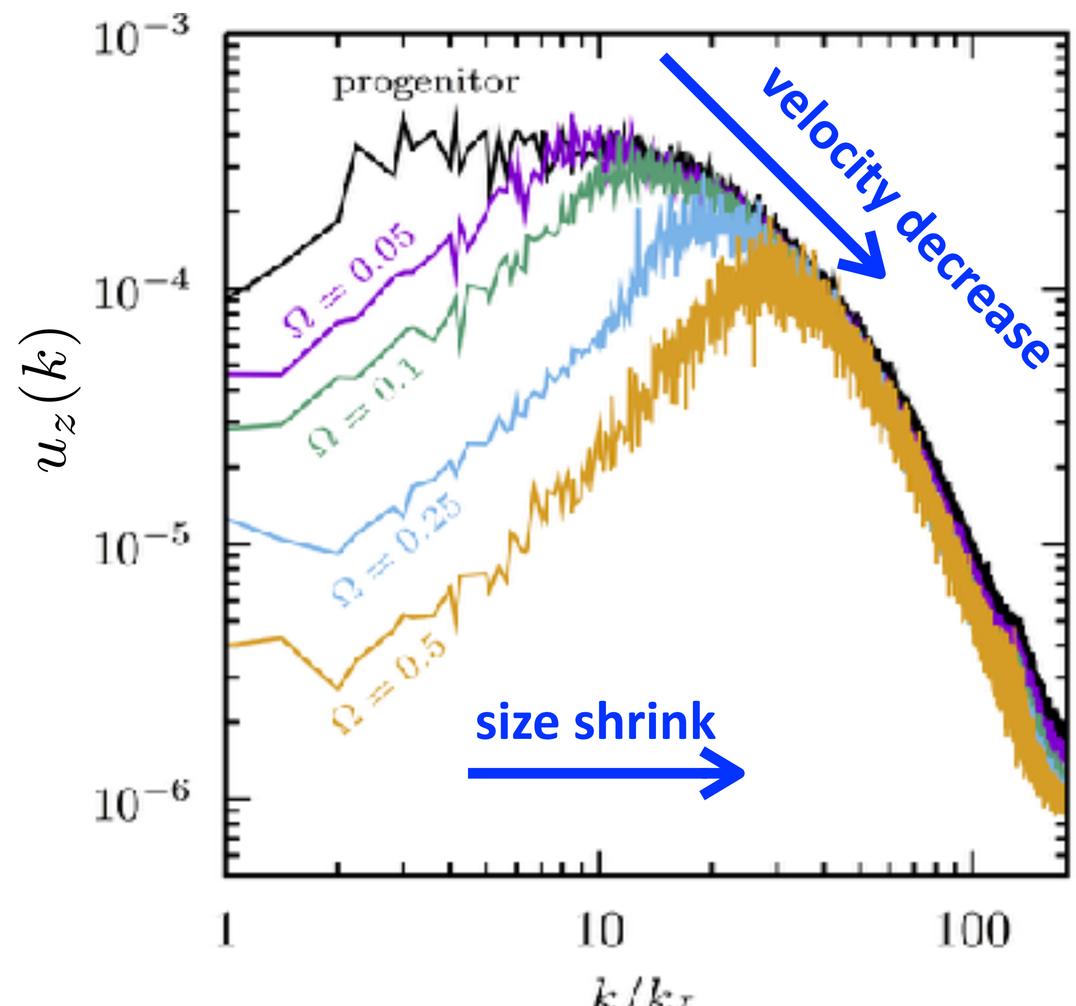
Surface distribution of the convective velocity for the models with the different Ω :



From the distribution and spectrum of u_z

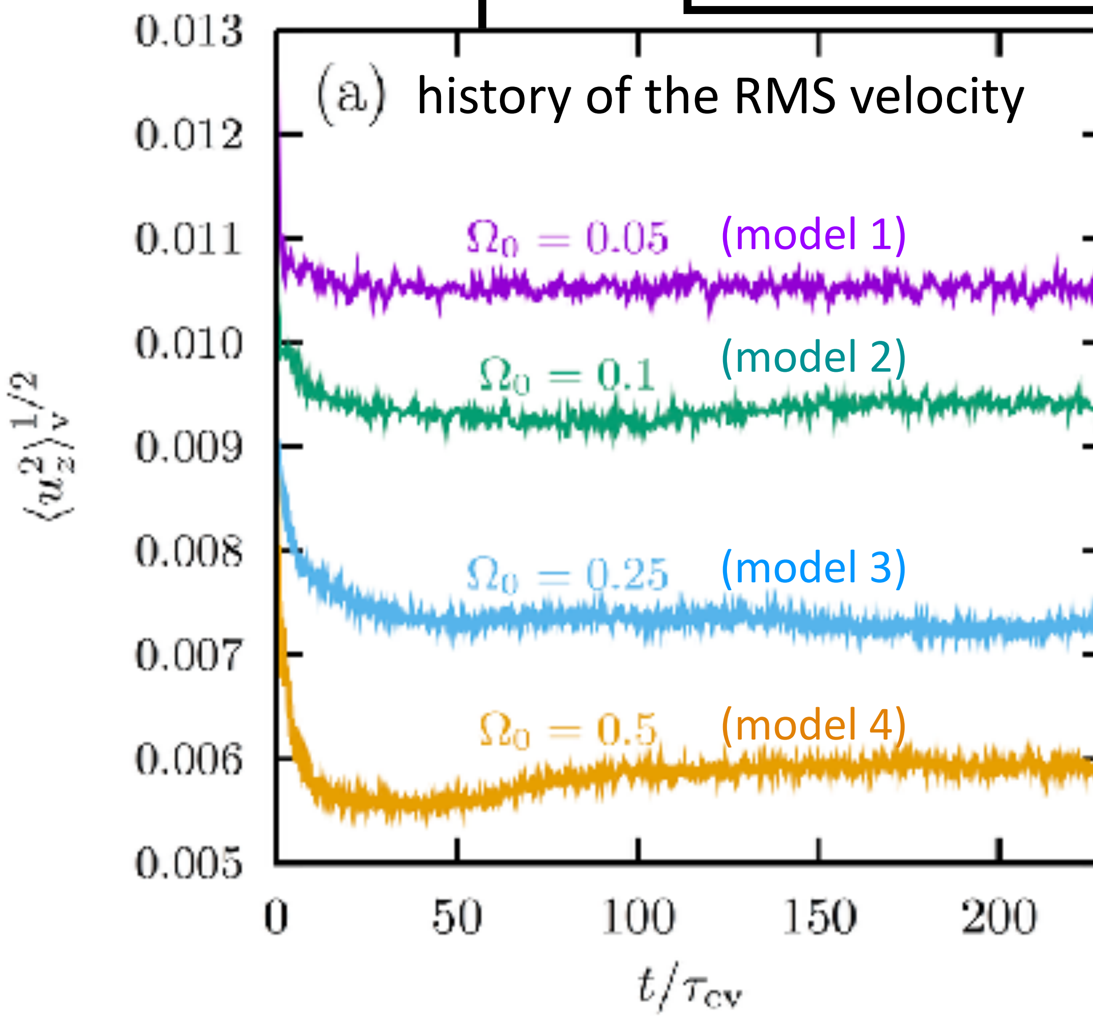
① Convective-cell shrinks with the increase of Ω

② Convective velocity decreases with the increase of Ω



$$\mathrm{Ro} \equiv \frac{u_{\mathrm{rms}}}{2\Omega d_{\mathrm{cz}}}$$

evaluated as



→ Ro = 0.09

→ Ro = 0.04

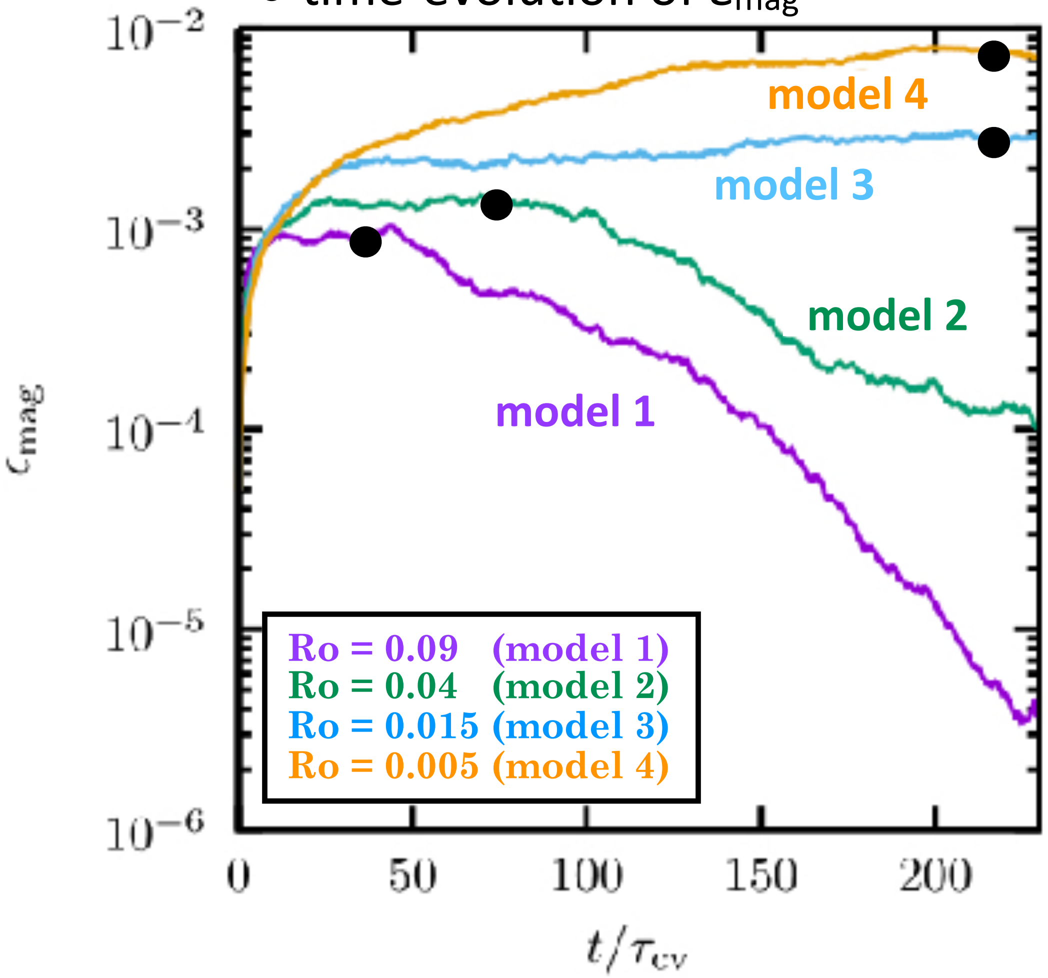
→ Ro = 0.015

→ Ro = 0.005

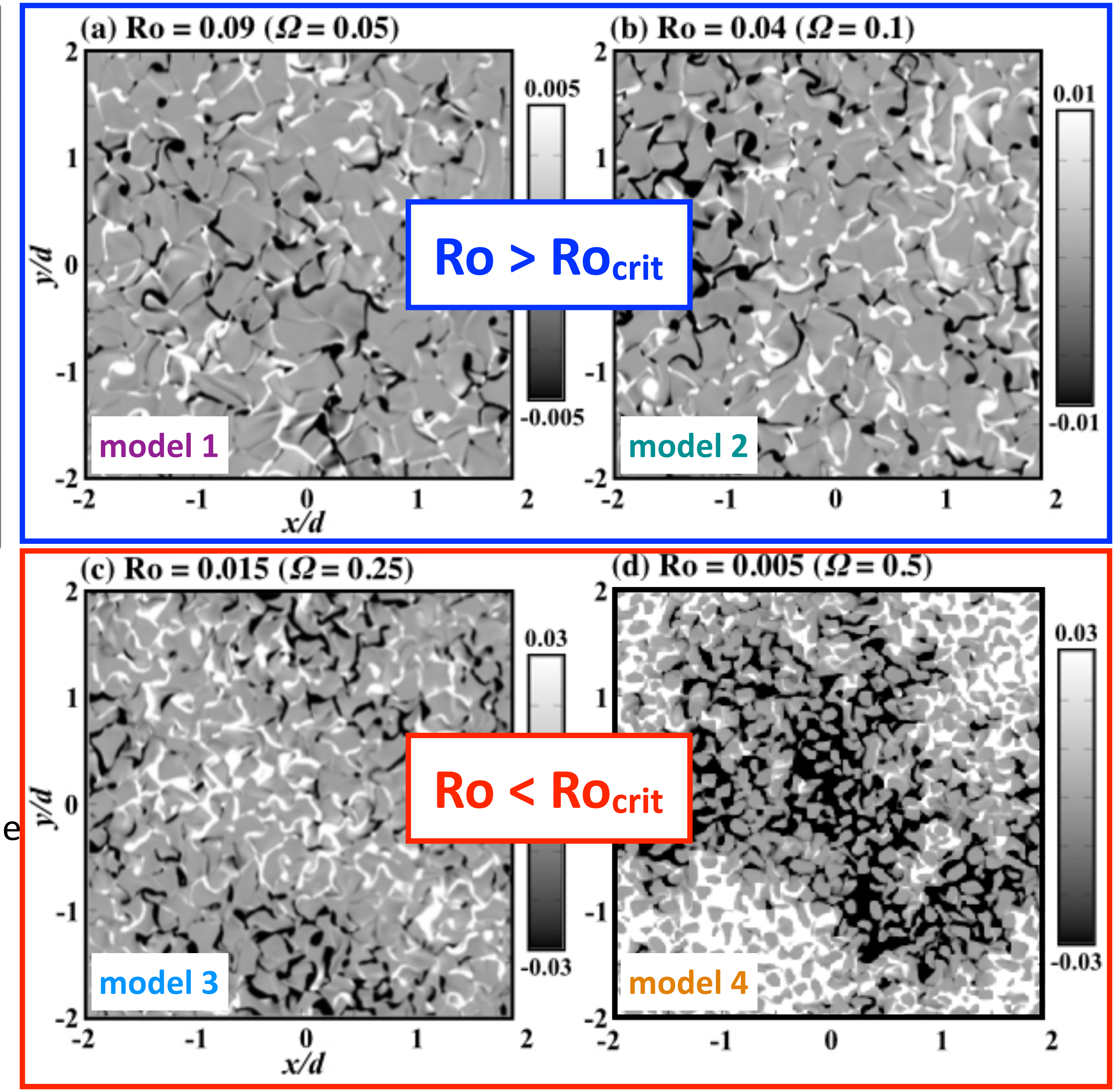
Ro-dependence of Turbulent Convective Dynamo①

Depending on the Rossby number, the dynamo properties also change:

- time-evolution of ϵ_{mag}



- distribution of the vertical field (Bz) @ CZ surface



• High Ro model:

- turbulent B-field becomes dominant
- weak large-scale B-field grows initially but is not sustained and decays with time

• Low Ro model:

- strong large-scale B-field grows and is sustained for sufficiently long-time
- turbulent and large-scale fields co-exist

There exists a critical Ro for the successful large-scale dynamo → Ro_{crit} ~ 0.015 - 0.04

YM&Sano16
YM&Sano18 in prep.

Ro-dependence of Turbulent Convective Dynamo②

Presence of the large-scale component can be confirmed from TD diagrams:

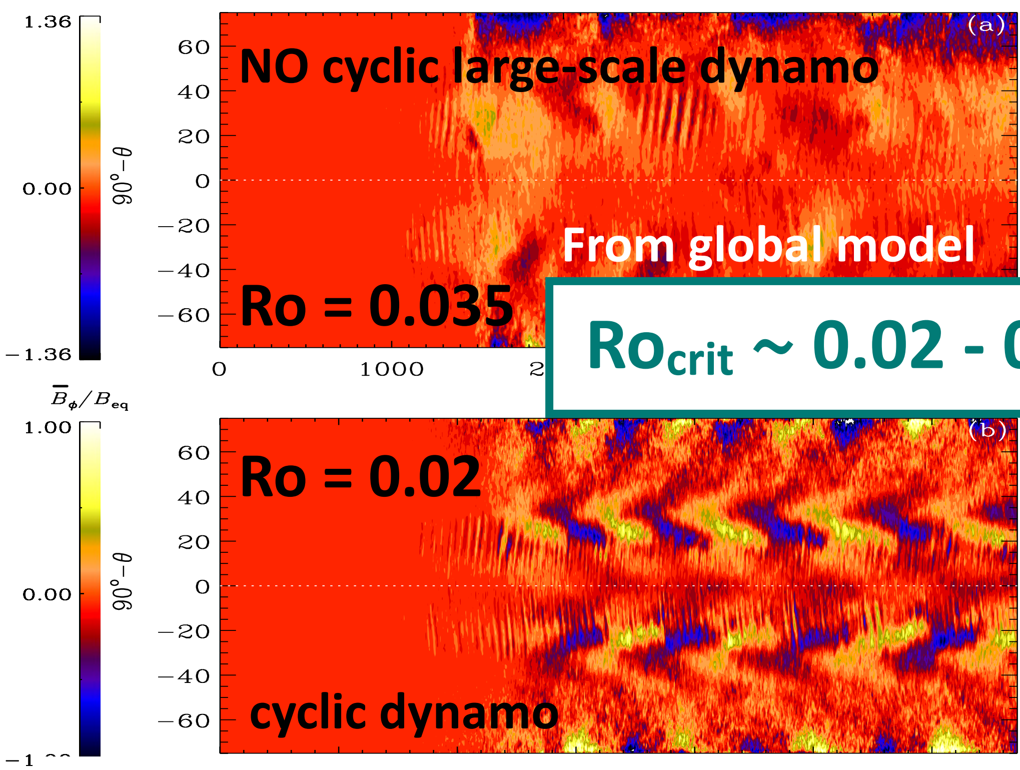
YM&Sano18 in prep.

- **Time-depth diagram for $\langle B_h \rangle$**

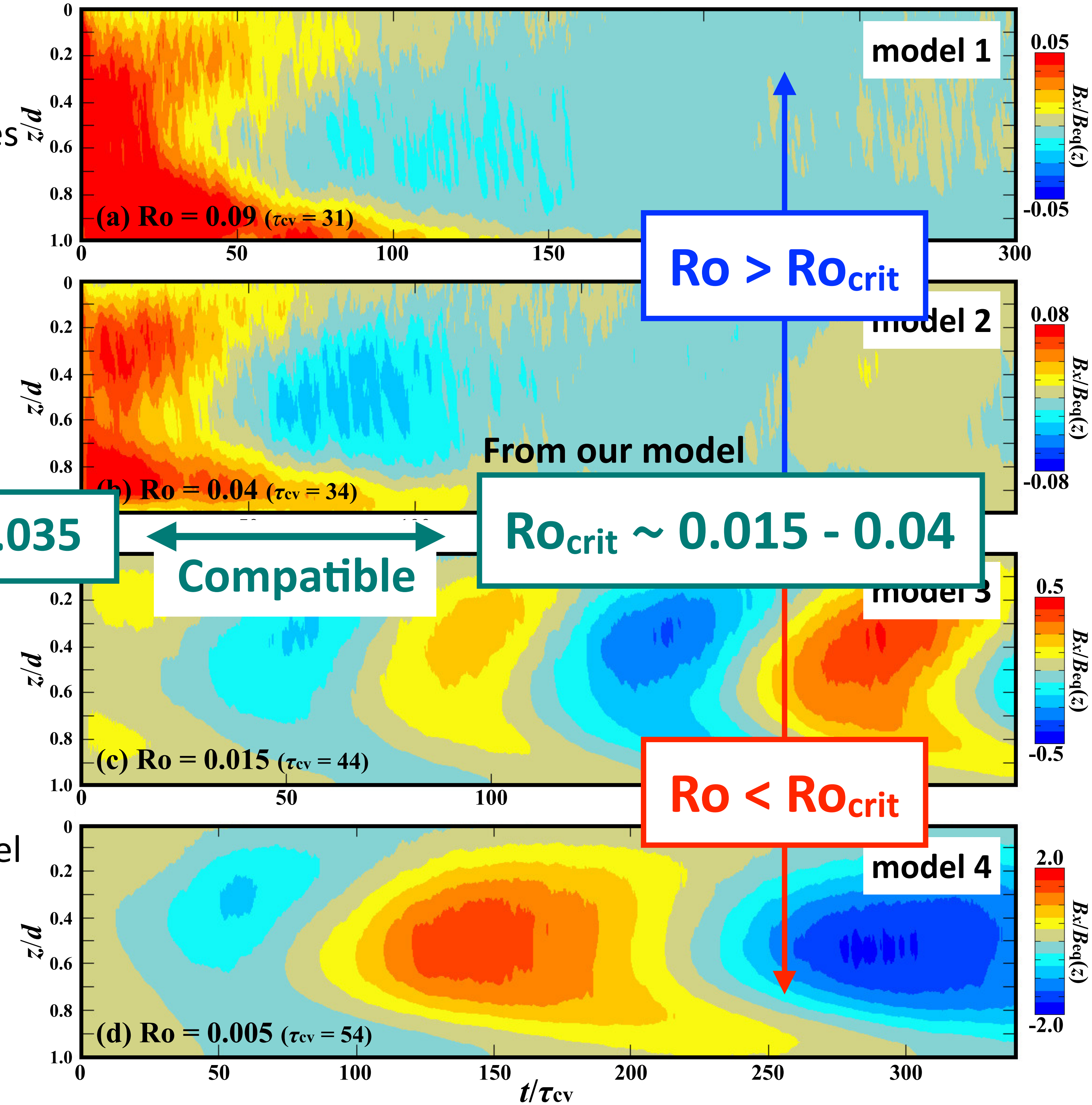
・High-Ro model :
$\langle B_h \rangle$ starts to grow, but decays as t passes

・Low-Ro model :
$\langle B_h \rangle$ grows and is then maintained

Kapyla+12 (see also, Warnecke+14)



Ro-dependence of global spherical model



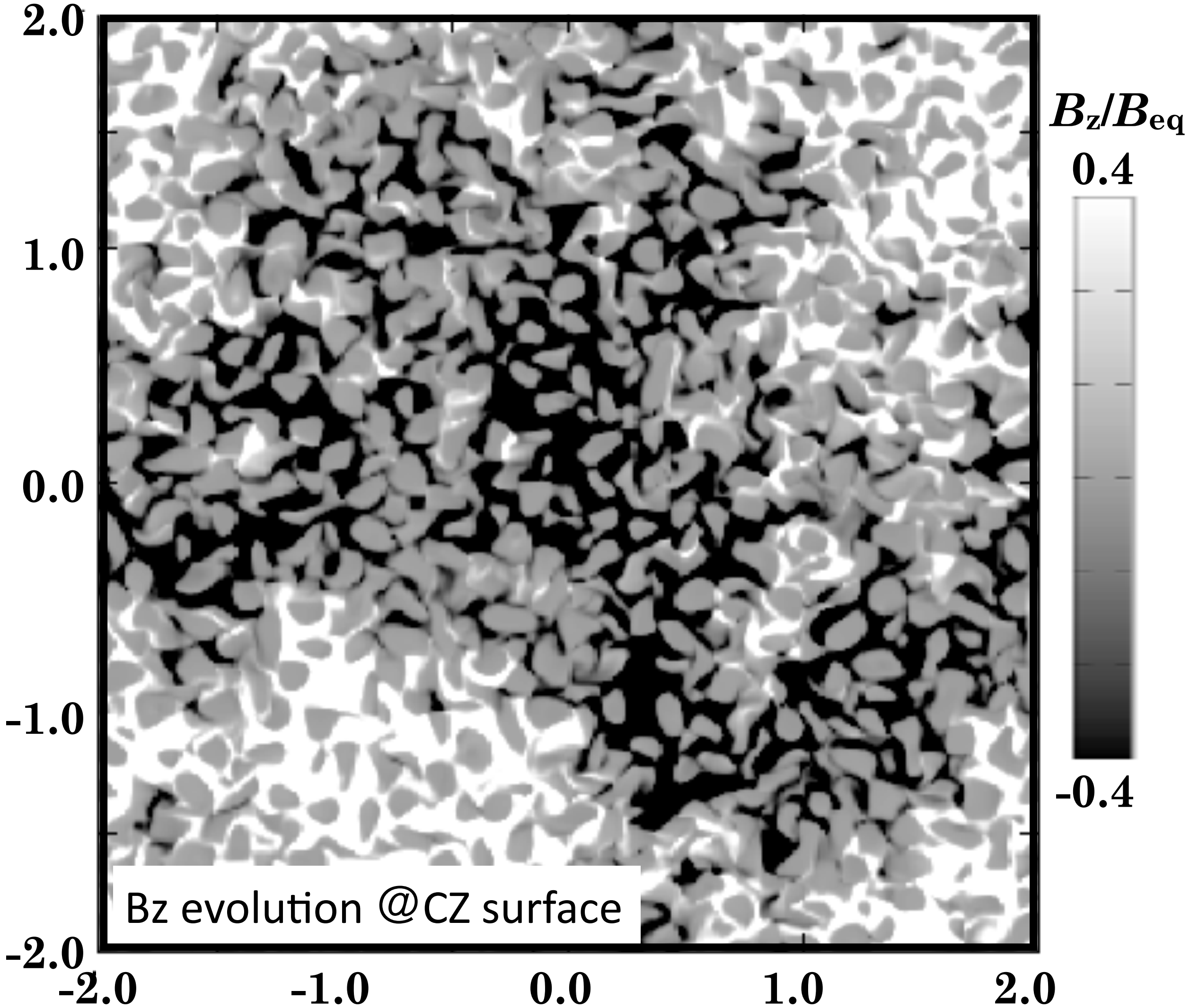
Ro_{crit} ~ 0.02 - 0.035 ⟷ **Compatible** ⟷ Ro_{crit} ~ 0.015 - 0.04

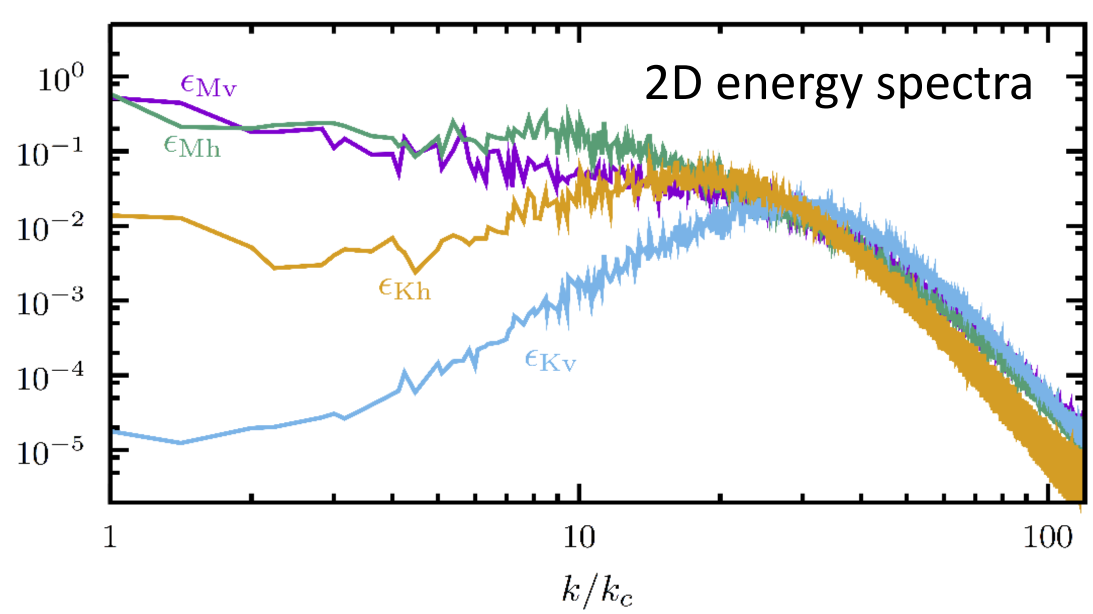
The mechanism which determines the success or failure of dynamo seems to be common in both global and our semi-global models because of the similar Ro_{crit}.

Ro-dependence of Turbulent Convective Dynamo③

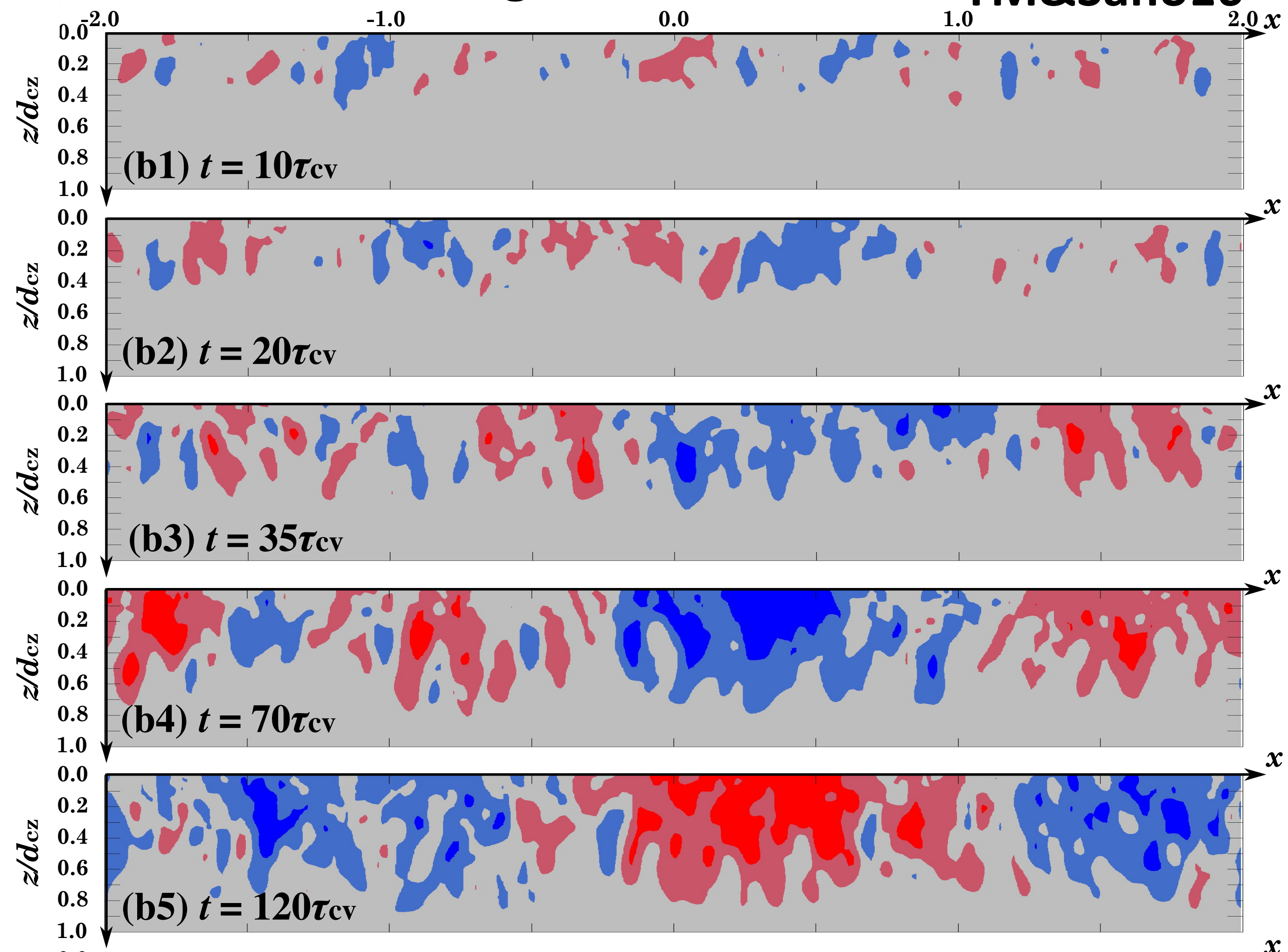
One interesting outcome : spontaneous formation of surface magnetic structure

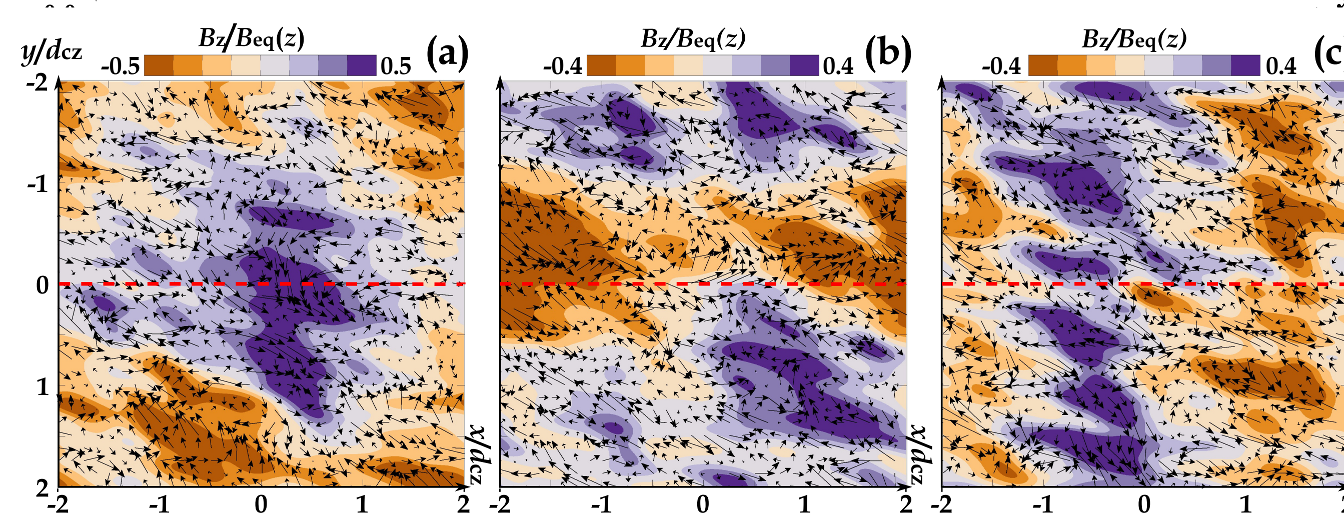
YM&Sano16





magnetic energy (B_z & B_h) peaks at the box size





- surface magnetic structure is developed and sustained
- band-like structure (see also, Mitra+14; Jabbari+17)
- common in the dynamo active phase of all the models

3. Mean-field Model Coupled with the DNS:

- How Does the "Ro" impact on the Success and Failure of the large-scale Dynamo ? -

How “Ro” impacts on the success and failure of dynamo ?

- **Summary of our MHD simulations**：
  - High Ro model : failed dynamo
    Low Ro model : successful large-scale dynamo (Ro_{crit} = 0.015 ~ 0.04)

consistent with global dynamo model

What is the “physical difference” between High Ro and Low Ro models ??

- **Mean-Field Dynamo Equation** (skip the introduction and derivation)：
  - Strong theoretical framework studying the large-scale dynamo in the turbulent flow:

(c.f., Krause & Radler 1980)

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B} - \eta_0 \boldsymbol{J}),$$

mean-field decomposition：
$\boldsymbol{u} = \langle \boldsymbol{u} \rangle + \boldsymbol{u}', \boldsymbol{B} = \langle \boldsymbol{B} \rangle + \boldsymbol{B}'$

smoothing approximation

Mean-Field (MF) Dynamo equation

no mean-flow

$$\frac{\partial \langle \boldsymbol{B} \rangle}{\partial t} = \nabla \times [\langle \boldsymbol{u} \rangle \times \langle \boldsymbol{B} \rangle + \mathcal{E}_t - \eta_0 \nabla \times \langle \boldsymbol{B} \rangle]$$

$$\mathcal{E}_t = \alpha \langle \boldsymbol{B} \rangle - \eta_t \nabla \times \langle \boldsymbol{B} \rangle$$ (simplest form)

turbulent α-effect：induction of mean B-field
turbulent diffusion：diffusion of mean B-field

Our strategy is to determine, from the simulation results,
(see YM & Sano 14b for details)

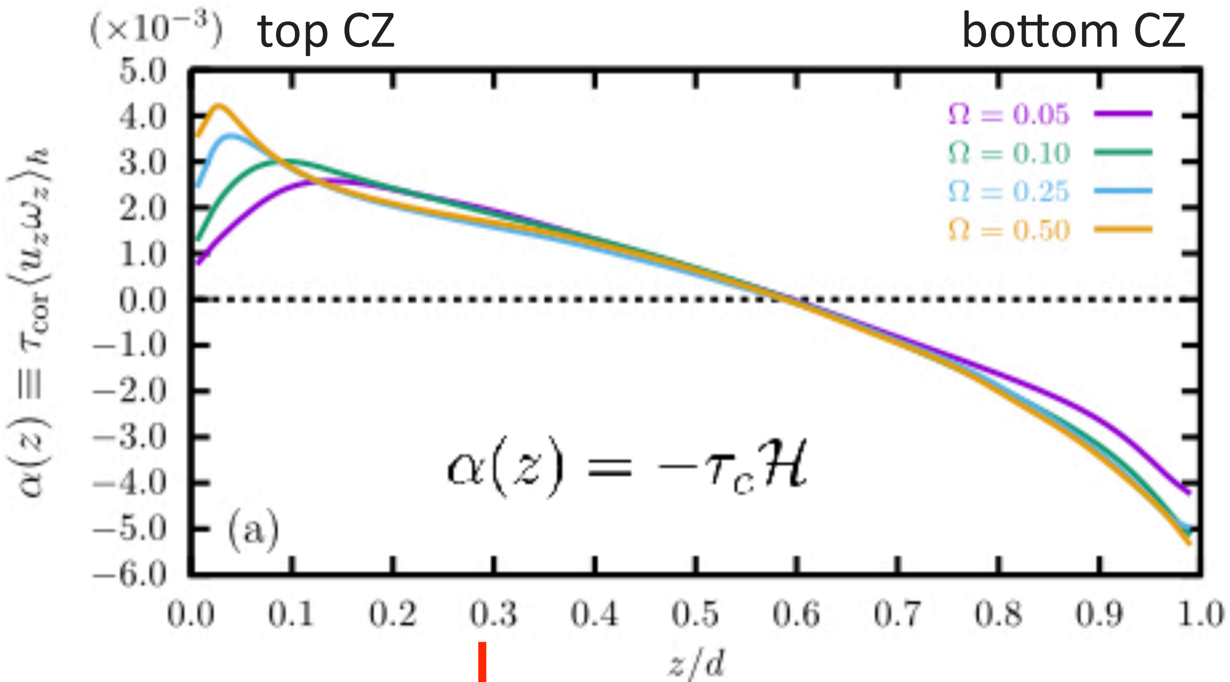
$$\alpha(z) = -\tau_c \mathcal{H} \quad , \quad \eta_t(z) = \tau_c \langle\langle u_z^2 \rangle\rangle \ ,$$

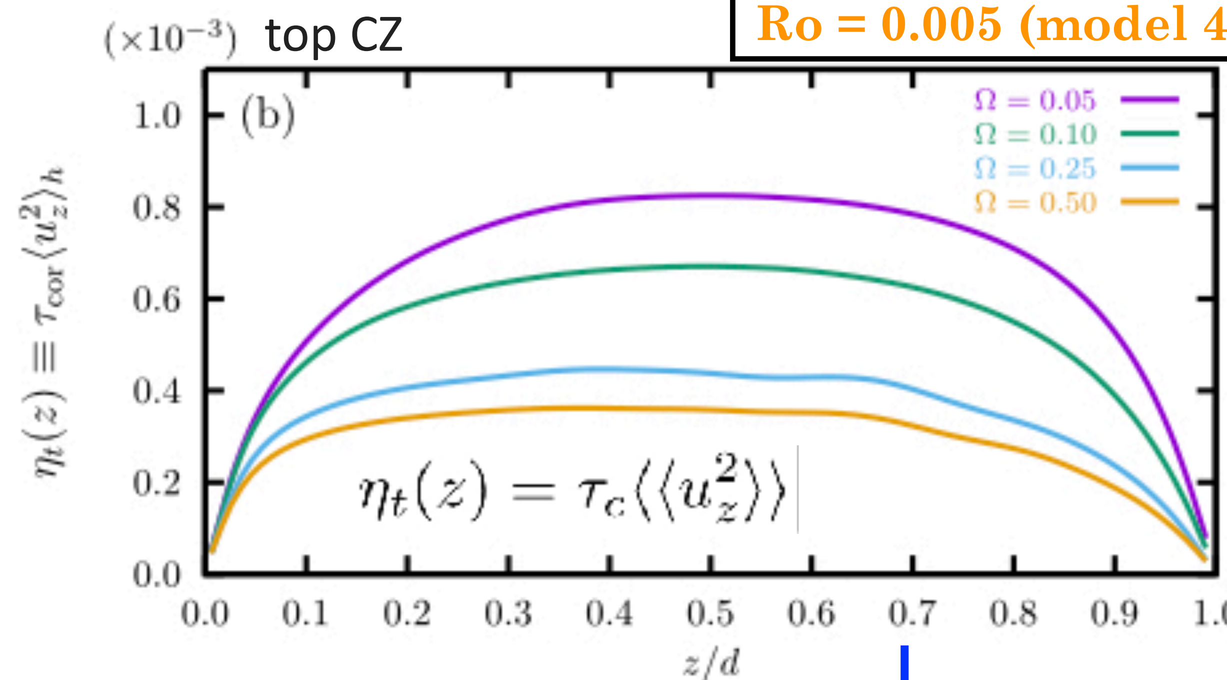
$(\tau_c = 2\pi H_p / u_z)$

Profiles of Dynamo Coefficients and Strategy for the Analysis

- Profiles of turbulent α-effect are similar in all the models except the top CZ
- turbulent diffusion effect becomes smaller with the increase of the Ω

Ro = 0.09 (model 1)
Ro = 0.04 (model 2)
Ro = 0.015 (model 3)
Ro = 0.005 (model 4)





With these coefficients, two-types of analyses are possible

substitute

Mean-Field (MF) Dynamo equation:

$$\frac{\partial \langle \boldsymbol{B} \rangle}{\partial t} = \nabla \times [\langle \boldsymbol{u} \rangle \times \langle \boldsymbol{B} \rangle + \mathcal{E}_t - \eta_0 \nabla \times \langle \boldsymbol{B} \rangle]$$

$$\mathcal{E}_t = \alpha \langle \boldsymbol{B} \rangle - \eta_t \nabla \times \langle \boldsymbol{B} \rangle$$

substitute

① Linear Analysis

- Dynamo
  = Instability of MFD eq.
- Compare the growth rate

② Non-Linear Analysis

- with physics-based model of non-linear quenching effect [α = α(B)]
- Compare the non-linear behavior

Local Linear Analysis and Dynamo Growth Rate

- When plane wave perturbation $\propto \exp[i(kr - \omega t)]$ is added, the dispersion equation is obtained :

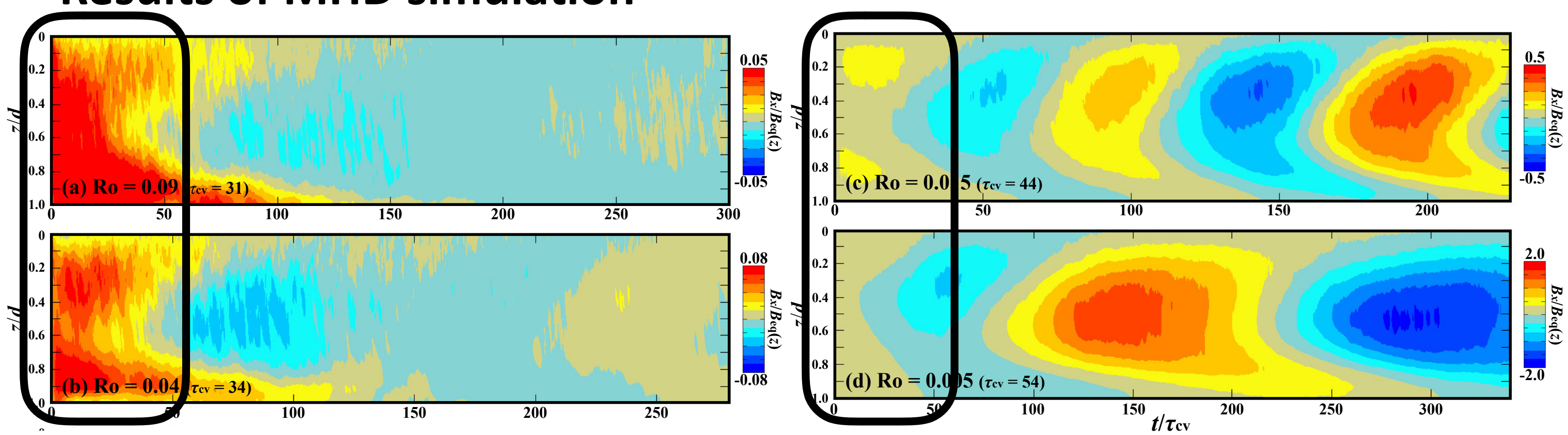
$$a_2\omega^2 + a_1\omega + a_0 = 0$$

$$\begin{cases} a_2 = 1 \\ a_1 = 2i\eta k^2 \\ a_0 = \alpha^2 k^2 - \eta^2 k^4 \end{cases}$$

(α2-type dynamo)

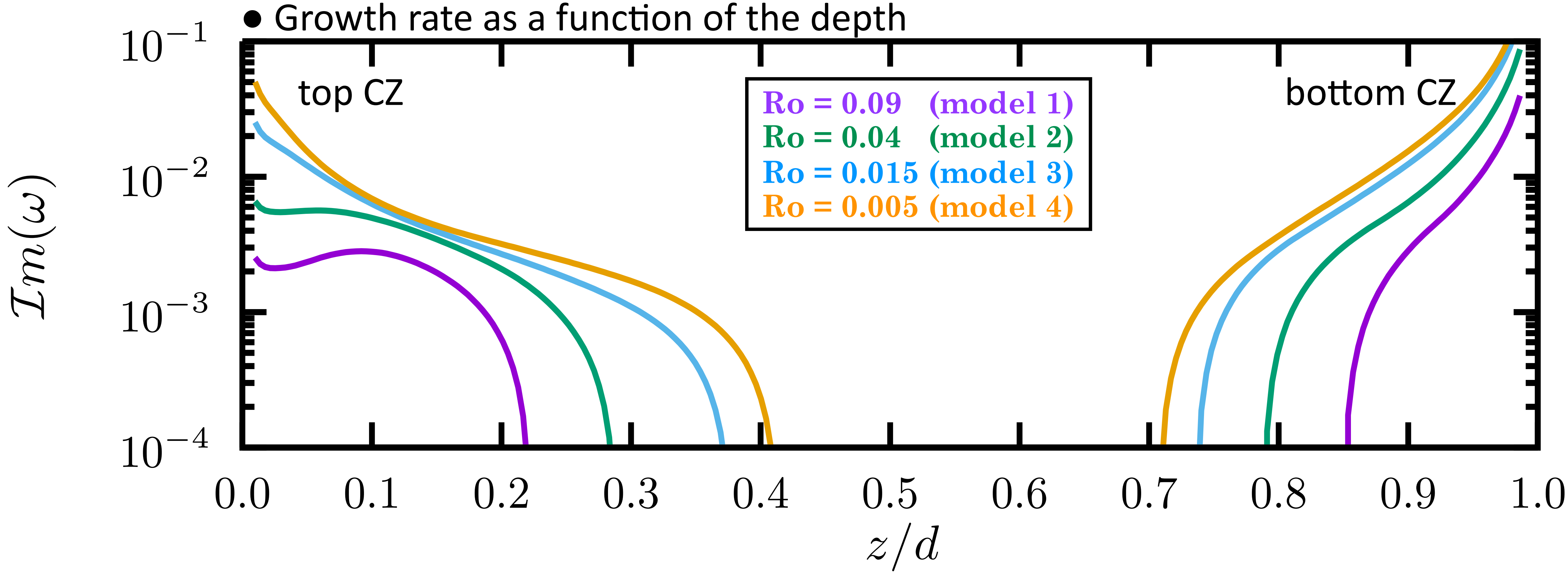
This local dispersion relation is applied to the global structure of α and η_t

Results of MHD simulation



The results of the linear analysis is consistent with the early evolutionary stage of the DNS

- Growth rate as a function of the depth



- All the models are linearly unstable to the dynamo both in the top and bottom CZ
- The growth rate is larger in the model with the smaller Ro (higher rotation)
- The dynamo unstable region is broader in the model with the smaller Ro (higher rotation)

MHD simulation v.s. Non-Linear Evolution of MFD model

MF Dynamo equation：

$$\frac{\partial \langle \boldsymbol{B} \rangle}{\partial t} = \nabla \times [\langle \boldsymbol{u} \rangle \times \langle \boldsymbol{B} \rangle + \mathcal{E}_t - \eta_0 \nabla \times \langle \boldsymbol{B} \rangle]$$

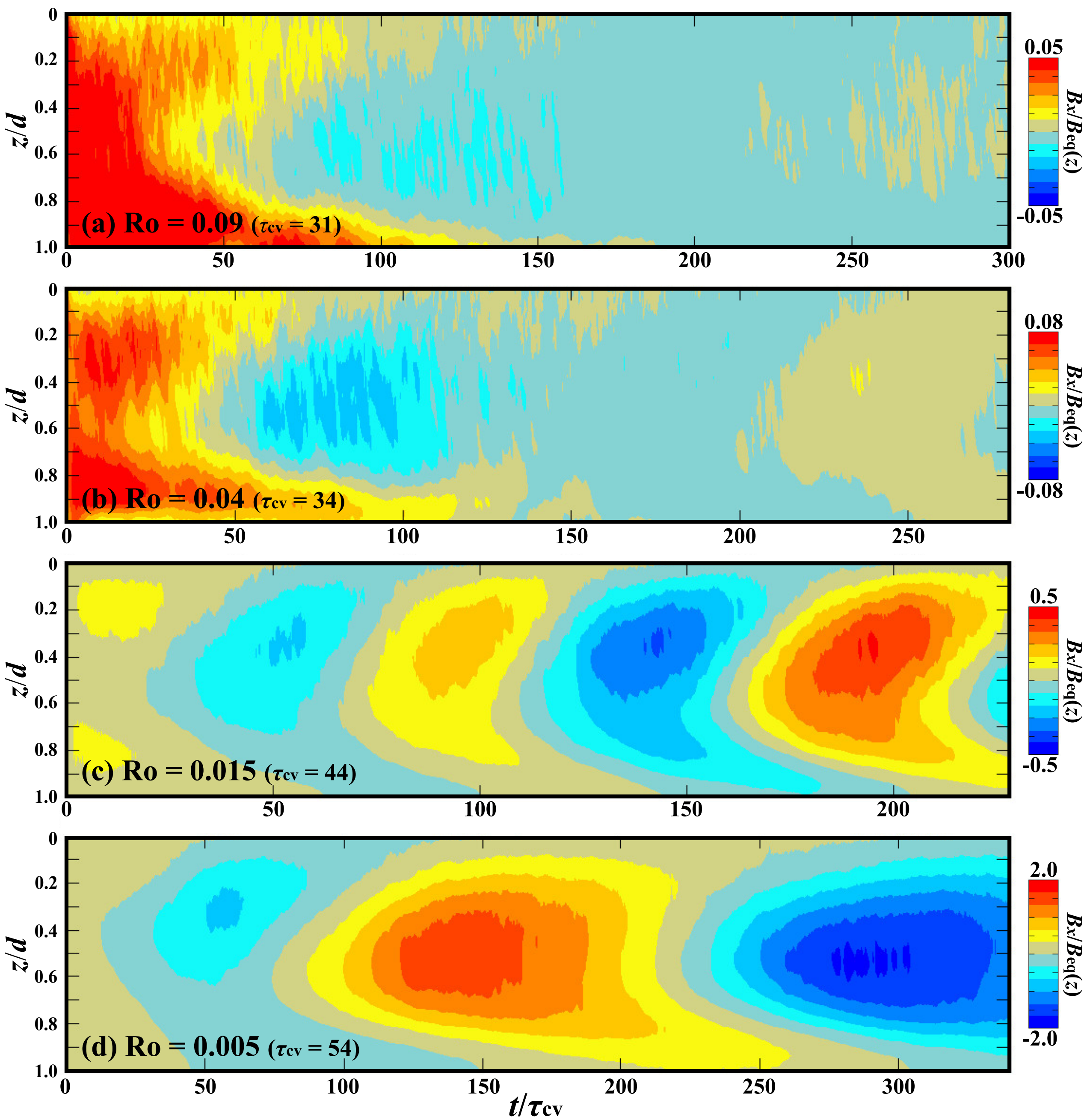
with $\mathcal{E}_t = \alpha \langle \boldsymbol{B} \rangle - \eta_t \nabla \times \langle \boldsymbol{B} \rangle$

+ nonlinear α-quenching (c.f., Brandenburg & Subramanian 05)

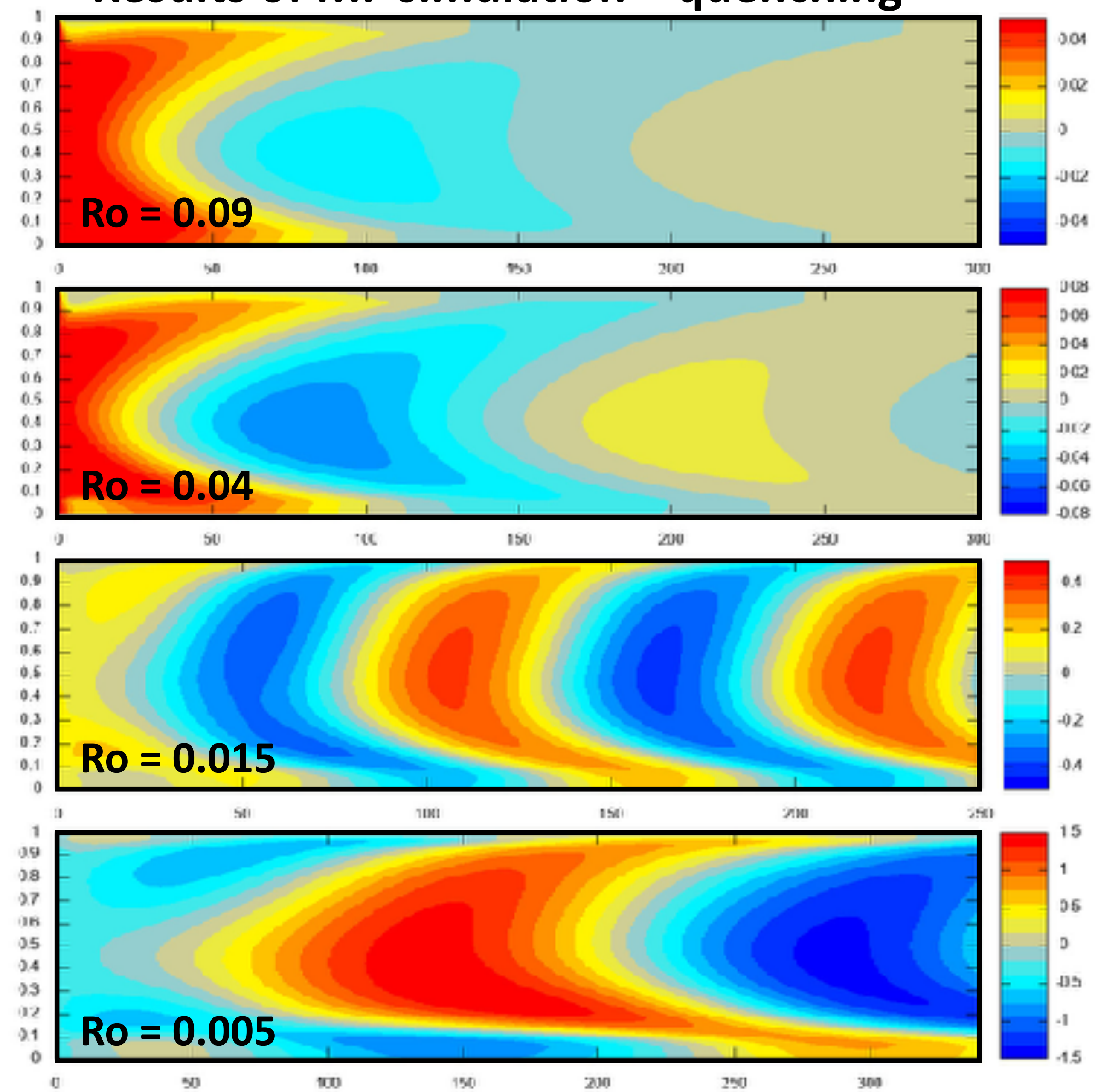
$$\frac{\partial \alpha}{\partial t} = -2\eta_k k_c^2 \left[\frac{\alpha \langle B \rangle^2 - \eta (\nabla \times \langle B \rangle) \cdot \langle B \rangle}{B_{\rm eq}^2} + \frac{\alpha - \alpha_k}{Re_M} \right]$$

(α-effect is suppressed with the increase of the B-field)

- **Results of our MHD simulation**



- **Results of MF simulation + quenching**



This suggests that the balance between turbulent α-effect and turbulent diffusion determines the success and failure of the large-scale dynamo at least in our simulation.

Extension of the MF Model coupled with the DNS to 3D :

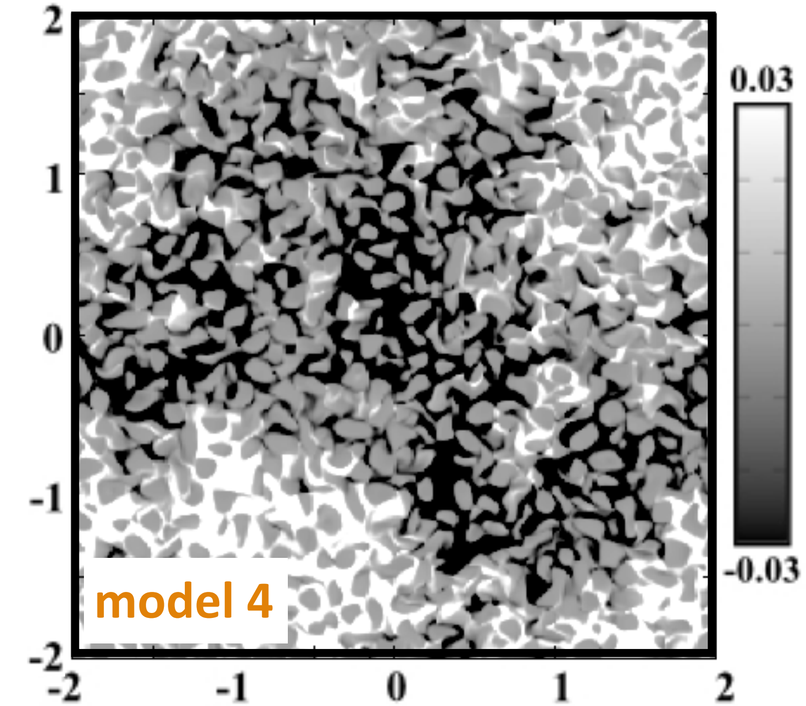
Surface structure formation may be also in the MF framework:

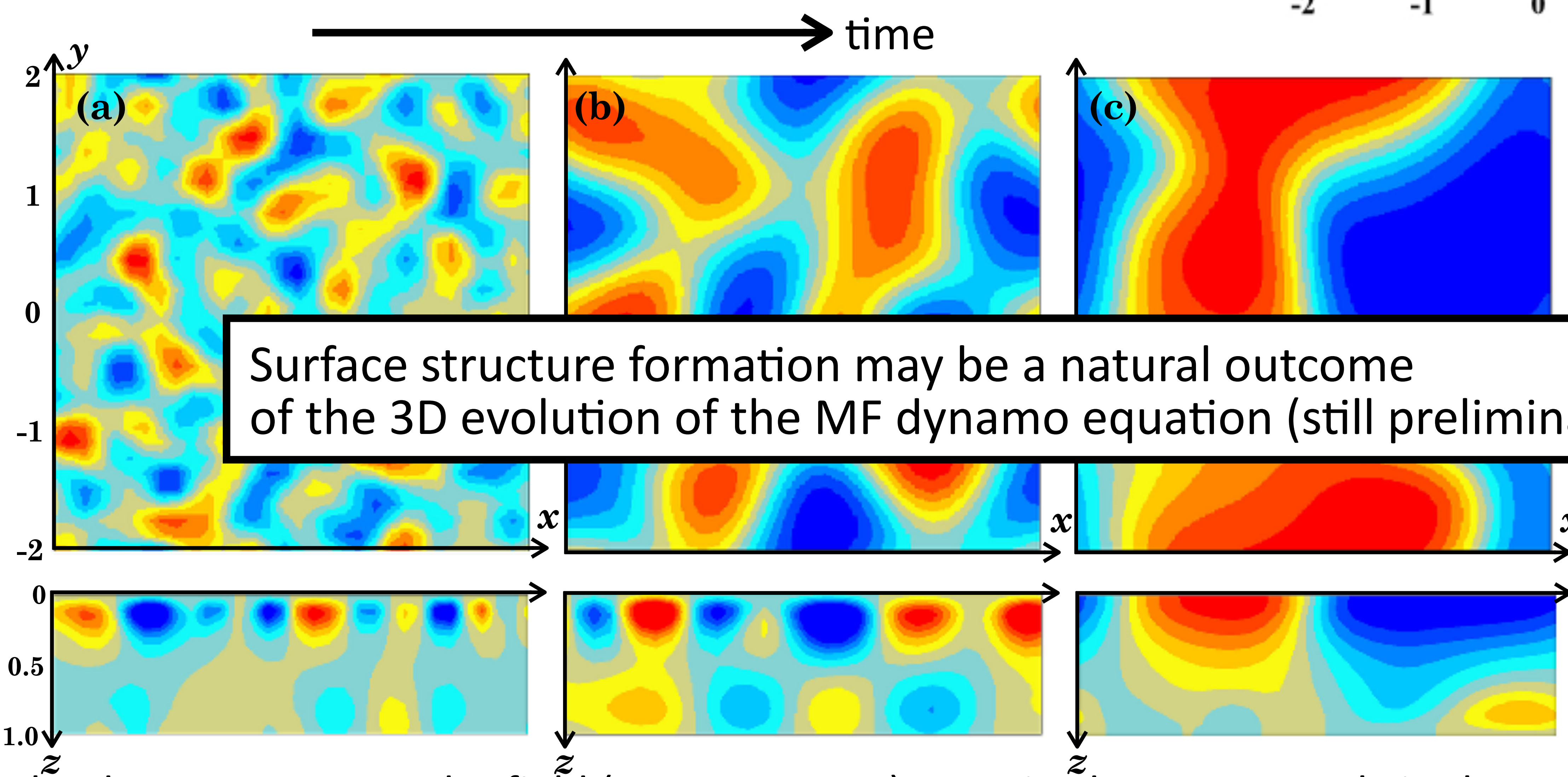
- Mean-field dynamo equation :

$$\frac{\partial \langle \boldsymbol{B} \rangle}{\partial t} = \nabla \times [\langle \boldsymbol{u} \rangle \times \langle \boldsymbol{B} \rangle + \mathcal{E}_t - \eta_0 \nabla \times \langle \boldsymbol{B} \rangle]$$

$$\mathcal{E}_t = \alpha \langle \boldsymbol{B} \rangle - \eta_t \nabla \times \langle \boldsymbol{B} \rangle$$

  - Just solve mean induction equation (in the vector potential form) in 3D in the similar way as the 1D.
  - no flow field except the given turbulent α and η_t



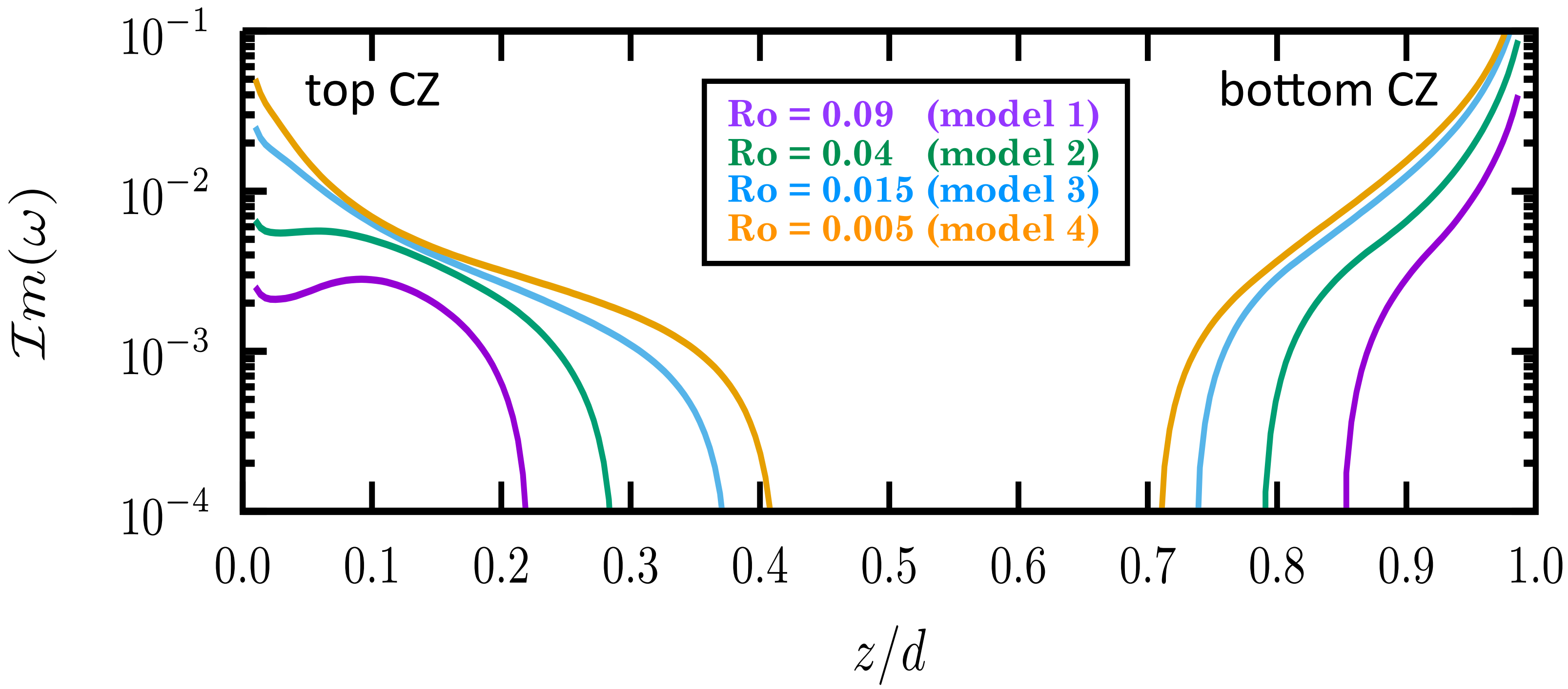


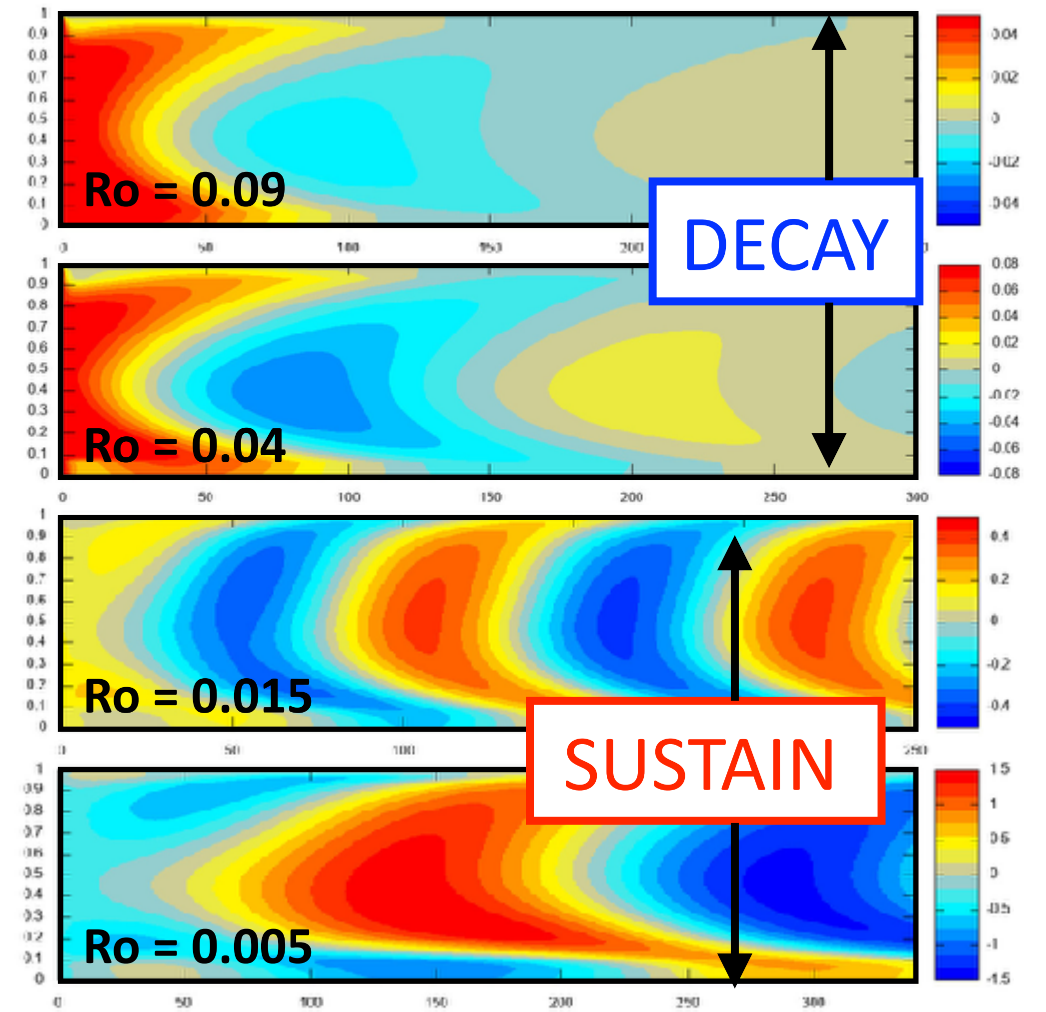
See, Jabbari+17 for the study of the similar surface structure formation in the forced-MHD turbulence

Surface structure formation may be a natural outcome of the 3D evolution of the MF dynamo equation (still preliminary)

The dynamo-generated B-field (Bz component) organized spontaneously in the upper CZ.

Why Does the High-Ro model fail to sustain the Dynamo ?





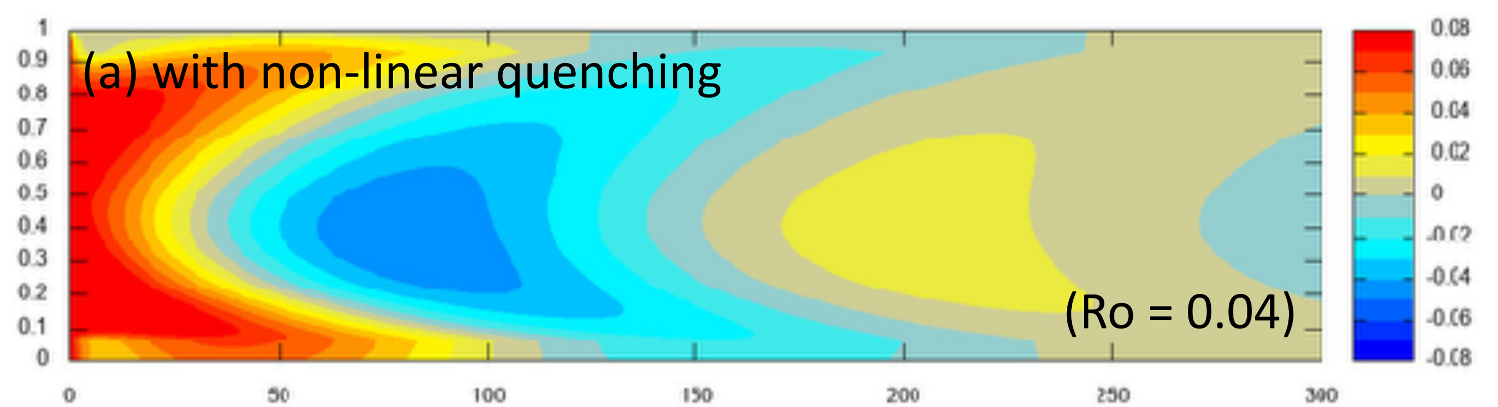
- In the linear theory, the difference between High and Low Ro models is only the size of the unstable domain (all the models are unstable to the dynamo linearly)
- None the less, the dynamo decays in the High Ro model. Some nonlinear effect kills the dynamo ?

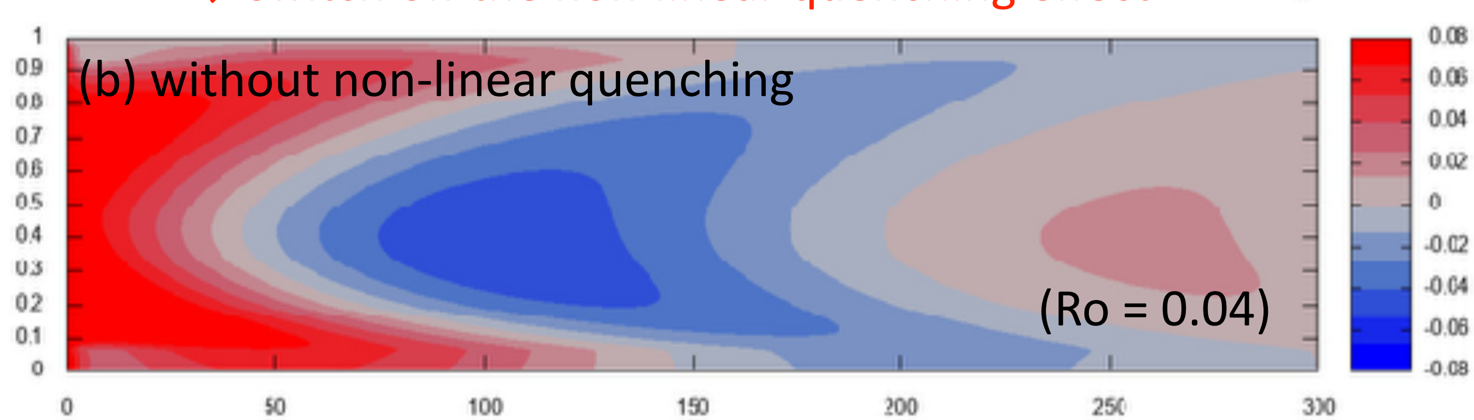
TEST CALCULATION:

- non-linear effect is controllable in the MFD model

$$\frac{\partial \alpha}{\partial t} = -2\eta_k k_c^2 \left[\frac{\alpha \langle B \rangle^2 - \eta \left(\nabla \times \langle B \rangle \right) \cdot \langle B \rangle}{B_{\rm eq}^2} + \frac{\alpha - \alpha_k}{Re_M} \right]$$

→ switch off the non-linear quenching effect





Even without the non-linear quenching effect, the dynamo in the high Ro model decays.

This implies that the convection structure in the High Ro model is unstable locally, but is stable globally to the dynamo.

A mechanism of the surface magnetic structure evolution

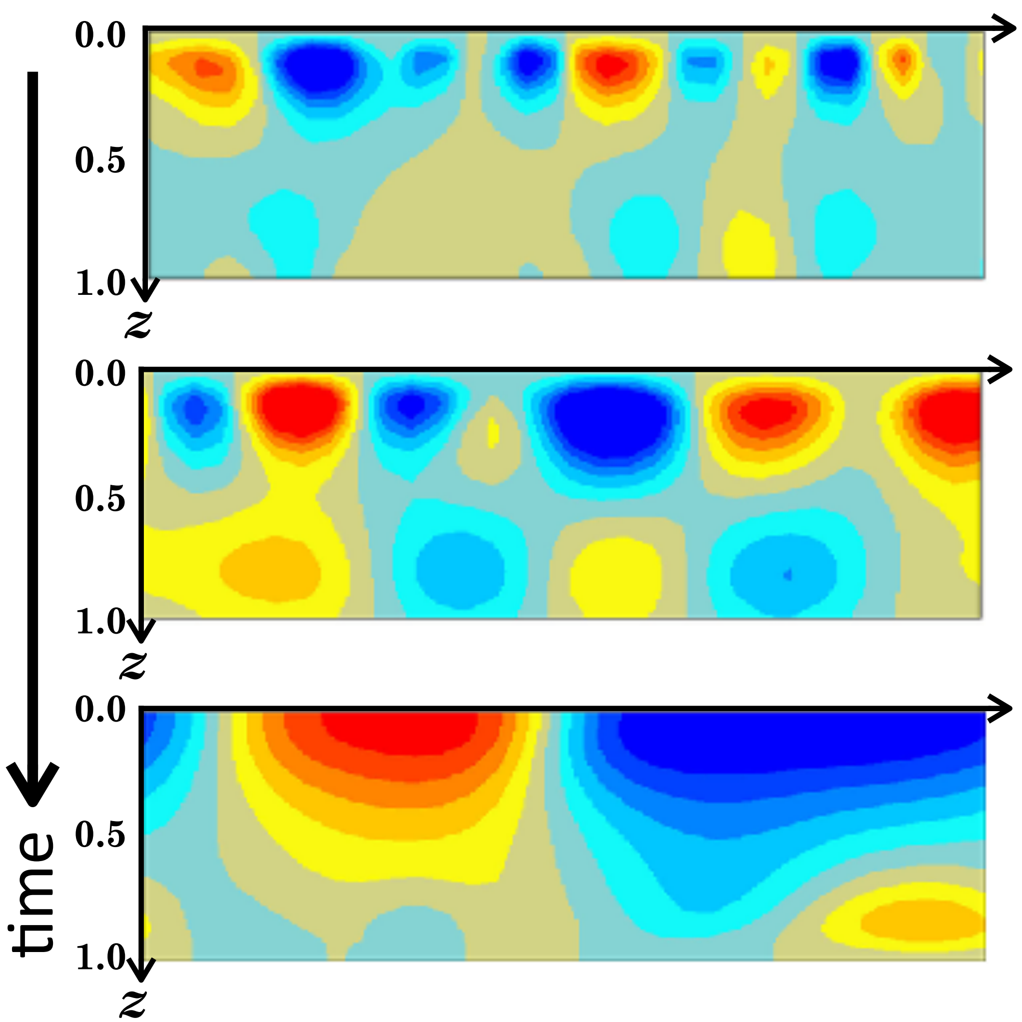
Based on the linear theory, we interpret the mechanism of the surface magnetic structure formation.

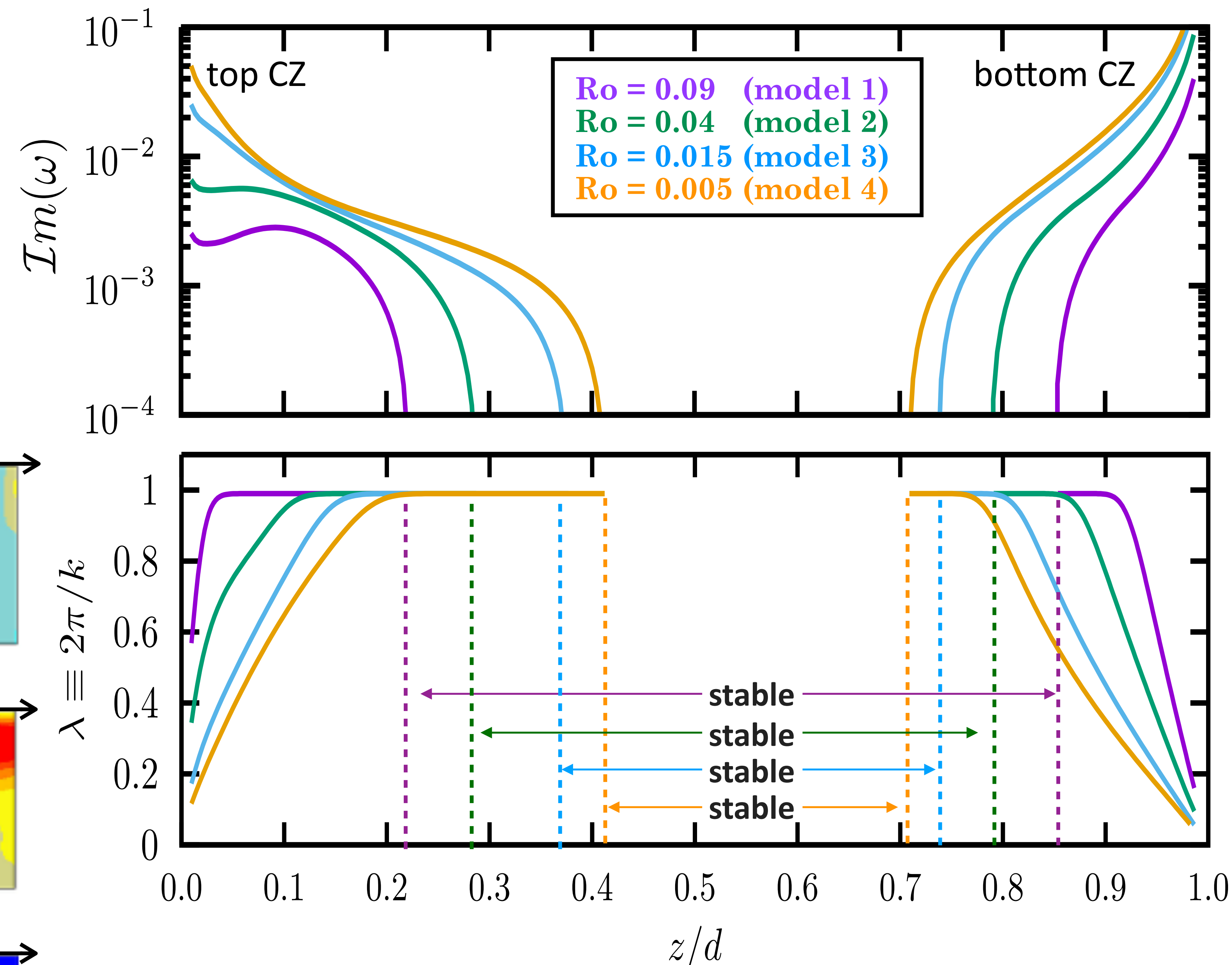
- The growth rate is higher and the unstable wavelength is shorter in the upper CZ

① the shorter λ mode grows initially in the upper CZ

② the longer λ mode grows gradually in the mid-CZ and then propagates upward

③ Finally, the band-like structure of the B-field is developed in the upper CZ

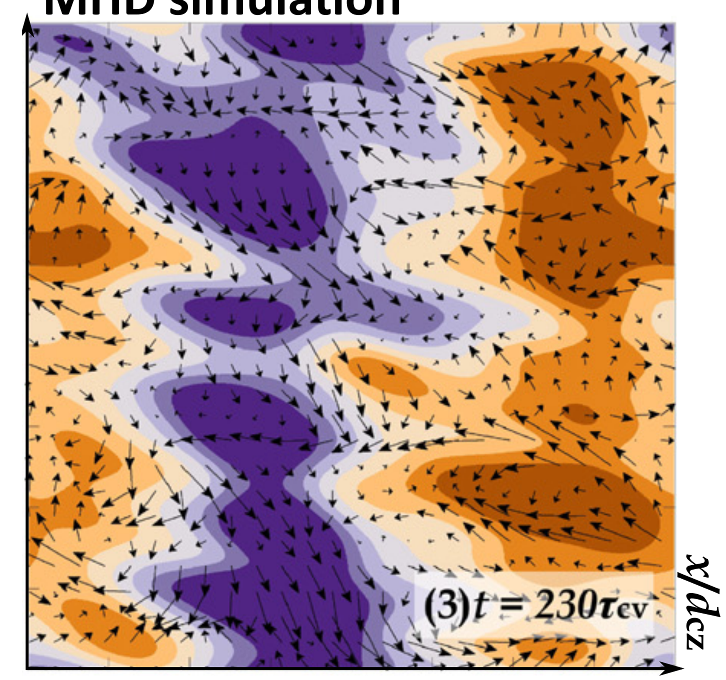




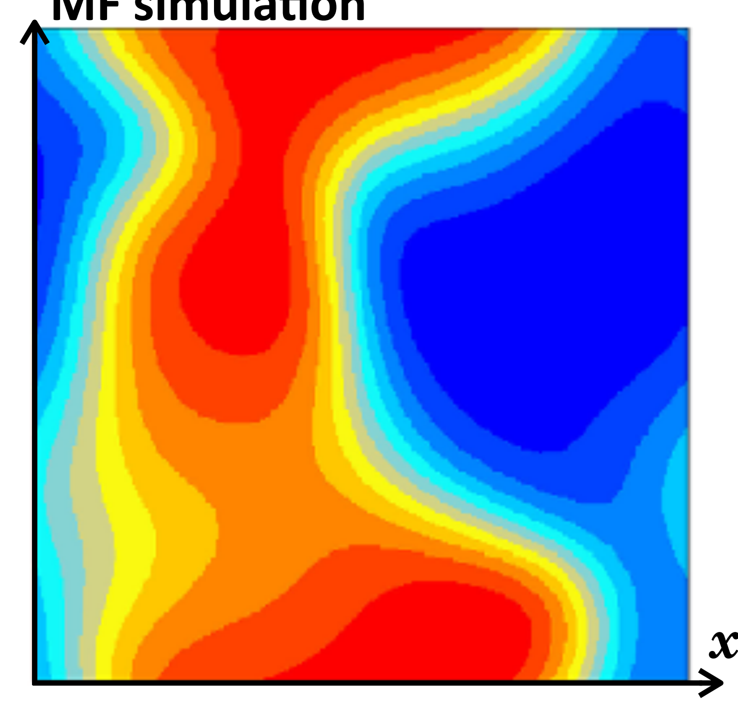
Summary

- "Ro" is a key for studying the stellar dynamo
  both observationally and theoretically
- **MHD simulation → Ro_{crit} = 0.015 ~ 0.04 for the successful large-scale dynamo**
  - turbulent α-effect seems to depend little on Ω
  - turbulent diffusion decreases with the increase of Ω
  - the dynamo behavior is controlled by the relationship bet. α and η_t
- **Mean-Field Dynamo Model Coupled with the DNS:**
  - Properties of the large-scale dynamo including the surface magnetic structure formation can be reproduced qualitatively

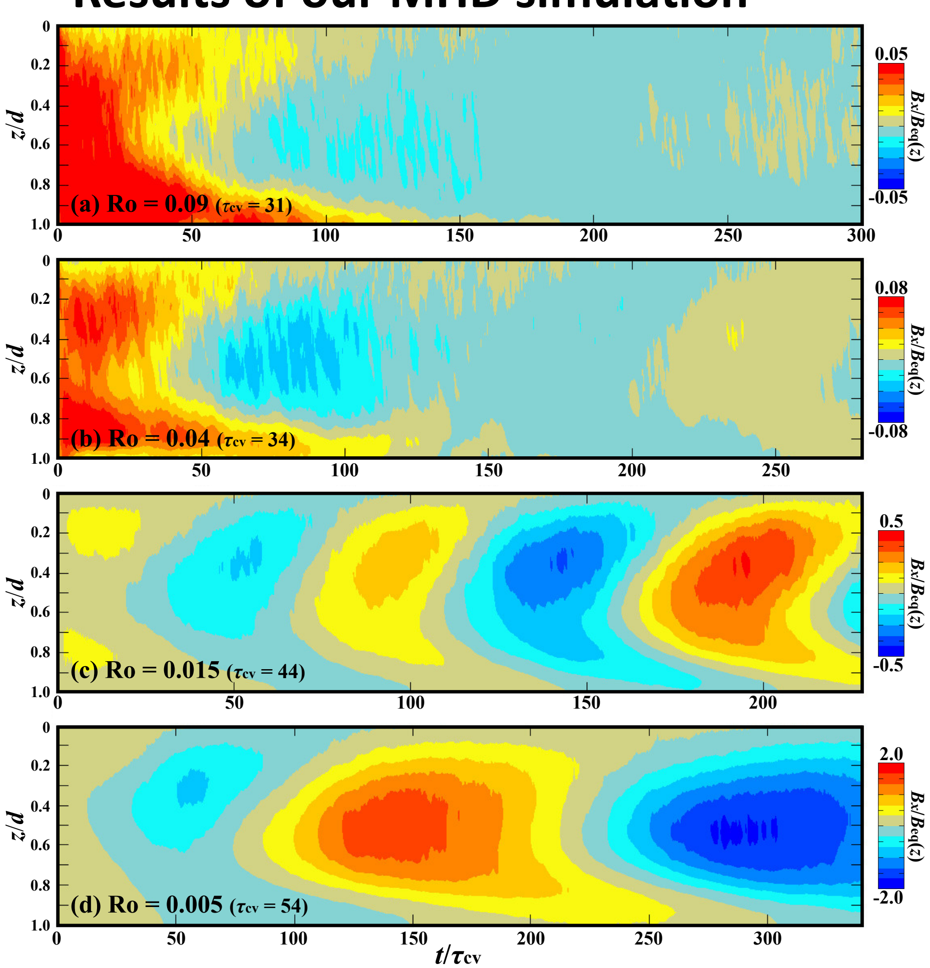
MHD simulation



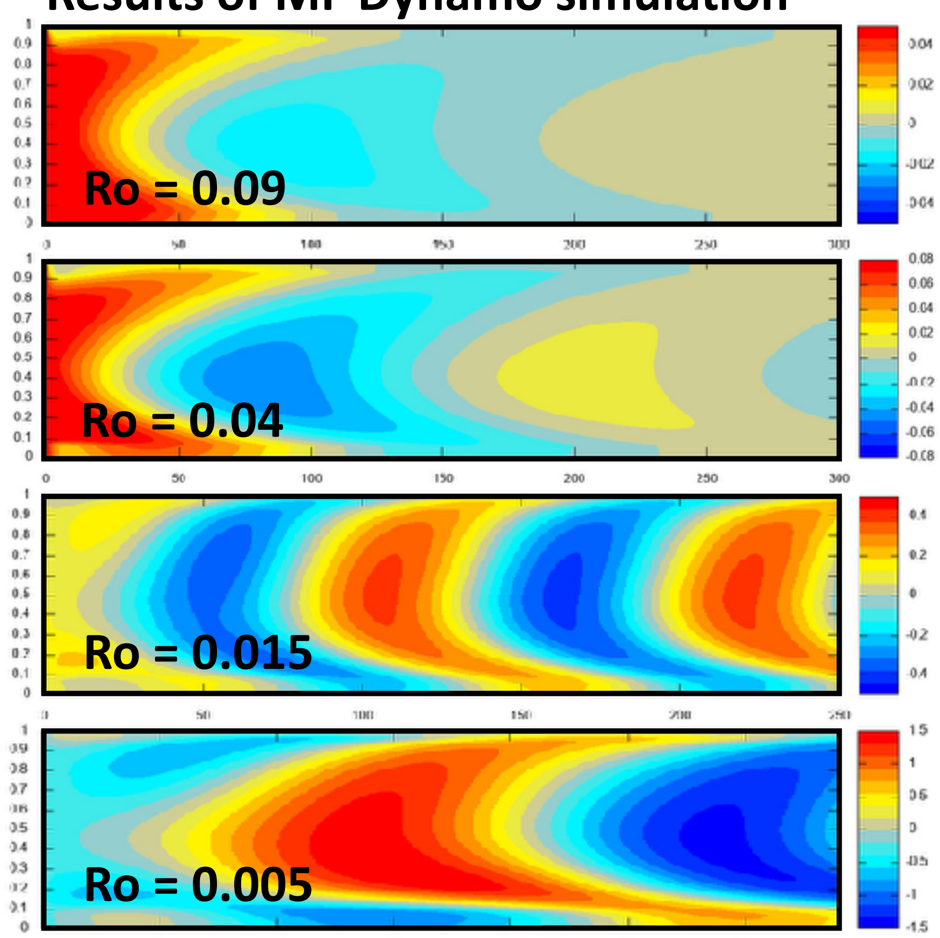
MF simulation



Results of our MHD simulation



Results of MF Dynamo simulation



Our large-scale dynamo in the Ek - RaEk$^{4/3}$ plane

Bushby et al. (2017) parametrically studied the success and failure of the large-scale dynamo: