Inverse transfer of Magnetic Energy in a decaying MHD system

I.T.A. Uni-Heidelberg

Kiwan Park

■ Large & small scale B field in space

- **Plasma & Magnetic field in space**

  Their mutual interactions, ubiquitous phenomena:
  → Evolution of B-field, constraint of Plasma

- **Role of B-field in plasma?**

  - Lorentz force constrains plasma

  - Stabilization or destabilization of a plasma system
    Pinch, Kruskal-Schwarzschild instability…

  - Transport of angular momentum (axisymmetric system)
    Braking fast spinning collapse or making accretion continue

  - Source of kinetic & thermal energy
    Magnetic reconnection

  -…

※ But, B-field is not a prerequisite for the evolution of a plasma system

■ Large & small scale B field in space

- **Origin of B-field?**

- Cosmological model (Primordial & Astrophysical model)

1. Primordial model
   a. When conformal invariance of EM fields was broken.
      (Inflation, Turner & Widrow 1988)
   b. Through cosmic phase transition
      Electro-Weak Phase Transition (EWPT),
      Quantum Chromo Dynamics transition (QCD)
      (Grasso & Rubinstein 2001)

2. Due to Plasma fluctuations (Astrophysical model)
   Biermann battery (Biermann 1950), Harrison effect (Harrison 1970)

$\rightarrow B \sim 10^{-62}G$ - $10^{-19}G$

- Observed B-field?

Dynamo

$\rightarrow B \sim 10^{-5}G$

■ Large & small scale B field in space

- **Requirements of LSD & SSD?**

  - LSD
    Helicity, Differential Rotation, Magneto-Rotational Instability
    → External forcing source

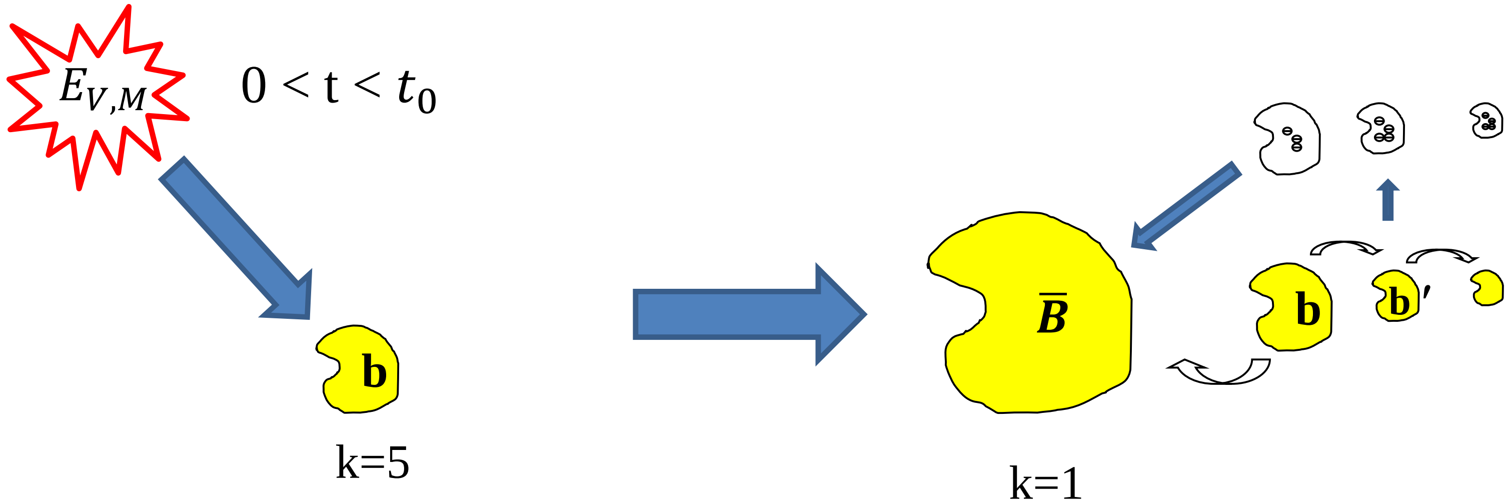
  - SSD
    No instability, no shear → Forced with nonhelical field

- **Question**

  1. Is LSD impossible without these specific forcing sources?
     - Dynamo theory: Impossible
       But some theories and simulations suggested and reported the possibility of LSD in a decaying MHD system.
       - Olesen 1997, Ditlevsen et al. 2004, Brandenburg et al. 2015, 2017, J. Zrake 2014, 2016, Park 2017a, 2017b
  2. How can we explain LSD?

■ Energy Spectrum in a decaying MHD system

- Simulation setting

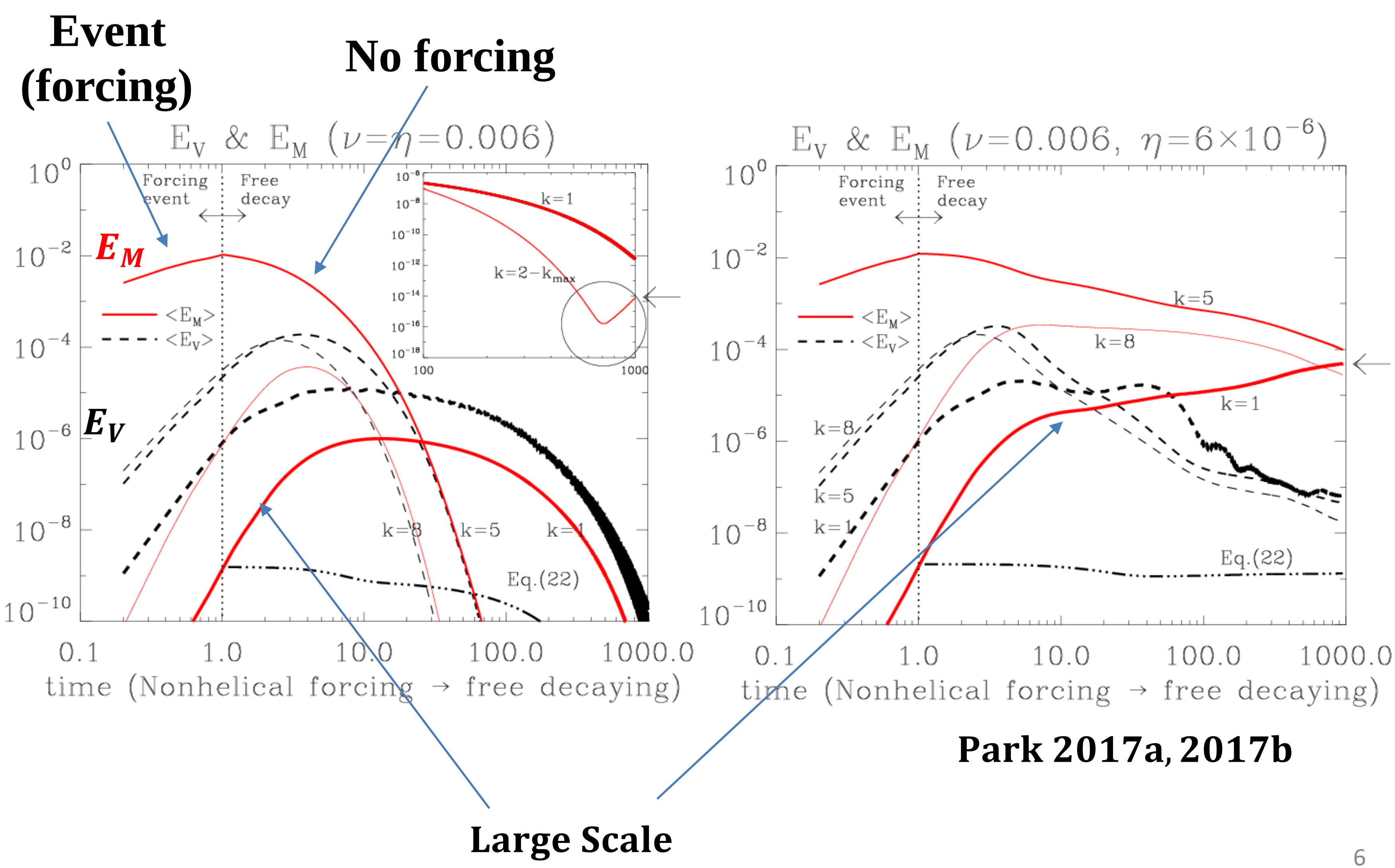


Ephemeral supply of E_V & E_M

Decaying (e.g. Supernovae etc.)

■ Energy Spectrum in a decaying MHD system

- **NonHelical** magnetic energy



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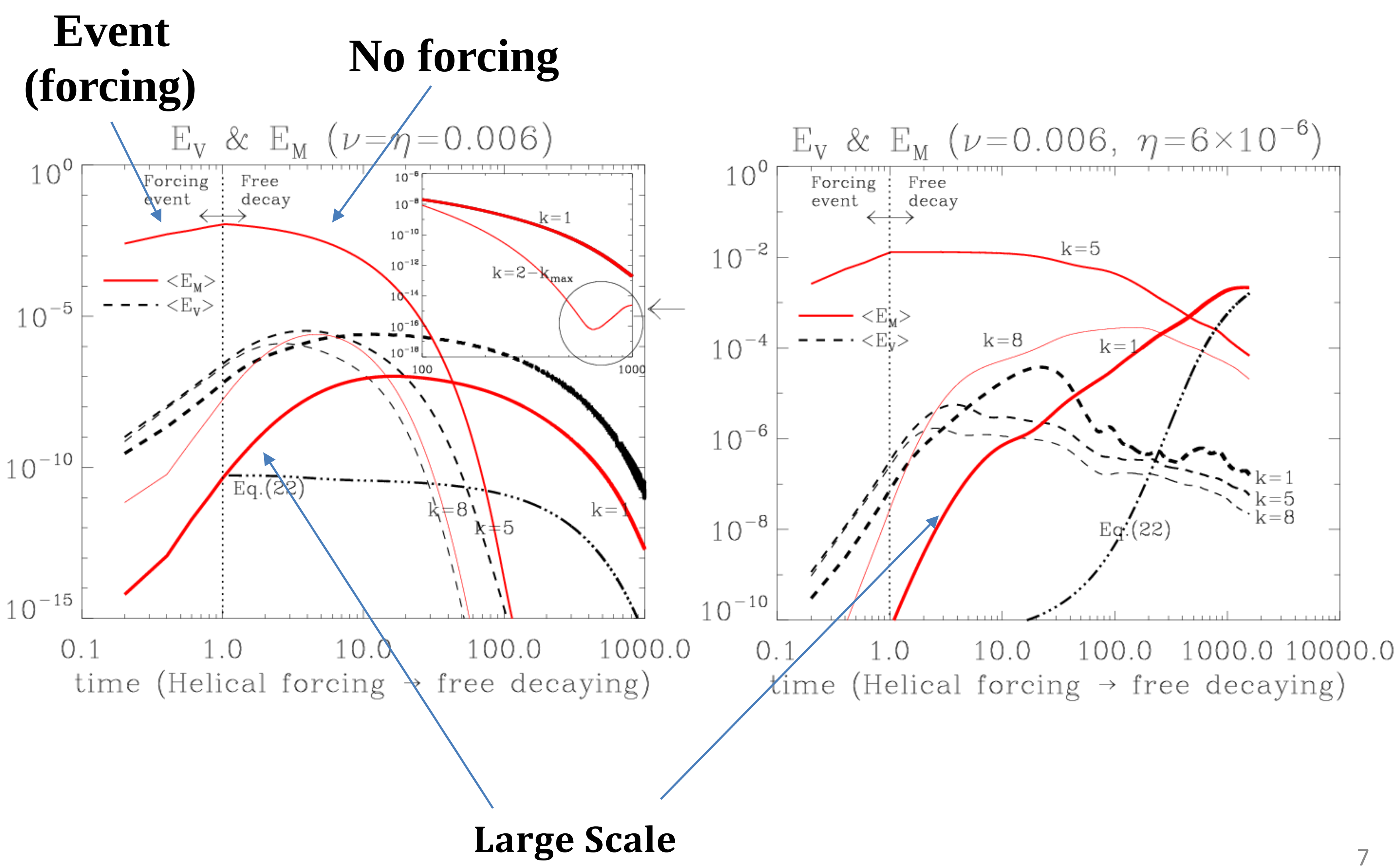
■ Energy Spectrum in a decaying MHD system

- **Helical** magnetic energy



- **How can we explain?**

(1) Helical case

- α effect

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times \langle \mathbf{u} \times \mathbf{b} \rangle + \eta \nabla^2 \overline{\mathbf{B}}$$
$$\Rightarrow \nabla \times \alpha \overline{\mathbf{B}} + (\eta + \beta) \nabla^2 \overline{\mathbf{B}}$$

$$\because \alpha \sim \int \langle \mathbf{j} \cdot \mathbf{b} \rangle - \langle \mathbf{u} \cdot \boldsymbol{\omega} \rangle \, d\tau \neq 0$$

(2) NonHelical case?

- No α effect $\Rightarrow \overline{\mathbf{B}} \rightarrow \mathbf{0}$

But, $\overline{\mathbf{B}}(t) \neq 0$,

■ Analytic solution

1. Helical initial E_{M0}

$$\bar{A}\cdot\left(\frac{\partial \bar{B}}{\partial t} = \nabla\times\alpha\bar{B} + (\beta+\eta)\nabla^2\bar{B}\right),\ \bar{B}\cdot\left(\frac{\partial \bar{B}}{\partial t} = \nabla\times\alpha\bar{B} + (\beta+\eta)\nabla^2\bar{B}\right)$$

$$\Rightarrow\begin{cases}2H_{ML} = (2E_{M0}+H_{M0})e^{2\int(\alpha-\beta-\eta)d\tau} - (2E_{M0}-H_{M0})e^{-2\int(\alpha+\beta+\eta)d\tau}\\ 4E_{ML} = (2E_{M0}+H_{M0})e^{2\int(\alpha-\beta-\eta)d\tau} + (2E_{M0}-H_{M0})e^{-2\int(\alpha+\beta+\eta)d\tau}\end{cases}$$

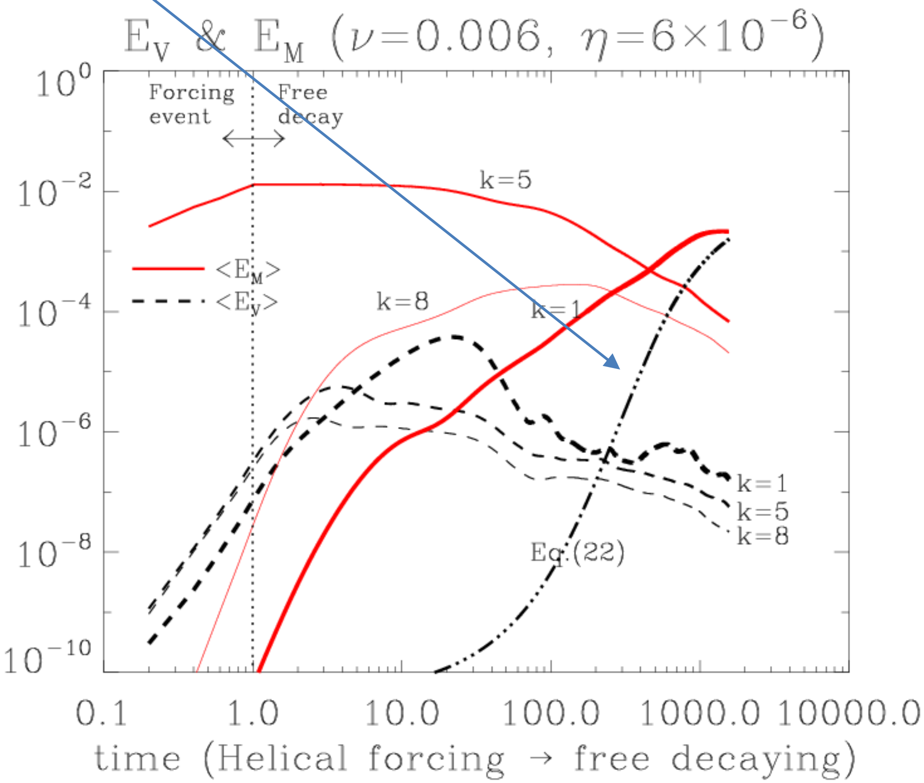
$\alpha = \frac{1}{3}\int\langle \mathbf{j}\cdot\mathbf{b}\rangle\, d\tau$

$\sim \sum\langle \mathbf{j}\cdot\mathbf{b}\rangle(k)\,\Delta t$

$k = 4-6,\ 7$

$\Delta t = t_n - t_{n-1}$

$\beta = \frac{1}{3}\int\langle u^2\rangle\, d\tau$



■ Analytic solution

2. Nonhelical initial E_{M0}

- **Statistical method for the ideal MHD system**

  a. 2D MHD (Fyfe & Montgomery 1976)

① **Conservation**

E, $\langle \mathbf{u} \cdot \mathbf{B} \rangle$, $\langle A^2 \rangle$ (stationary system)

② **Gibbs distribution function**

$$Z_0 \exp[-(\alpha E + \beta \langle \mathbf{u} \cdot \mathbf{B} \rangle + \frac{\gamma \langle A^2 \rangle}{2})]$$

③ $E_M = \left(\alpha - \frac{\beta^2}{4\alpha} + \frac{\gamma}{k^2}\right)^{-1}$

④ With β=0 (P = 0, No cross helicity) and $\gamma < 0$

$E_M = \frac{1}{\left(\alpha - \frac{|\gamma|}{k^2}\right)} \rightarrow E_{M,\,peak}$ at k_{min} ← Inverse transfer of E_M.

b. 3D MHD (Frisch et al. 1975)

※ Valid for a decaying MHD system? Assumption is not valid.

- **Scaling invariant method (Olesen 1997)**

$$\frac{\partial \boldsymbol{B}}{\partial \boldsymbol{t}} = \boldsymbol{\nabla} \times \langle \mathbf{u} \times \boldsymbol{B} \rangle + \boldsymbol{\eta} \boldsymbol{\nabla}^2 \boldsymbol{B}$$

① <u>Scale invariants</u> : r → lr, t → l^{1-h}t, l^hu, l^hb, …

$$\rightarrow E_{V,M}(k/l, l^{1-h}\mathrm{t}) = E_{V,M}(k, \mathrm{t})$$

② Assuming $E_{V,M}(k, \mathrm{t}) \rightarrow k^{-1-2h}\psi_{V,M}(k, \mathrm{t})$

$$\rightarrow k\frac{\partial \psi_{V,M}}{\partial k} + (h-1)t\frac{\partial \psi_{V,M}}{\partial t} = 0$$

③ $E_{V,M}(k, \mathrm{t}) \rightarrow k^{-1-2h}\psi_{V,M}(k^{1-h}t)$ where $k^{1-h}t = const.$
If $1 - h > 0$ → Inverse transfer of $E_{V,M}(k, \mathrm{t})$.

④ $\int E_{V,M}(k)\, dk = E_{V,M}$? Unreliable… (Ditlevsen et al. 2004)

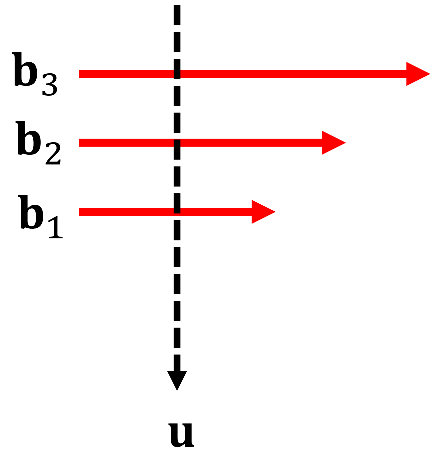
- **Can we use another intuitive or theoretical model to explain the inverse transfer of nonhelical E_M in a decaying MHD system?**
- Field structure model, α theory, Qaasi Normal approx.

- **Field Structure model**

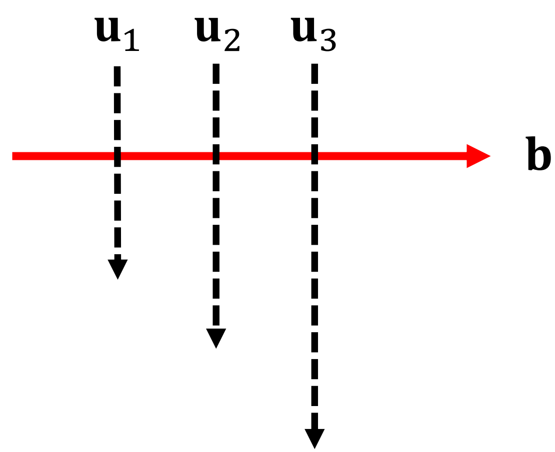
Nonhelical B field

$$\frac{\partial \mathbf{B}}{\partial t} = \mathbf{B}\cdot\nabla\mathbf{u} - \mathbf{u}\cdot\nabla\mathbf{B} + \eta\nabla^2\mathbf{B}$$

$$-\mathbf{u}\cdot\nabla\mathbf{B} > 0$$

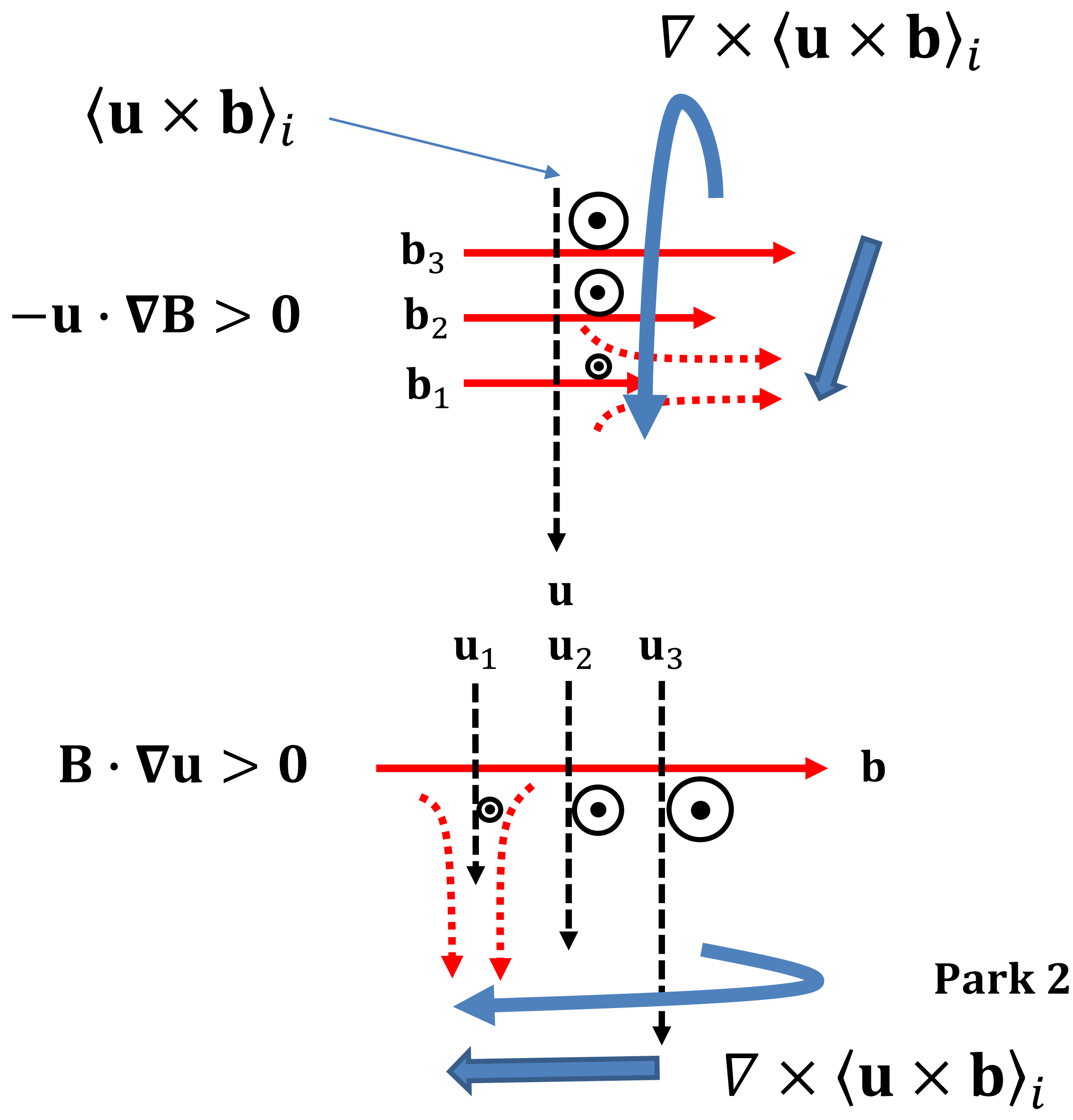


$$\mathbf{B}\cdot\nabla\mathbf{u} > 0$$



• Inhomogeneous EMF

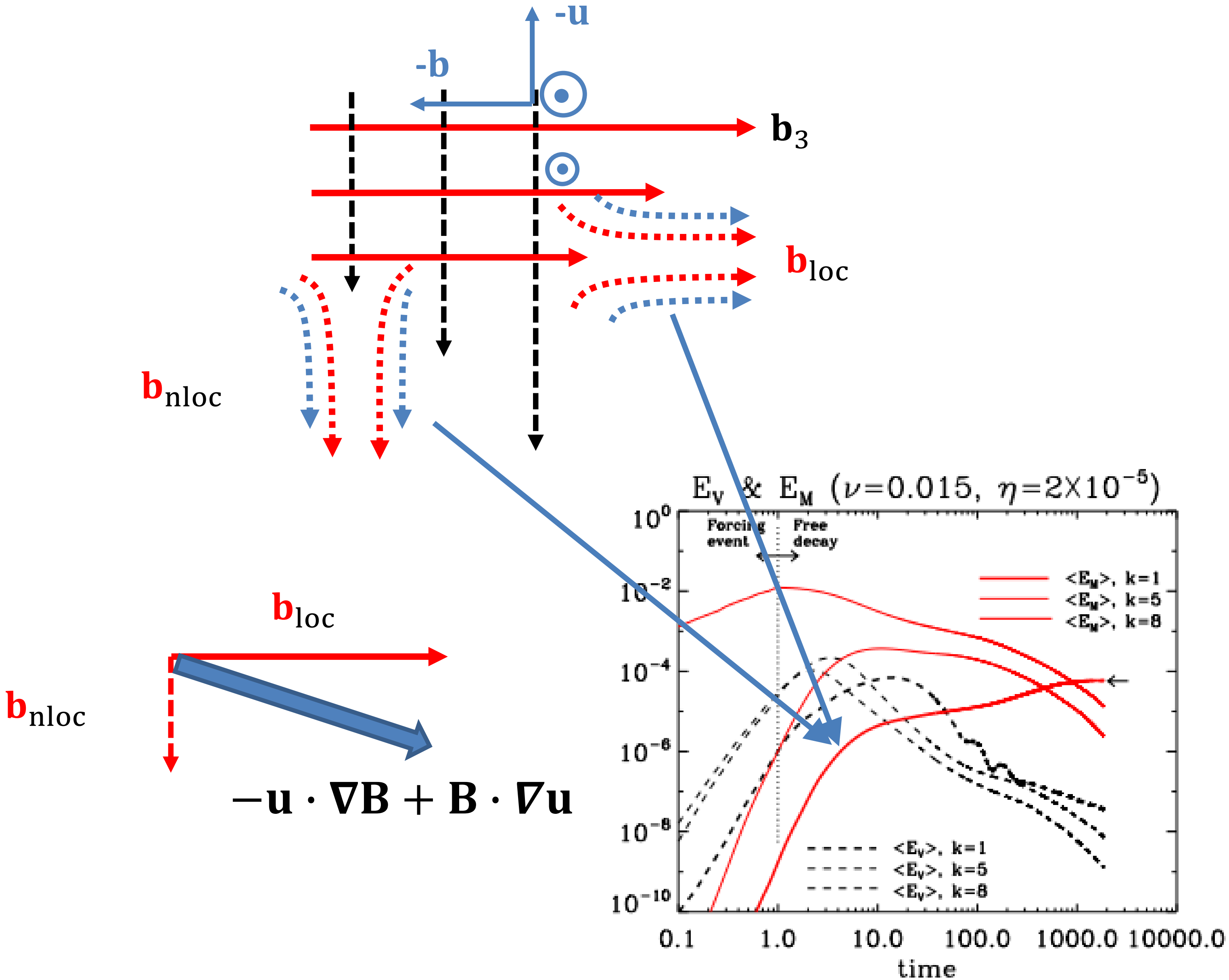
Nonhelical B field



Park 2017a, 2017b

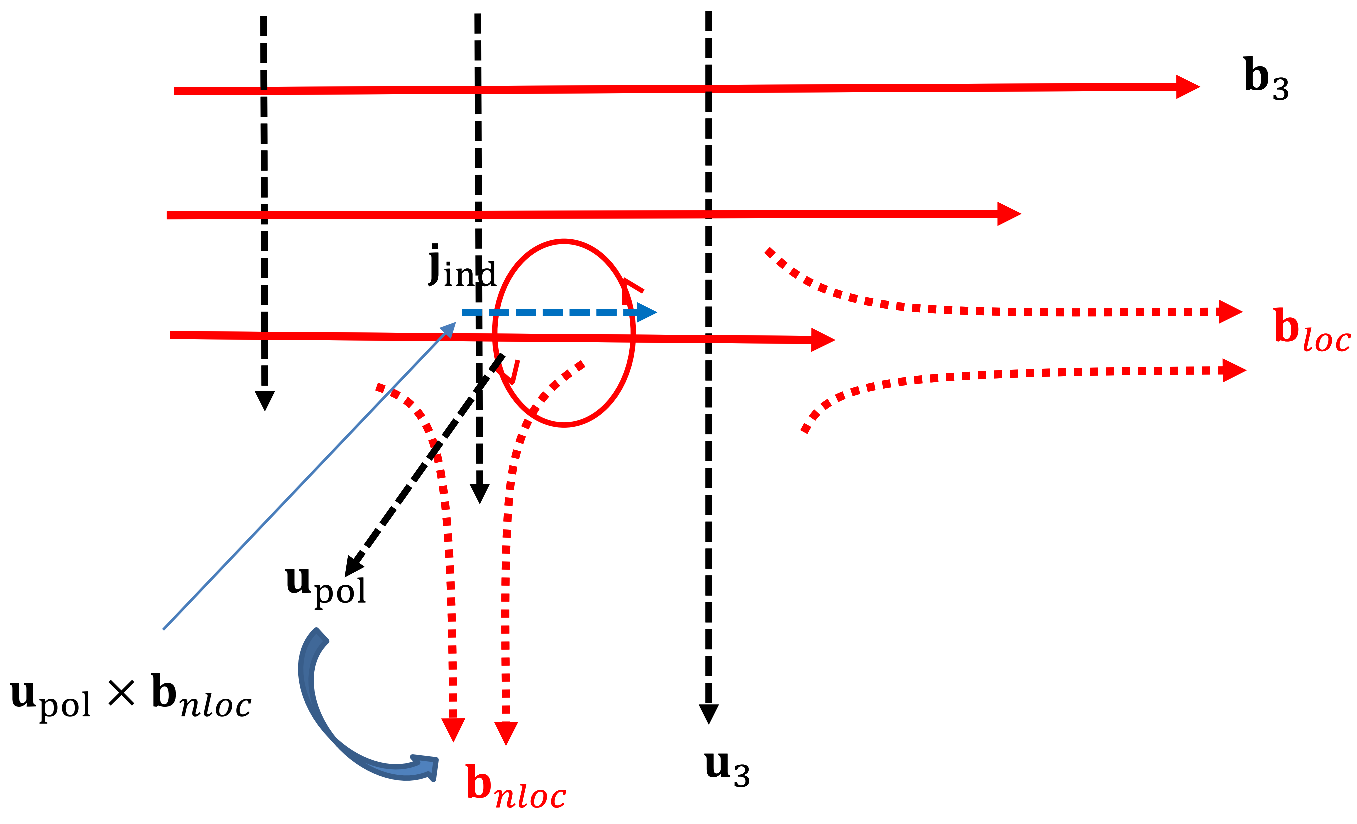
- **Dissipation effect**

Nonhelical B field

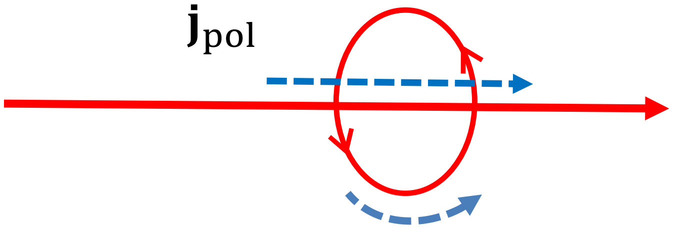


•Helical Field effect

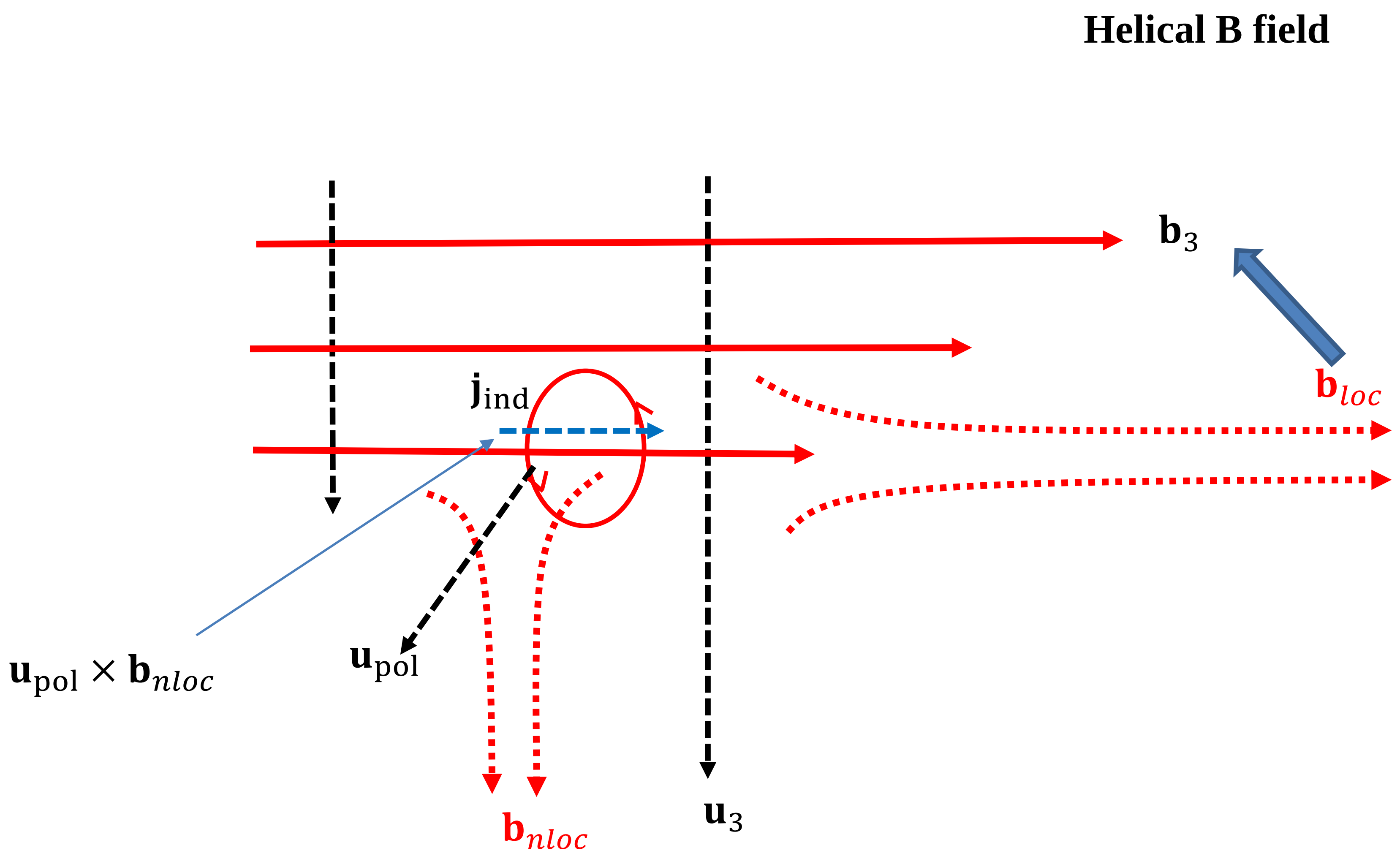
Helical B field



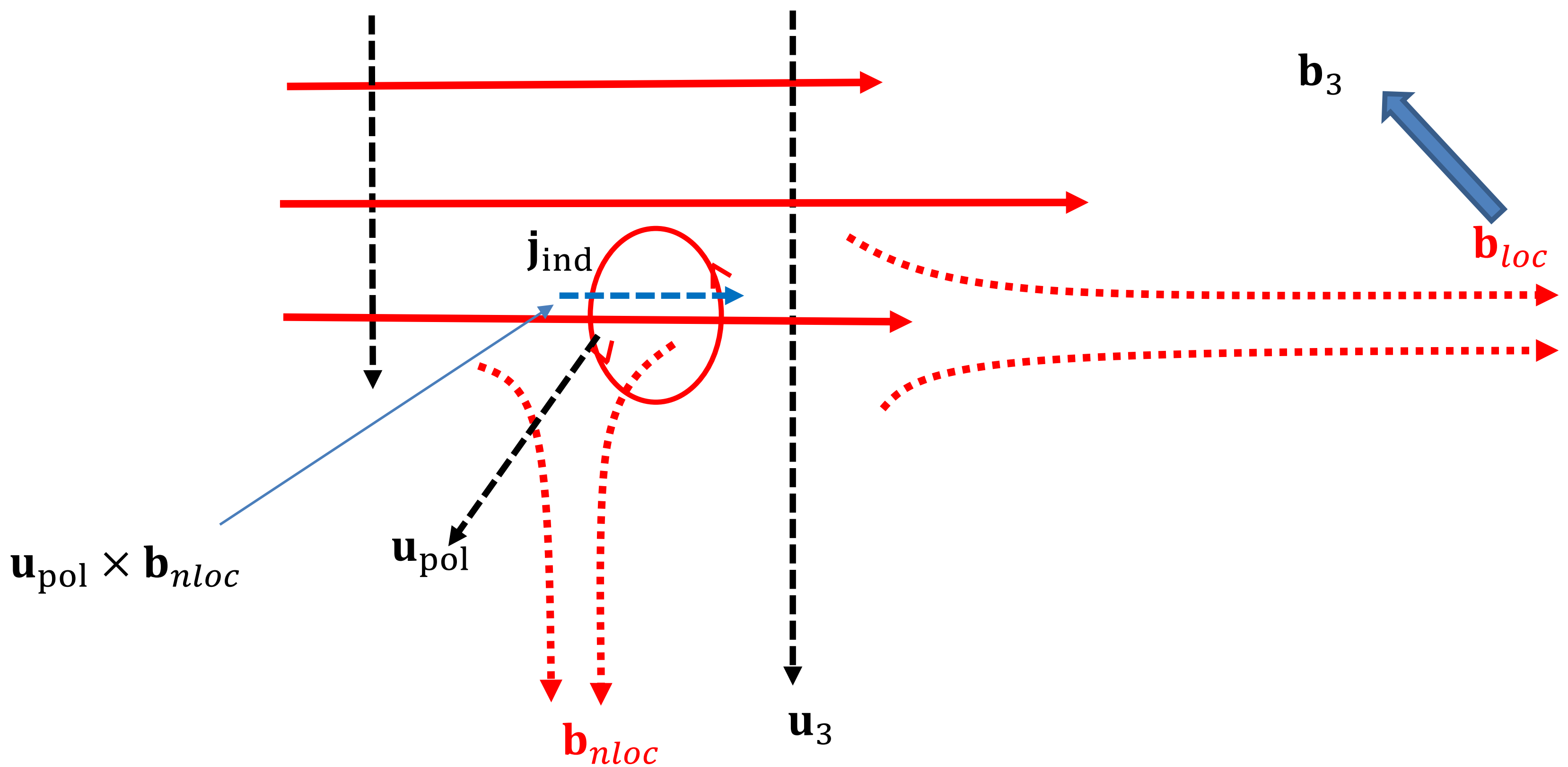
Helical B field



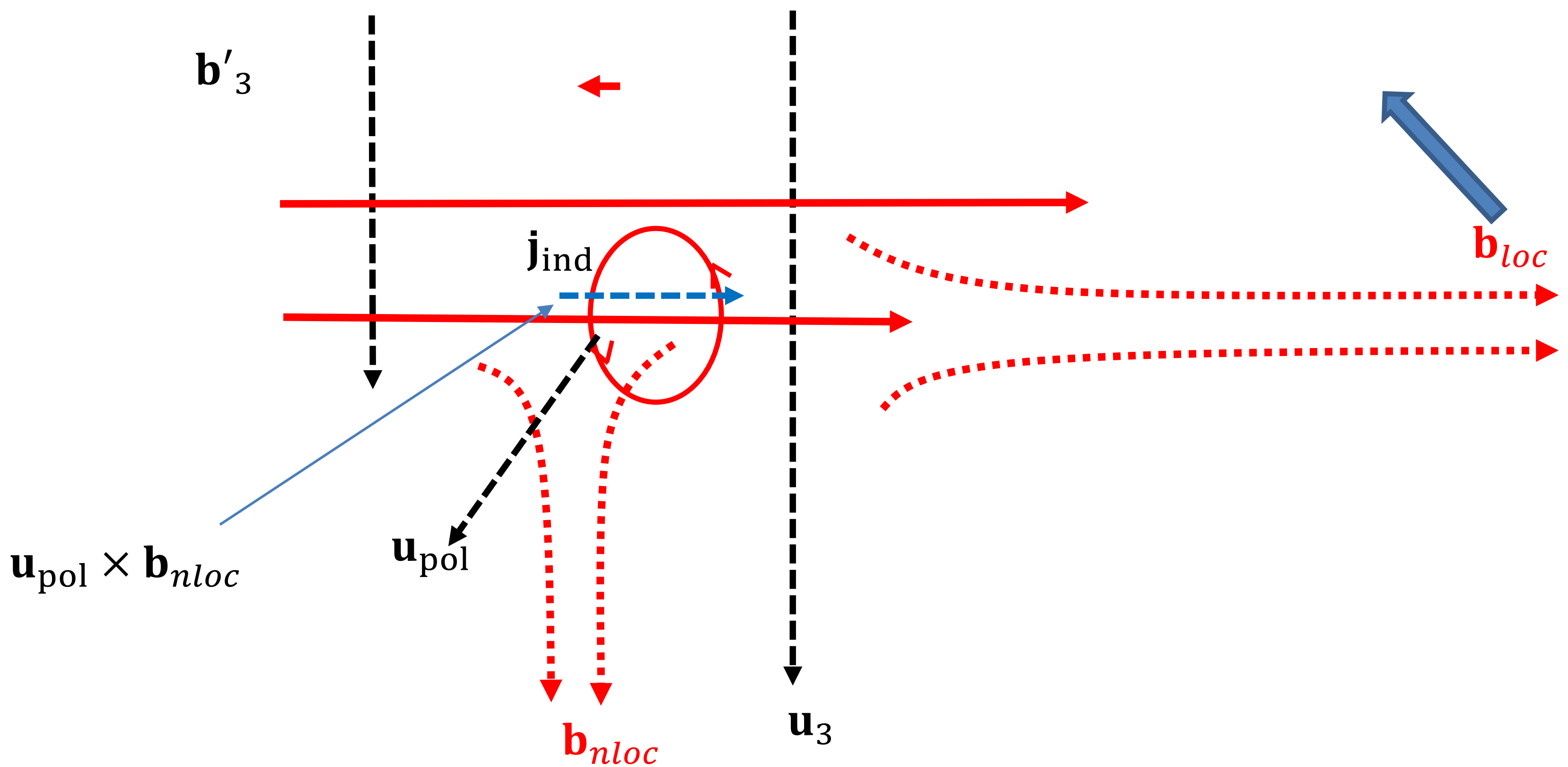
$$\langle \mathbf{u} \cdot \boldsymbol{\omega} \rangle < \mathbf{0} \rightarrow \langle \mathbf{A} \cdot \mathbf{B} \rangle > \mathbf{0} \Rightarrow \alpha^2 \text{dynamo}$$

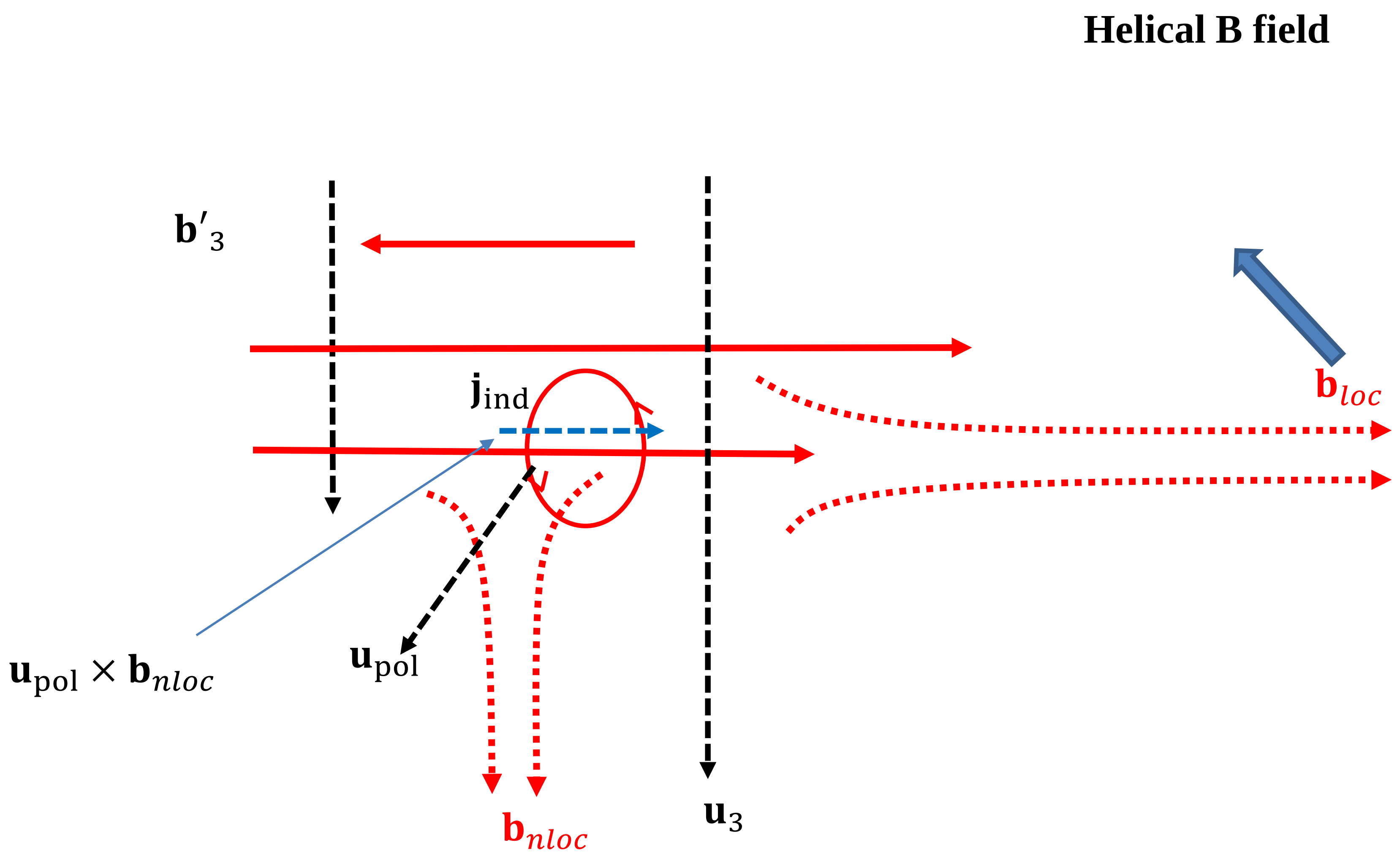

Helical B field
$\mathbf{b}_3$
$\mathbf{b}_{loc}$
$\mathbf{j}_{\mathrm{ind}}$
$\mathbf{u}_{\mathrm{pol}} \times \mathbf{b}_{nloc}$
$\mathbf{u}_{\mathrm{pol}}$
$\mathbf{b}_{nloc}$
$\mathbf{u}_3$

Helical B field

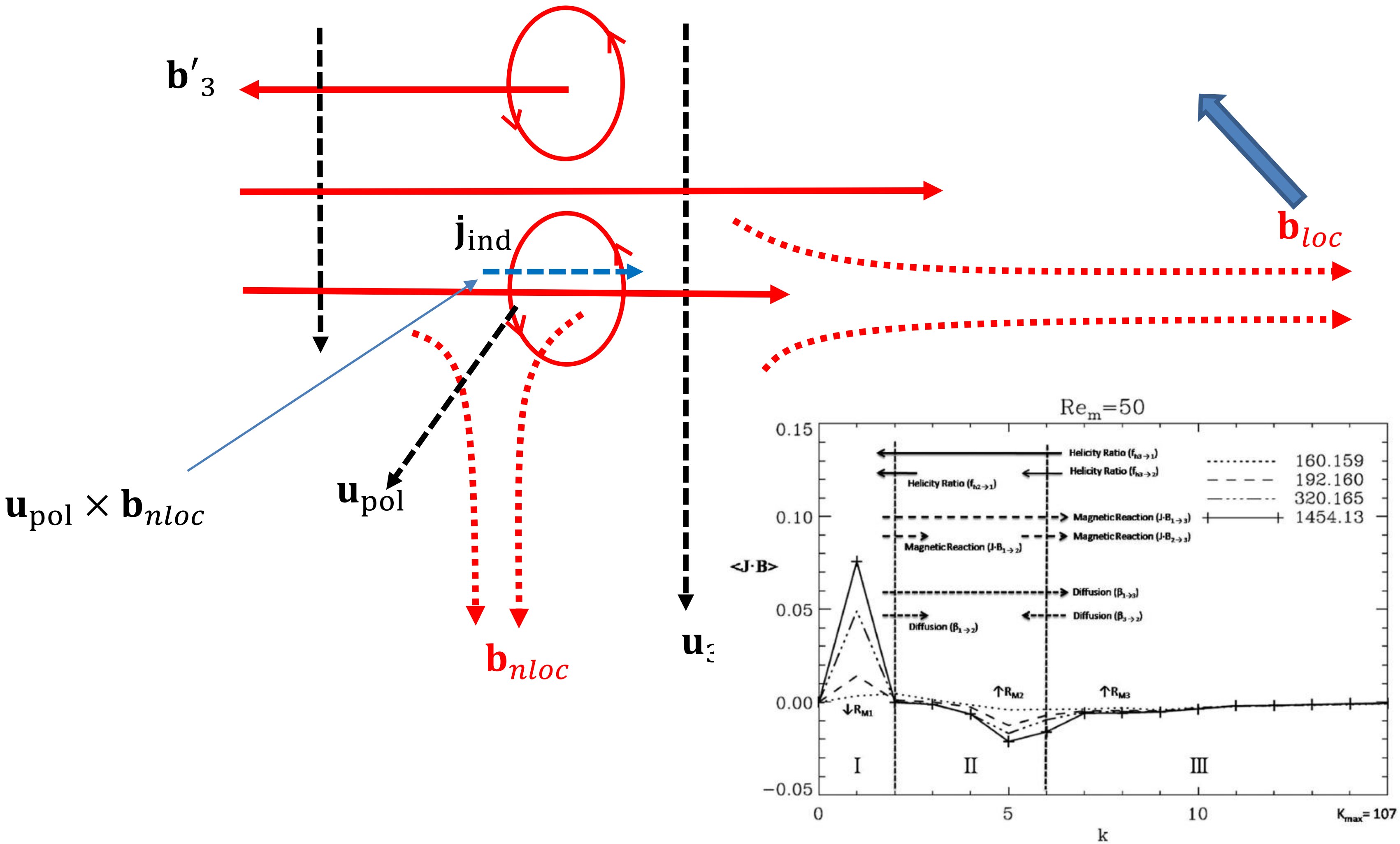


Helical B field

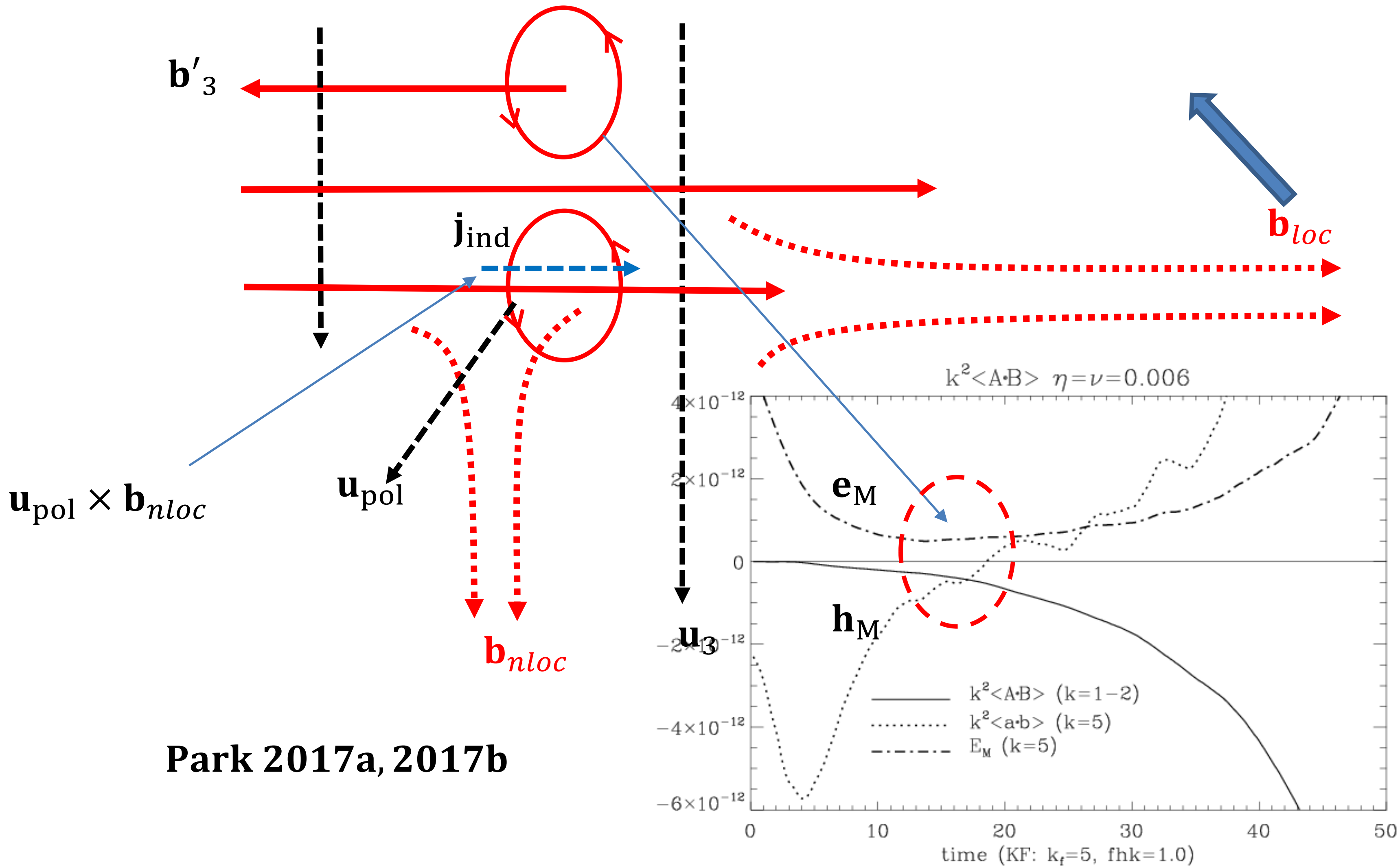



Helical B field
b′3
jind
bloc
upol × bnloc
upol
bnloc
u3

Helical B field



Helical B field



Park 2017a, 2017b

■ **Resuming the analytic solution for the nonhelical**

- **EDQNM**
  (Eddy Damped Quasi Normalized Markovian approx.)

v• v•

$$\frac{\partial \mathrm{v}}{\partial \mathrm{t}} = -\mathrm{v}\cdot\nabla\mathrm{v} - \nabla\mathrm{P} + \mathrm{B}\cdot\nabla\mathrm{B} + \upsilon\nabla^2\mathrm{v}$$

$$\frac{\partial \mathrm{B}}{\partial \mathrm{t}} = \nabla\times(\mathrm{v}\times\mathrm{B}) + \lambda\nabla^2\mathrm{B}$$

B• B•

$$\langle \mathrm{vv}\rangle = (4\pi\mathrm{k}^2)^{-1}E_V$$

$$\langle \mathrm{BB}\rangle = (4\pi\mathrm{k}^2)^{-1}E_M$$

- **Analytic solution II (EDQNM)**

$$\frac{\partial E_M(k)}{\partial t} = -\int dpdr\theta_{kpr}^{\eta\eta\nu}(t)\frac{p^2}{r}z(1-x^2)E_M(r)E_M(k)$$

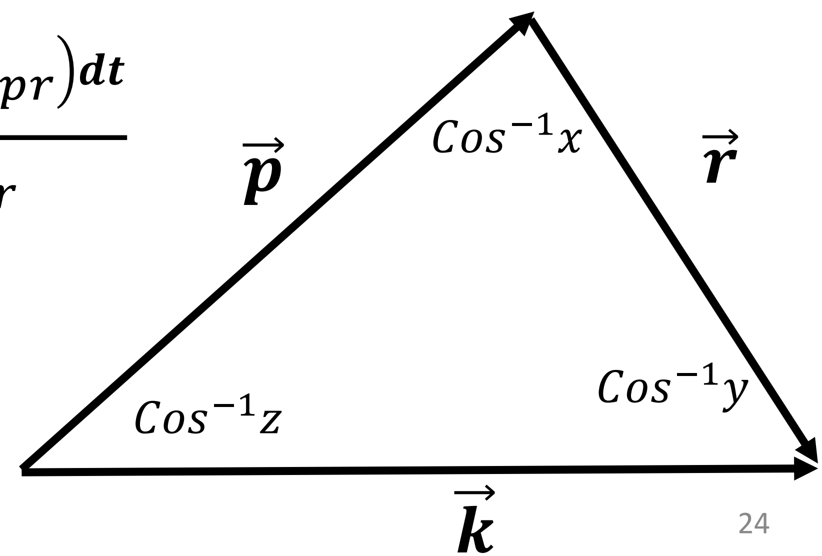
$$-\int dp\, dr\theta_{kpr}^{\eta\eta\nu}(t)\frac{r^2}{p}(y+xz)E_V(p)E_M(k)$$

$$+\int dpdr\theta_{kpr}^{\eta\eta\nu}(t)\frac{k^3}{pr}(1+xyz)E_V(p)E_M(r)$$

$$\theta_{kpr}^{\nu\eta\eta}(t) = \frac{\mathbf{1}-\boldsymbol{e}^{-\int\left(\nu p^2+\eta k^2+\eta r^2+\mu_{kpr}\right)\boldsymbol{dt}}}{\nu p^2+\eta k^2+\eta r^2+\mu_{kpr}}$$

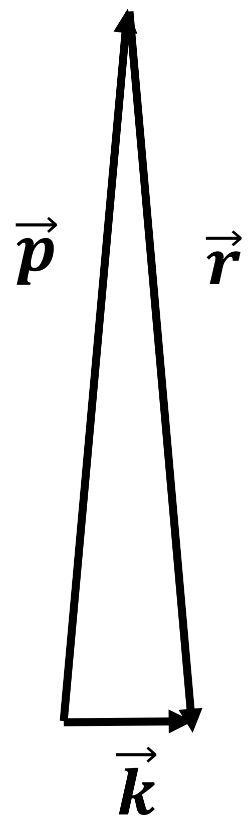
(triad relaxation time)

k = p + r



• Analytic solution II (EDQNM)

p ~ r ≫ k



$x{\sim}1, y{\sim}z{\sim}0$

$\eta \rightarrow \mathbf{0}$, p ~ r ≫ k = 1

$$\theta_{kpr}^{\nu\eta\eta}(t) = \frac{\mathbf{1-e^{-\int\left(\nu p^2+\eta k^2+\eta r^2+\mu_{kpr}\right)dt}}}{\nu p^2+\eta k^2+\eta r^2+\mu_{kpr}} \rightarrow \frac{\mathbf{1}}{\nu p^2}$$

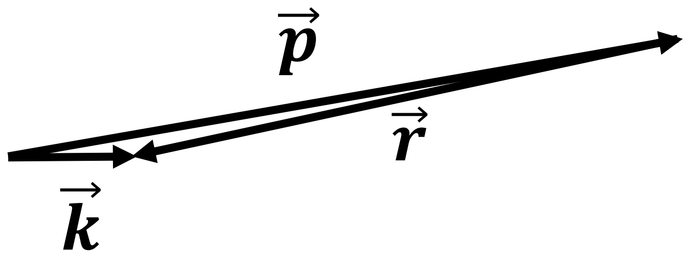
$$\frac{\partial E_M(k,t)}{\partial t} = -\int dpdr\,\theta_{kpr}^{\eta\eta\nu}(t)\frac{p^2}{r}z(1-x^2)E_M(r)E_M(k)$$
$$-\int dp\,dr\,\theta_{kpr}^{\eta\eta\nu}(t)\frac{r^2}{p}(y+xz)E_V(p)E_M(k)$$
$$+\int dpdr\,\theta_{kpr}^{\eta\eta\nu}(t)\frac{k^3}{pr}(1+xyz)E_V(p)E_M(r)$$

$$\frac{\partial E_M(k,t)}{\partial t} \sim \frac{\mathbf{1}}{\nu p^4}E_V(p)E_M(p)$$

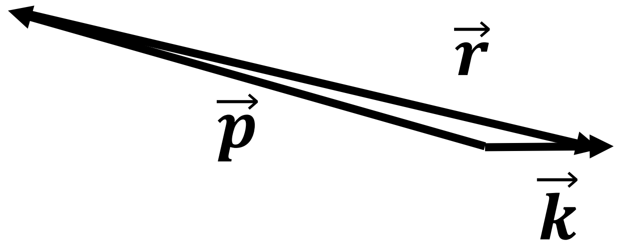
$$\rightarrow E_M(k,t) \sim \int \frac{\mathbf{1}}{\nu p^4}E_V(p)E_M(r)\,\mathrm{d}\tau\ (\mathrm{k}=1)$$

- **Analytic solution II (EDQNM)**

p ~ r ≫ k



$x \sim z \sim 1, y \sim -1$

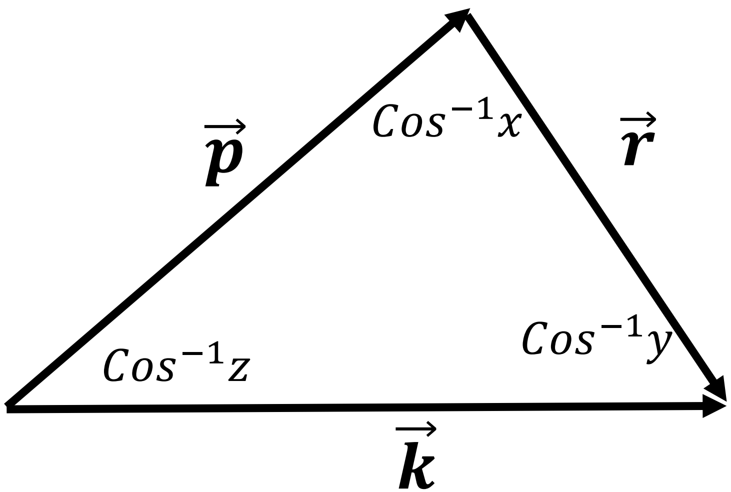


$x \sim y \sim 1, z \sim -1$

$$\frac{\partial E_M(k,t)}{\partial t} = -\int dp dr \theta_{kpr}^{\eta\eta\nu}(t) \frac{p^2}{r} z(1 - x^2) E_M(r) E_M(k)$$
$$- \int dp\, dr \theta_{kpr}^{\eta\eta\nu}(t) \frac{r^2}{p} (y + xz) E_V(p) E_M(k)$$
$$+ \int dp dr \theta_{kpr}^{\eta\eta\nu}(t) \frac{k^3}{pr} (1 + xyz) E_V(p) E_M(r)$$
$$\sim \mathbf{0}$$

→ No inverse transfer from $E_V(p)E_M(r)$

- **Analytic solution II (EDQNM)**



$$p \sim r \sim k, x \sim y \sim z \sim \frac{1}{2}$$

$$E_M(k,t) \sim \frac{Q}{P} - \frac{Q}{P^2}\,\mathrm{e}^{-\int P\,\mathrm{d}t}$$

$$\mathrm{P} = \mathrm{f}\,(E_M(p,t), E_V(r,t))$$

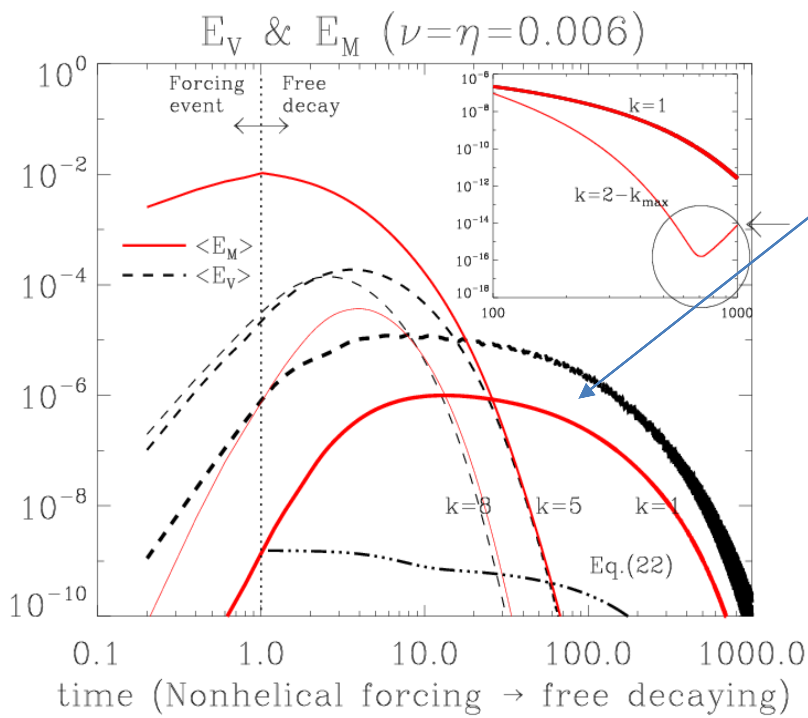
$$\mathrm{Q} = \mathrm{g}\,(E_M(p,t), E_V(r,t))$$

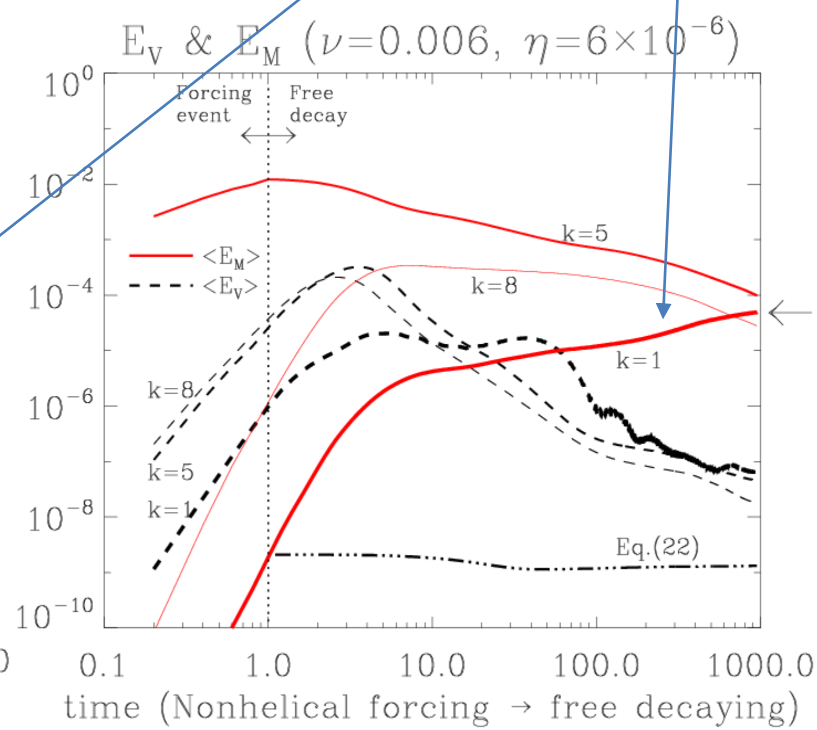
$$E_{M,\,tot}(k,t) \;\geq \int \frac{\mathbf{1}}{\nu p^4} E_V(p) E_M(r)\;\mathrm{d}\tau + \frac{Q}{P} - \frac{Q}{P^2}\,\mathrm{e}^{-\int P\,\mathrm{d}t}$$

- **Analytic solution II (EDQNM)**

If $\nu = \eta \ \ (p_{r_M} = 1, \text{Son1999})$

$$\theta^{\nu\eta\eta}_{kpr}(t) = \frac{1 - e^{-\int\left(\nu p^2 + \eta k^2 + \eta r^2 + \mu_{kpr}\right)dt}}{\nu p^2 + \eta k^2 + \eta r^2 + \mu_{kpr}} \rightarrow \frac{1}{\nu(2p^2 + r^2)} \left(< \frac{1}{\nu p^2}\right)$$





▪ Summary

1. **Inverse transfer of E_M in a decaying MHD system**

2. **Helical $\mathrm{E_M}$: α effect**

3. **Nonhelical $\mathrm{E_M}$:**
   - Statistical method: conserved system variables
   - Scaling invariant method:
   $\mathrm{E_M}(k, t) = k^{-1-2h}\psi_{V,M}(k^{1-h}t),\ k^{1-h}t = const.$

4. **Field structure from magnetic induction equation**
   $-\mathbf{u}\cdot\nabla\mathbf{B},\ \mathbf{B}\cdot\nabla\mathbf{u}$

5. **EDQNM**
   $p, r \longrightarrow k\ (p + r = k),\ \theta_{kpr}^{\nu\eta\eta}(t), x, y, z \Rightarrow \mathrm{E_M}(k, t)$