

# Modelling the Line-of-Sight Projection and Filtering-Induced Leakage in Time-Distance Helioseismology

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In current approaches to time-distance helioseismology, the line-of-sight projection effect on the travel-times is not fully taken into account. Furthermore, filtering of full-disc data induces leakage due to the projection onto the CCD, which has so far not been accounted for. We develop a theoretical approach to consider these effects when computing sensitivity functions. As the formulas obtained do not seem to give results for spherical Born approximation sensitivity functions in a reasonable computation time, we develop tests to estimate the strength of the effects.

The theoretical development presented in this Bachelor thesis is a contribution to the current scientific debate on adequately correcting the center-to-limb variation of observed helioseismic travel-times (Zhao et al, 2012; Kholikov et al., 2014). With the help of this development, time-distance helioseismology can be extended to distances of larger heliocentric angles, i.e. deeper probing, taking the effects of the line-of-sight-projection into account.

Using the Born approximation, Böning (2013) computed sensitivity functions in spherical geometry which model the effect of deep solar interior flows on helioseismic travel-times. Our work extends the sensitivity functions of Böning (2013) with an implementation of the line-of-sight operator:

$$C_0(\mathbf{r}_1, \mathbf{r}_2, \omega) = S(\omega) \sum_l f(l, \omega)^2 \left( \sum_{n\hat{n}} \frac{R_{ln}(r_s) R_{l\hat{n}}(r_s) R_{ln}(r_1) R_{l\hat{n}}(r_2)}{(\sigma_{ln}^{2*} - \omega^2)(\sigma_{l\hat{n}}^2 - \omega^2)} \right) \frac{2l+1}{4\pi} P_l(\cos \Delta_{1,2}) \quad (1)$$

↓

$$C_0(\mathbf{r}_1, \mathbf{r}_2, \omega) = S(\omega) \sum_l \mathbf{1} \left( \sum_{n\hat{n}} \frac{R_{ln}(r_s) R_{l\hat{n}}(r_s) R_{ln}(r_1) R_{l\hat{n}}(r_2)}{(\sigma_{ln}^{2*} - \omega^2)(\sigma_{l\hat{n}}^2 - \omega^2)} l_1^j l_2^k D_{j,ln}^{r,1} D_{k,l\hat{n}}^{r,2} \right) \left[ \frac{2l+1}{4\pi} P_l(\cos \Delta_{1,2}) \right] \quad (2)$$

↓

$$C_0(\mathbf{r}_1, \mathbf{r}_2, \omega) = S(\omega) \sum_{lm, l\hat{m}} f(l, \omega) f(\hat{l}, \omega) \left( \sum_{\hat{l}\hat{m}, n\hat{n}} \frac{R_{\hat{l}n}(r_s) R_{l\hat{n}}(r_s) R_{\hat{l}n}(r_1) R_{l\hat{n}}(r_2)}{(\sigma_{\hat{l}n}^2 - \omega^2)^* (\sigma_{l\hat{n}}^2 - \omega^2)} \mathbb{L}_{\hat{l}\hat{m}n}^{lm} \mathbb{L}_{l\hat{m}\hat{n}}^{l\hat{m}} \right) Y_{lm,1}^* Y_{l\hat{m},2} \quad (3)$$

Equation (1) shows the zero order cross-correlation in the case of Böning (2013). The changes to the zero order model, which were necessary to incorporate the line-of-sight projection without and with filtering ( $f(l, \omega)$ ), are marked red in Equations (2) and (3), respectively. The line-of-sight projection is incorporated in Equation (2) directly via  $l_1^j l_2^k$  and the differential operators  $D_{j,ln}^{r,1} D_{k,l\hat{n}}^{r,2}$  and in Equation (3) via the multiplication with leakage matrices  $\mathbb{L}_b^a$ .

## References

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