Supression of kinematic dynamo by shear flow

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Abstract

A kinematic dynamo action is studied in a spherical shell with a small scale prescribed velocity field. Velocity field is considered to be axisymmetric and nature of flow is steady and strongly helical. Flow is chosen in such a way so that we have dipole/quaderupole decoupling for magnetic field B. The effects of large scale shear on dynamo is studied for different m modes for large Reynolds number R_m. Specifically, we have studied the flow in the presence of radial and latitudinal shears in azimuthal direction, respectively. We study the behaviour of growth rate and structures of magnetic field B. We observe that the structure of the magnetic field is distorted and the growth rate of the magnetic field is found to decrease for large R_m, which indicates that the dynamo is suppressed in the presence of shear.

1. Introduction

We solve induction equation $\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u_t} \times \mathbf{B}) + R_m^{-1}\nabla^2 \mathbf{B}$ in spherical shell in the prescribed axisymmetric velocity field $\mathbf{u_t} = v_t \hat{\mathbf{e}}_\phi + \nabla \times (\psi_s \hat{\mathbf{e}}_\phi)$. Here v_t represents the zonal flow and ψ_s represents the meridional circulation. Total zonal flow is

$$v_t = v_s + r \sin\theta v_l \qquad (1)$$

where, v_s is the small scale velocity field similar to [1] and v_l represents large scale shear in the azimuthal direction given as

$$v_s = \sin\left(\frac{(r-r_i)}{(r-r_o)} N_r \pi\right) \sin\theta \cos(N_\theta \theta), \qquad v_l = \begin{cases} (r-0.75) & \text{is the radial shear} \\ (\cos^2\theta - 0.5) & \text{is the lattitudinal shear} \end{cases} \qquad (2)$$

ψ_s is the small scale meridional circulation

$$\psi_s = \frac{1}{N_\theta} r \cos\theta \sin\left(\frac{(r-r_i)}{(r-r_o)} N_r \pi\right) \sin\theta \cos(N_\theta \theta) \qquad (3)$$

Here, r_i and r_o are the inner boundary and outer boundary of the sphere, respectively. N_r and N_θ are the number of cells in r and θ, respectively. We study the two flows by varying $R_m \in [100, 10000]$ for different fixed m values. Two forms of flows are

Flow1	Flow2
$\mathbf{u_t} = (v_s + r\sin\theta(r-0.75))\hat{\mathbf{e}}_\phi + \nabla \times (\psi_s \hat{\mathbf{e}}_\phi)$	$\mathbf{u_t} = (v_s + r\sin\theta(\cos^2\theta - 0.5))\hat{\mathbf{e}}_\phi + \nabla \times (\psi_s \hat{\mathbf{e}}_\phi)$

We use pseudo spectral spherical code for numerical solutions by decomposing magnetic field into its poloidal and toroidal components which are further expanded in their spectral coefficients such as Legendre function in θ and Fourier expansion in ϕ and radial structures are expanded in Chebyshev polynomial [2].

2. Results for Flow1

- Growth rates of magnetic field are plotted as function of R_m for different m modes in Fig.1.
- Growth rates are found to reach a maximum value and decrease for large R_m indicating a slow dynamo in the presence of radial shear.

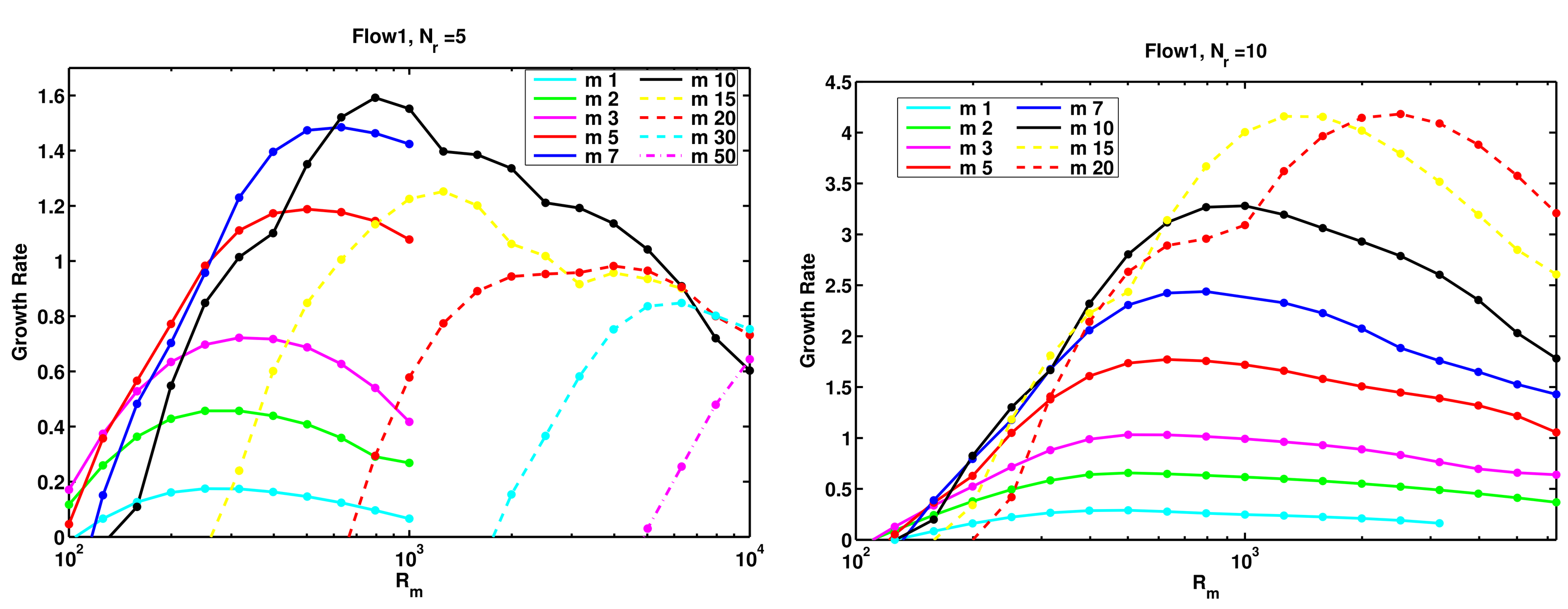

Figure 1: *Left: Growth rate as a function of R_m for Flow1 for $(N_r, N_\theta) = (5, 20)$. Right: Growth rate as a function of R_m for Flow1 for $(N_r, N_\theta) = (10, 40)$*

- For fixed $R_m = 1000$ and $m = 10$, contour plots of B_ϕ for $(N_r, N_\theta) = (5, 20)$ (Fig.2 left) and for $(N_r, N_\theta) = (10, 40)$ (Fig.2 middle) show that the structure of magnetic field is distorted and is concentrated in the middle near the inner boundary of the shell.
- Magnetic energy is plotted as a function of L for $(N_r, N_\theta) = (5, 20)$ (red) and $(N_r, N_\theta) = (10, 40)$ (blue), respectively. Magnetic energy decreases with L in both the cases.

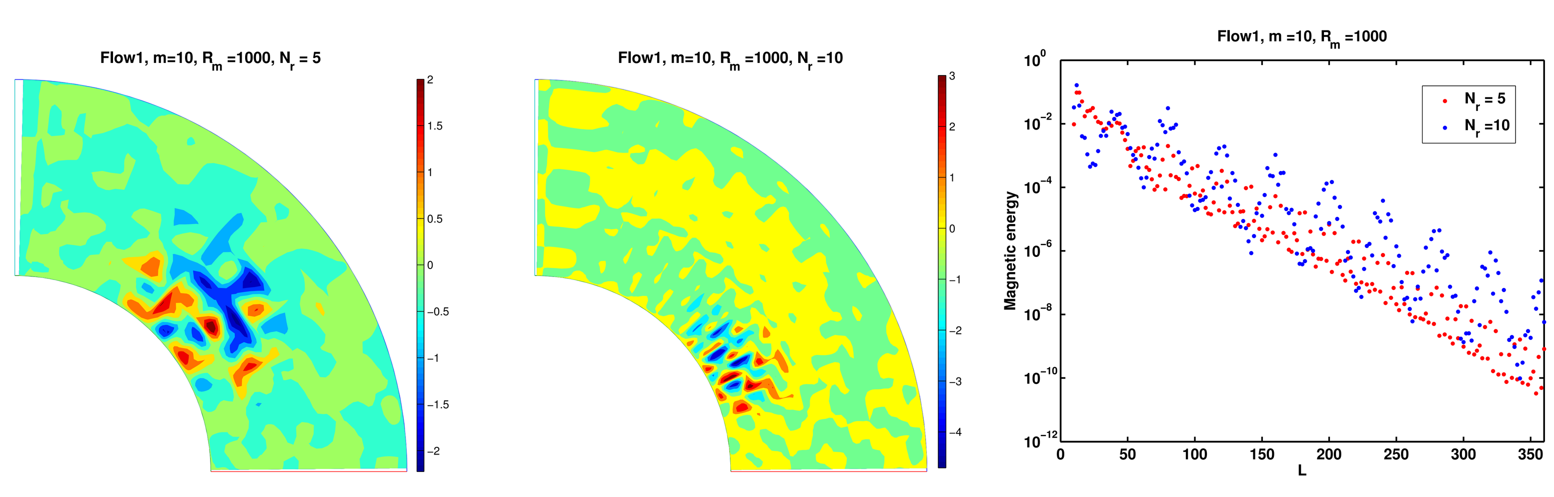

Figure 2: *Left-Middle: Contour plots of B_ϕ, Right: Behavior of magnetic energy with L*

3. Results for Flow2

- Growth rates are found to decrease for large R_m for all values of m (Fig.3), and these results are almost similar to Flow1.

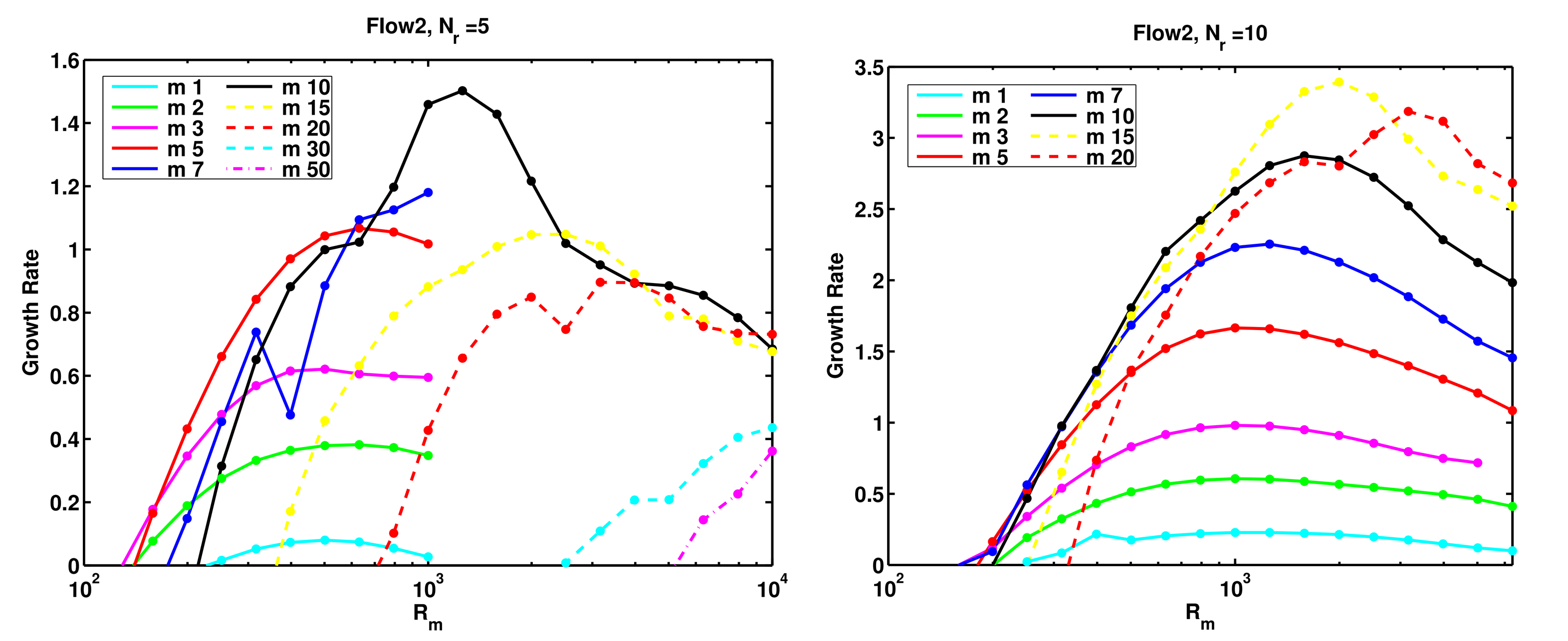

Figure 3: *Left: Growth rate as a function of R_m for Flow2 for $(N_r, N_\theta) = (5, 20)$. Right: Growth rate as a function of R_m for Flow2 for $(N_r, N_\theta) = (10, 40)$*

- The contour plots for B_ϕ for $(N_r, N_\theta) = (5, 20)$ (Fig.4 left) and for $(N_r, N_\theta) = (10, 40)$ (Fig.4 middle) show that magnetic field is concentrated in the middle (away from edges) near the inner boundary for the most excited modes.
- Magnetic energy is found to decrease with L for $(N_r, N_\theta) = (5, 20)$ (red) and $(N_r, N_\theta) = (10, 40)$ (blue), respectively (see Fig.4: Right). Magnetic energy is smoother than in the case of Flow1.

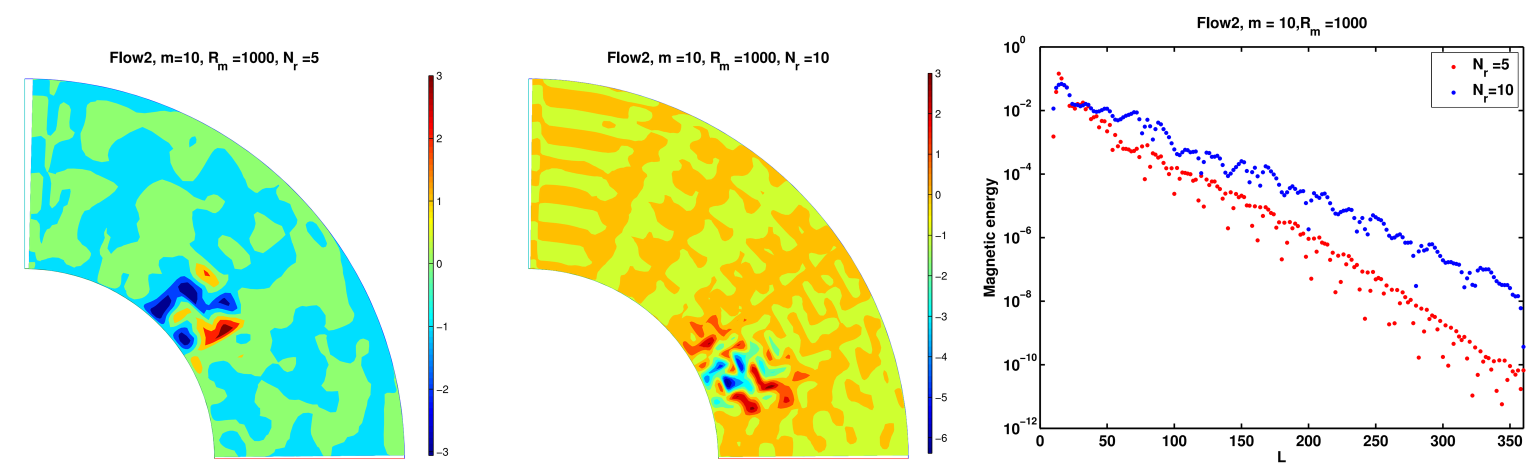

Figure 4: *Left-Middle: Contour plots of B_ϕ, Right: Behavior of magnetic energy with L*

4. Behaviour of growth rates with shear parameter for Flow1 & Flow2

- Growth rate is found to decrease as we increase the relative amplitude of shear owing to the fact that shear suppresses the growth rate and slows down the dynamo action.

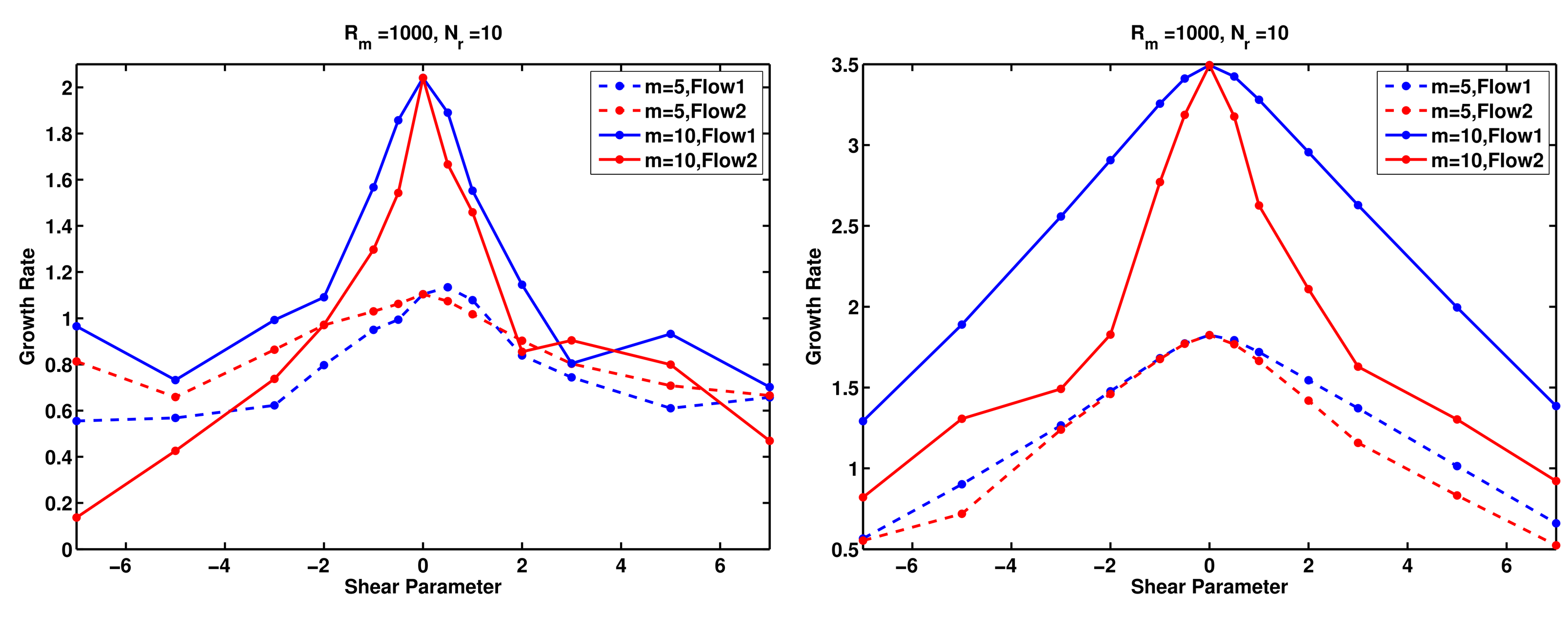

Figure 5: *Left-Right: Growth rates as a function of shear parameter for Flow1 and Flow2*

5. Conclusions

- Large scale shear in azimuthal direction inhibits the growth of magnetic field and slows down the dynamo action in case of small scale steady flow.
- Evidence for a similar dynamo quenching by shear flow in meridional direction has been found.
- These results are in agreement with [3]

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