Dynamical model for spindown of solar type stars

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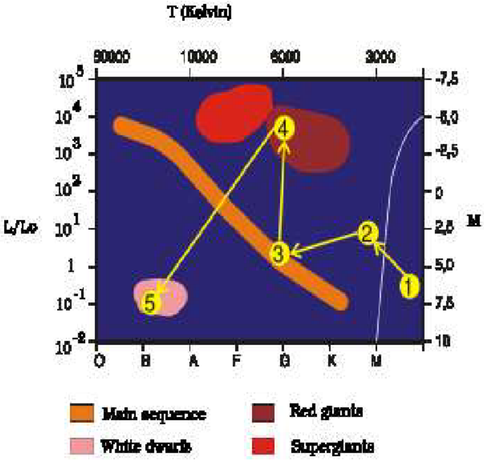
Outline

Evolution of a Star

1. Collapsing cloud of gas
2. Contracting protostar
3. Main Sequence
4. Expansion to red giant
5. White dwarf



Observations

- Observationally, cycle period depends upon the stellar rotation period P_{rot} as $P_{cyc} \propto P_{rot}^{n}$, $n = 1.25 \pm 0.5$[Noyes et. al;1984].

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- Exponent n = 0.80 for fast rotators and 1.15 for slow rotators [Saar & Brandenburg; 1998, 1999, Charbonneau & Saar; 2001, Saar; 2002]

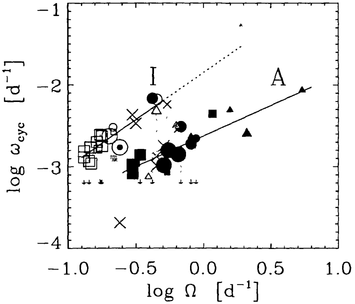
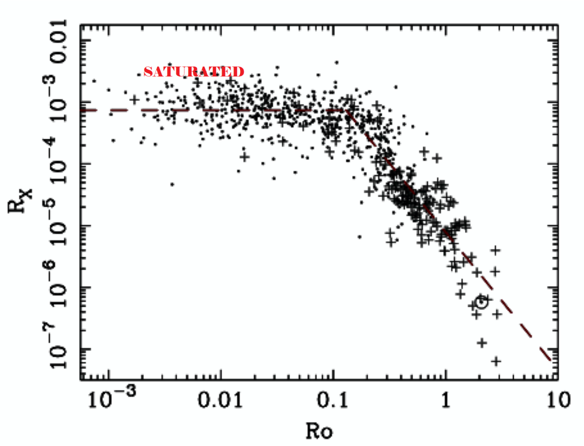


Figure: *Two branches of stars.*

Courtsy: Saar,2002

- Saturation of Magnetic activity for high rotation rate



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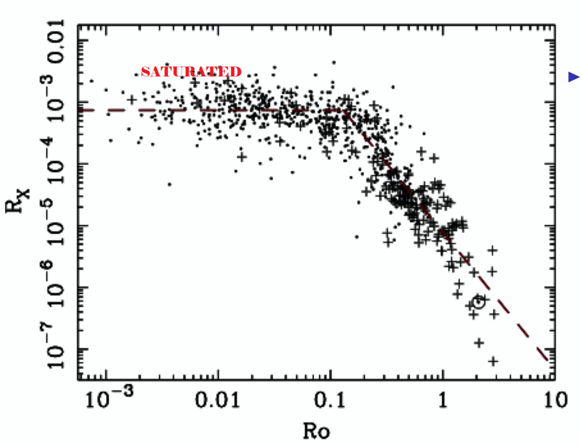
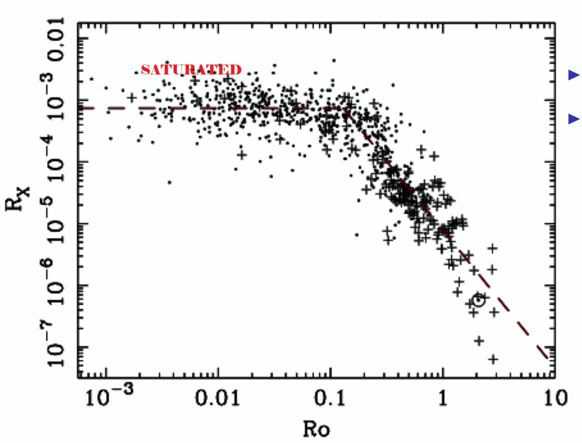


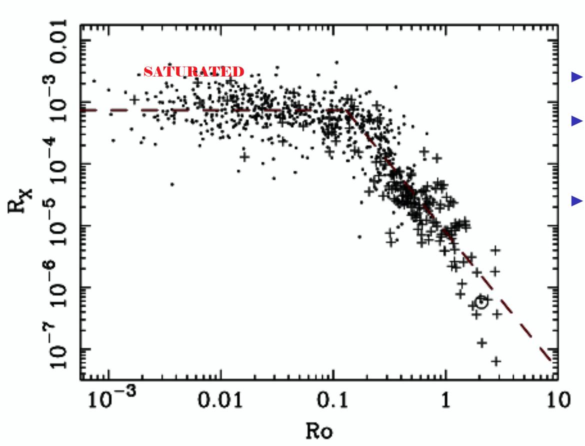
- saturated region

- Saturation of Magnetic activity for high rotation rate



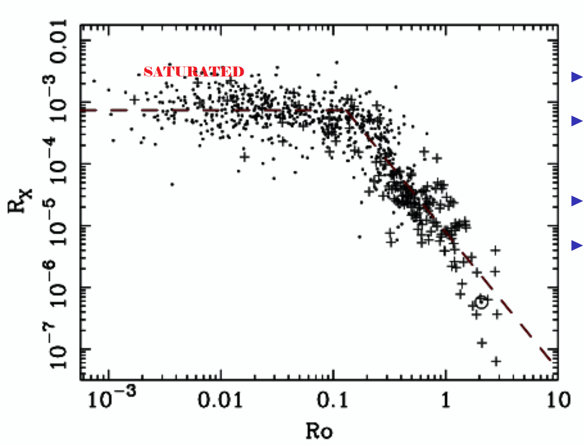
- saturated region
- exponential spin down,i.e., $\Omega \propto e^{-at}$

- Saturation of Magnetic activity for high rotation rate



- saturated region
- exponential spin down,i.e., $\Omega \propto e^{-at}$
- linear growth

- Saturation of Magnetic activity for high rotation rate



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- saturated region
- exponential spin down,i.e., $\Omega \propto e^{-at}$
- linear growth
- Power law spin down, i.e., $\Omega \propto t^{\frac{-1}{2}}$ (*Skumanich, 1972):*)

Previous Work and Spin Evolution Models

- 1D evolution equation for the rotation of the core + envelope subject to momentum loss by stellar wind (Keppens et al. 1991, Leprovost & Kim,2010)

Model

- We propose a self-contained model for the evolution of rotation and magnetic field.
- We extend our previous work (Sood & Kim; 2013,2014, Cattaneo,Jones & Weiss;1983)in which a simple model is constructed by taking B $= (0, B(t)e^{ikx}, ikA(t)e^{ikx})$ and Lorentz force that generates the differential rotation $\frac{\partial W}{\partial z} = w_0 + w(t)exp(2ikx)$ then dimensionless system in the presence of equation of evolution is given as:

$$\partial_t A = \frac{2\Omega^2 B}{1+\kappa_1(|B|^2)} - [1+\lambda_1(|B|^2)]A, \quad (1)$$

$$\partial_t B = i(1+w_0)A - \frac{1}{2} i A^* w - [1+\lambda_2(|B|^2)]B, \quad (2)$$

$$\partial_t w_0 = \frac{1}{2} i(A^* B - AB^*) - \nu_0 w_0, \quad (3)$$

$$\partial_t w = -iAB - \nu w, \quad (4)$$

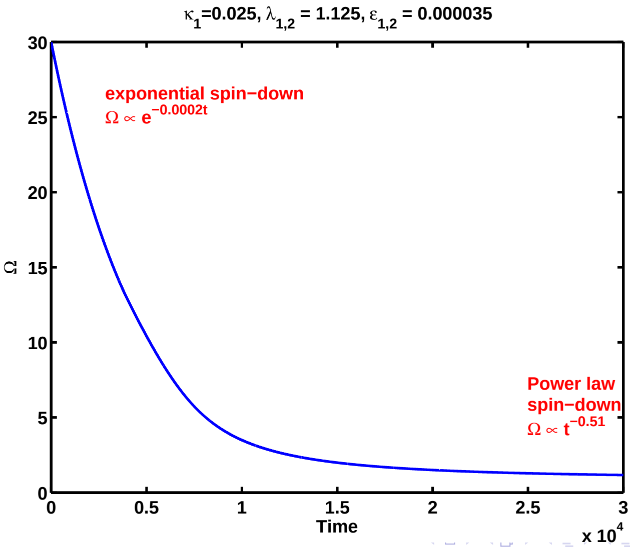
$$\partial_t \Omega = -\varepsilon_1 |B|^2 \Omega - \varepsilon_2 \frac{|A|^2}{\Omega^2} \Omega \quad (5)$$

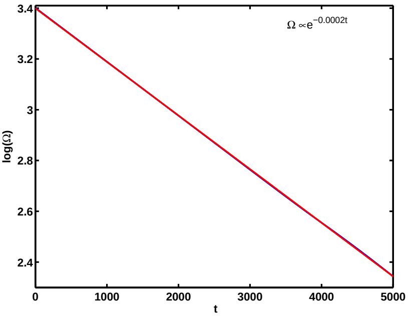
- A is poloidal field, B is toroidal field, w_0 is mean differential rotation, w is fluctuating differential rotation and Ω is the rotation rate.
- κ_1, λ_1,λ_2 are positive. ε_1,ε_2 represent loss of angular momentum.

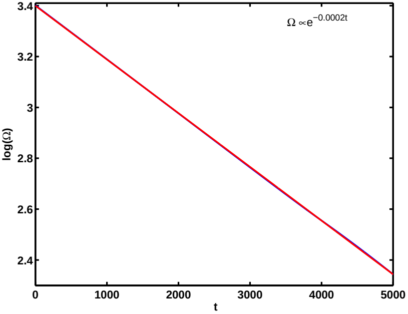
Results

- We have solved the system by taking $\kappa_1 = 0.0125, \lambda_1 = \lambda_2 = 1.125$ and $\varepsilon_1 = \varepsilon_2 = 0.000035$ whereas $v_0 = 35$ and $v = 0.5$

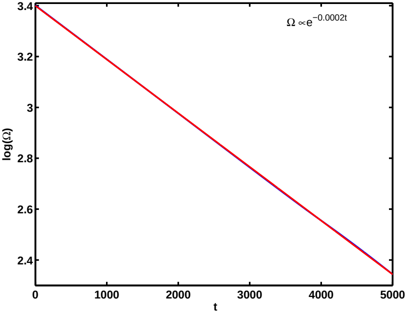
- Relationship of rotation rate Ω with time




Ω ∝e−0.0002t
log(Ω)
3.4
3.2
3
2.8
2.6
2.4
0
1000
2000
3000
4000
5000
t

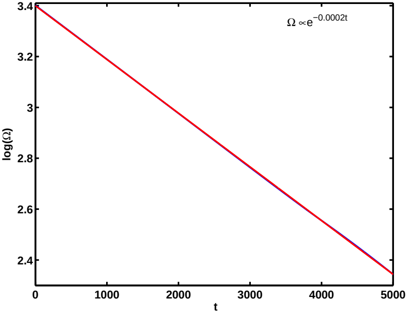


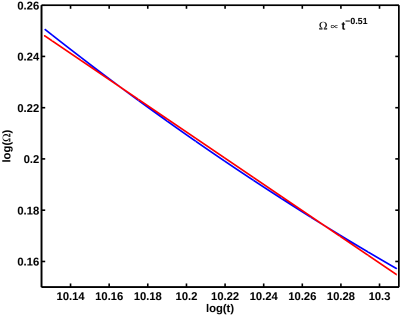
Exponential spin down for fast rotators, $\Omega \propto e^{-0.0002t}$



Exponential spin down for fast rotators, $\Omega \propto e^{-0.0002t}$

Consistent with observations





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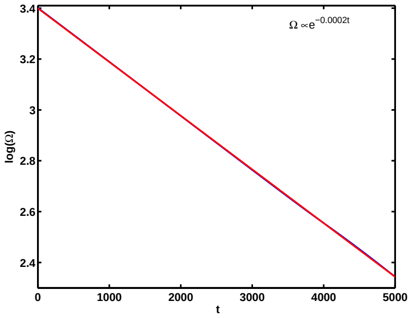

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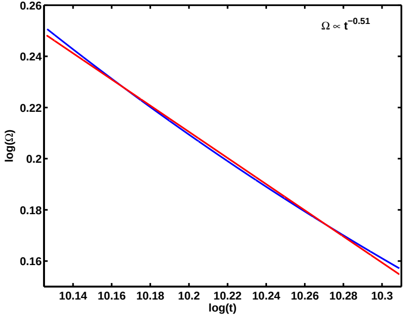
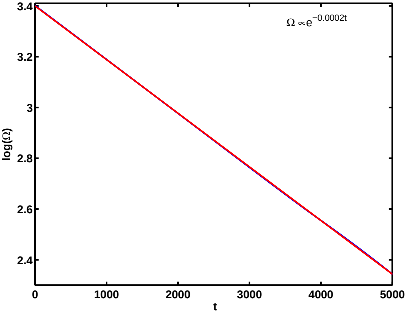


Power law spin down, $\Omega \propto t^{-0.51}$



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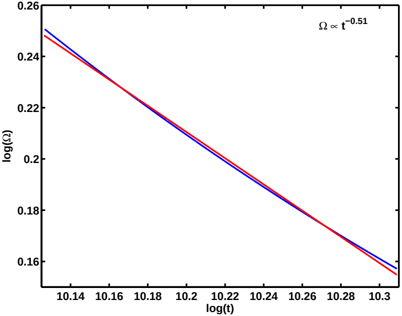
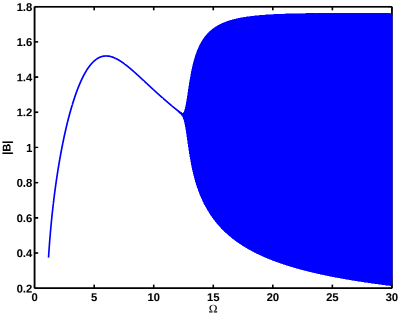


Power law spin down, $\Omega \propto t^{-0.51}$

consistent with observations

- Behavior of magnetic field strength with rotation rate



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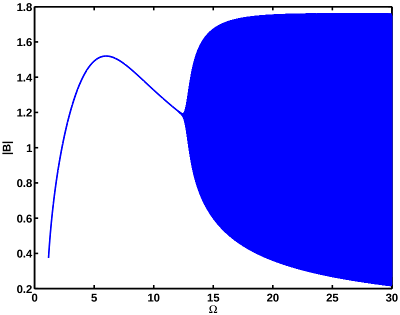
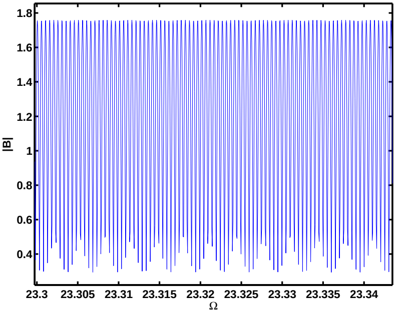
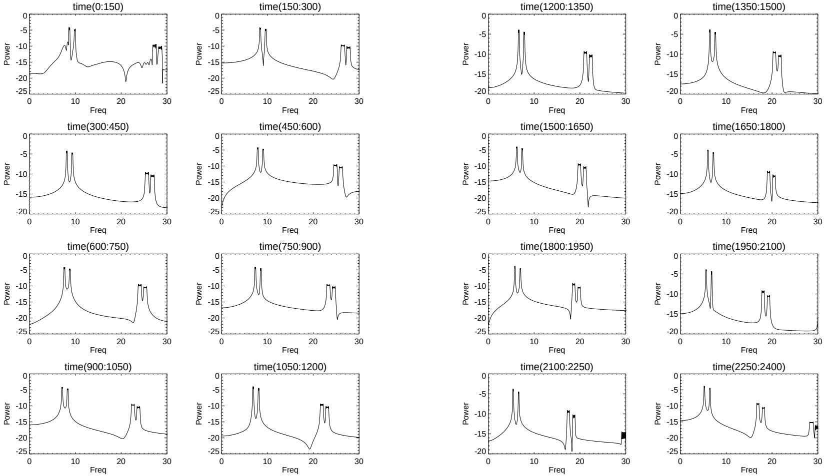
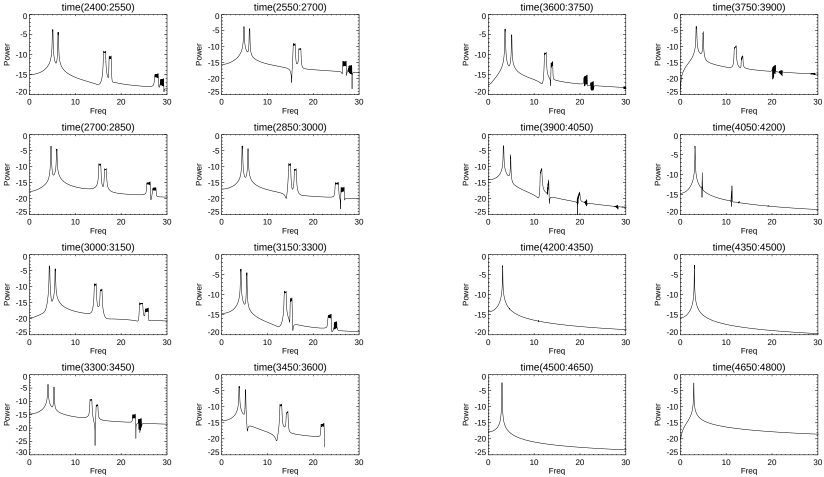


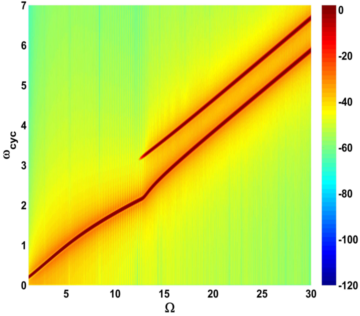


Frequency spectra for $\Omega\epsilon[30, 12.5]$




time(2400:2550)
time(2550:2700)
time(3600:3750)
time(3750:3900)
time(2700:2850)
time(2850:3000)
time(3900:4050)
time(4050:4200)
time(3000:3150)
time(3150:3300)
time(4200:4350)
time(4350:4500)
time(3300:3450)
time(3450:3600)
time(4500:4650)
time(4650:4800)
Power
Freq

Spectrogram of time series of magnetic field B



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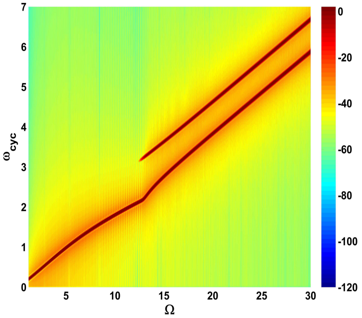

- $\omega_{cyc} \propto \Omega^{1.16}$ for stars with in the range $25.6 \leq P_{rot} < 23$.

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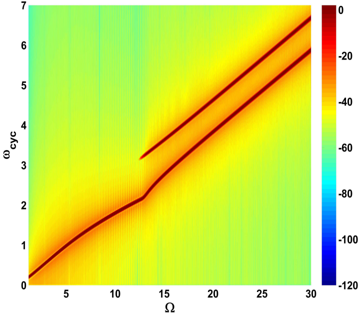
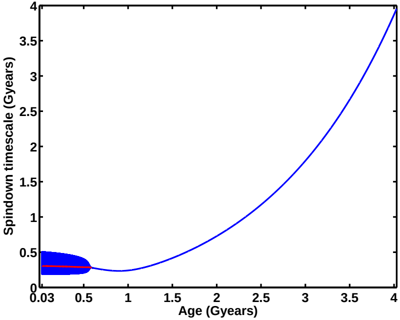
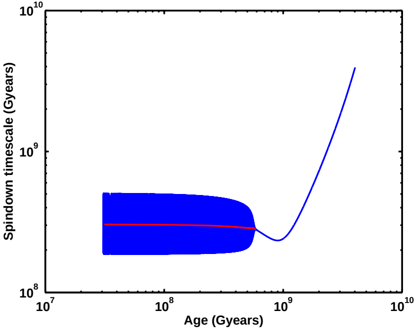


- $\omega_{cyc} \propto \Omega^{1.16}$ for stars with in the range $25.6 \leq P_{rot} < 23$.
- $\omega_{cyc} \propto \Omega^{0.85}$ for stars with in the range $2 \leq P_{rot} \leq 1$.

Here, we show timescale as function of t in linear and log scale.





Conclusions

- For higher rotation rate exponential law $\Omega \propto e^{-0.0002t}$
- For slow rotation rate power law $\Omega \propto t^{-0.51}$
- $|B|$ increases with Ω for slow rotation regime but saturates for hight rotation regime.
- Frequency analysis shows only one peak for slow rotation rate but for higher rotation rate we found two peaks of frequency of maximum intensity.
- for fast rotators, $\omega_{cyc} \propto \Omega^{0.85}$ and for slow rotators, $\omega_{cyc} \propto \Omega^{1.16}$
- A region is found for $\Omega \in [5.8, 12.5]$ or time $\in [4200, 7441]$ where we have decreasing behavior of $|\mathrm{B}|$ and timescale.

- Sood & Kim (2013,2014) have already explained the role of nonlinear interactions in the self-regulation of dynamo.
- In present work, we emphasize how nonlinear transport coefficient could be important in the spin-down of star.