Measurement of the amplitude of the solar cross-covariance function

Kaori Nagashima[1], Laurent Gizon[1,2],

Aaron C. Birch[1], Damien Fournier[2]

1.Max-Planck-Institut für Sonnensystemforschung

2. Georg-August-Universität Göttingen

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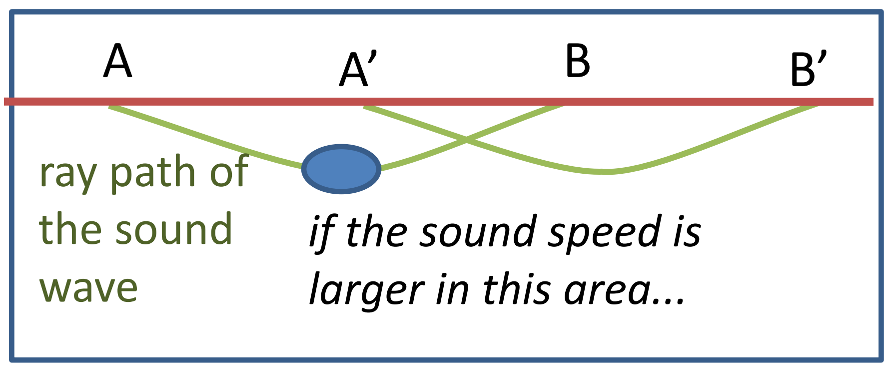
Introduction 1.
Local helioseismology
Time-Distance Analysis

Measure travel times of the waves — How?

=> Detect local sound speed anomaly and/or flows

Example 1



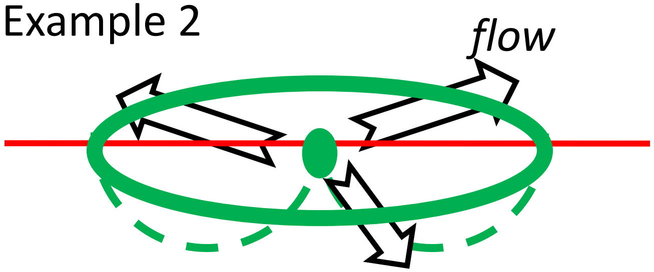
<u>If there is sound speed anomaly below the surface</u>

Travel time from A to B

∧

Travel time from A’ to B’

Example 2



<u>If there is flow below the surface</u>

When it is a divergent flow

outward travel time < inward travel time

Introduction 2.
Time-Distance Analysis

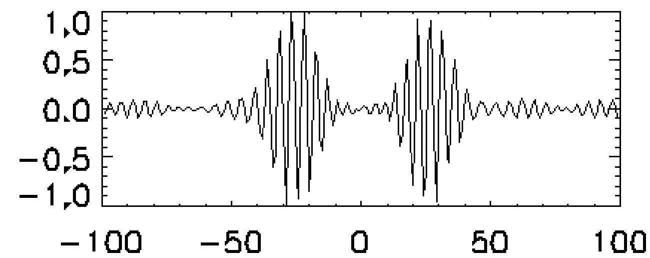
- What do we actually do to measure the travel time?

1. Calculate the cross-covariance function of the wavefield $v(\vec{x}.t)$

$$C(\vec{\Delta},\tau;\vec{x}) = \int v(\vec{x},t)v(\vec{x}+\vec{\Delta},t+\tau)dt$$

Δ, τ: Distance & time shift between the two points

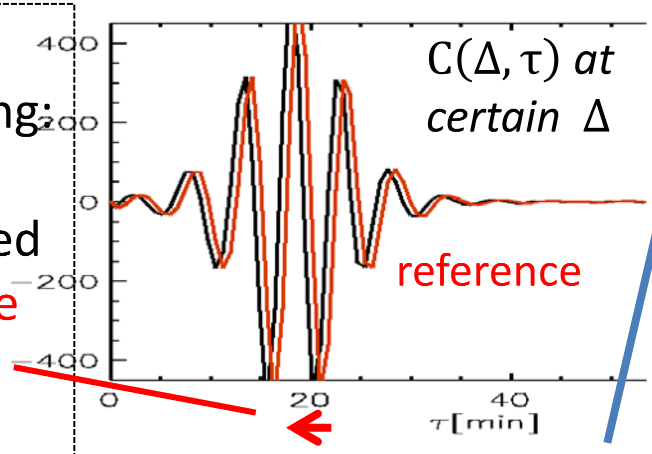
Example of $C(\tau)$ at a fixed Δ



(1'. Average *C* over some area to increase S/N)

2. Calculate the travel time using the cross-covariance function

e.g. One-parameter fitting: Measure the "shift" compared to the reference (Gizon & Birch, 2002, 2004)



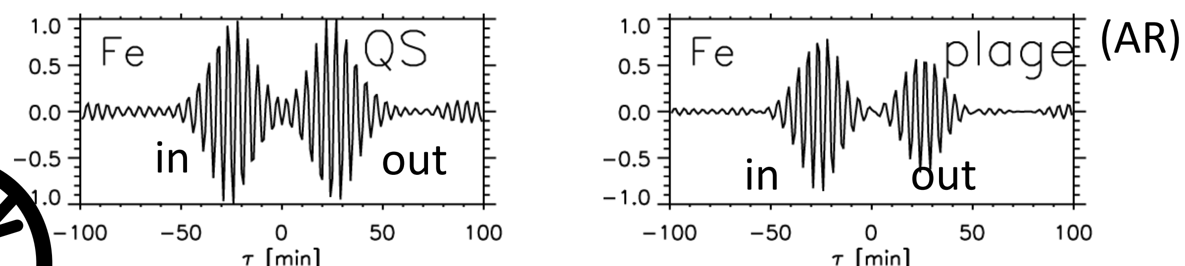
e.g. 5-paramter fitting: a Gabor wavelet function (e.g., Kosovichev & Duvall 1996) .

$$C(\tau) = a\cos\left(\omega\left(\tau - \tau_{\mathrm{ph}}\right)\right)e^{-\delta\omega^2\left(\tau-\tau_{\mathrm{gr}}\right)^2}$$

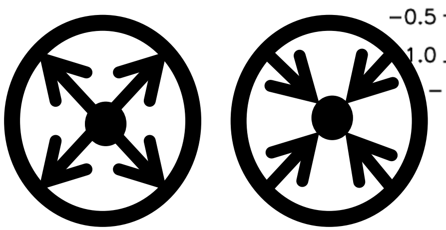
(phase) travel time is used for inversion

Introduction 3. Motivation

- So far we extract only "travel time" from cross-covariance function to infer the Solar interior.
- But ***the cross-covariance function has much more information other than the travel time***.
  - For example, the center-to-annulus cross-covariance functions in Quiet Sun and AR have clear difference in amplitude/shape.



Hinode Fe I Dopplergram
(Nagashima 2010 PhD thesis)



Parameter(s) other than travel time?

- Including parameters other than travel time in time-distance helioseismology analyses might improve the analyses.
- The travel time is the primary parameter of the cross-covariance function, but what is the second one?
  - How about Amplitude? (-> excitation/attenuation?)

Here we try to fit a reference function with two parameters: travel time and amplitude

Formulation

Our definition of Amplitude and Travel time of $C(t)$:

a and τ that minimize

$$X(a,\tau) = \int dt\, \boxed{f(t)} \left(C(t) - \boldsymbol{a}\, \boxed{C^{\mathrm{ref}}(t-\boldsymbol{\tau})} \right)^2 .$$

window function

reference function

cf. travel time only (one-parameter fitting)

Definition of the travel time (Gizon & Birch 2002):

τ that minimizes

$$X(\tau) = \int dt\, f(t) \left(C(t) - C^{\mathrm{ref}}(t-\boldsymbol{\tau}) \right)^2$$

Formulation

Our definition of Amplitude and Travel time of $C(t)$:

a and τ that minimize

$$X(a,\tau) = \int dt\, f(t) \left(C(t) - \boldsymbol{a}\, C^{\mathrm{ref}}(t - \boldsymbol{\tau}) \right)^2 .$$

This is two-parameter fit, but we can choose the window function $f(t)$ so that we have no cross-terms :

$\boxed{\delta\tau = \int dt\, W_\tau(t)\, \delta C(t)}$, where $W_\tau(t) = \dfrac{-f(t)\dot{C_0}(t)}{\left[\int dt'\, f(t')\, \dot{C_0}^2(t')\right]}$

(This weight function is same as one-parameter fitting by Gizon & Birch 2002)

$\boxed{\delta a = \int dt\, W_a(t)\, \delta C(t)}$,where $W_a(t) = \dfrac{f(t)C_0(t)}{\left[\int dt'\, f(t')\, C_0{}^2(t')\right]}$

* $\delta C(t) = C(t) - C_0(t)$, and we choose $C_0(t) = C^{\mathrm{ref}}(t - \tau_0)$

Let’s see the observation data!

Example 1: QS (supergranular pattern)

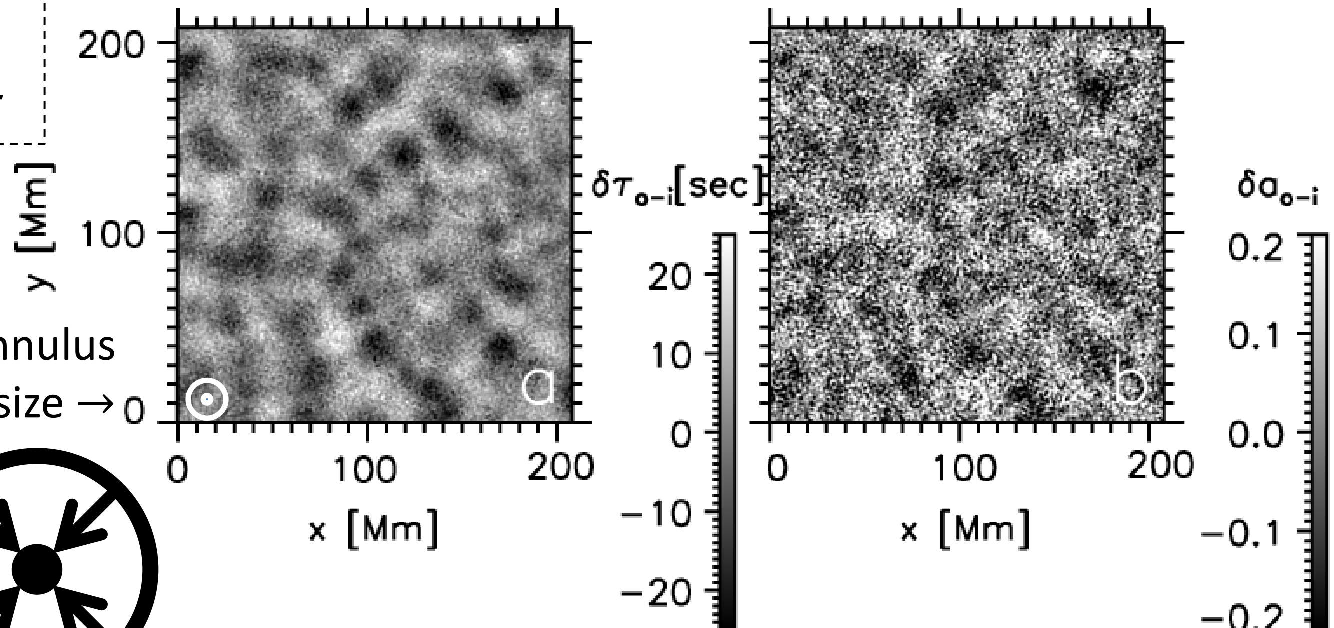
Example 2: around the AR

Example of amplitude measurement: *Supergranular pattern*

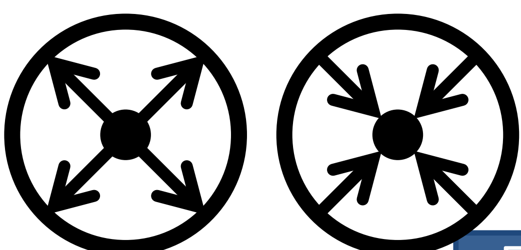
HMI 9hr Doppler,
p1-ridge filter used.
Δ =10Mm,
Center-to-Annulus Cross-covariance function=> $\boldsymbol{a}$ **and** $\boldsymbol{\tau}$

Out-In Travel-time difference

Out-In Amplitude difference



Actual annulus size →

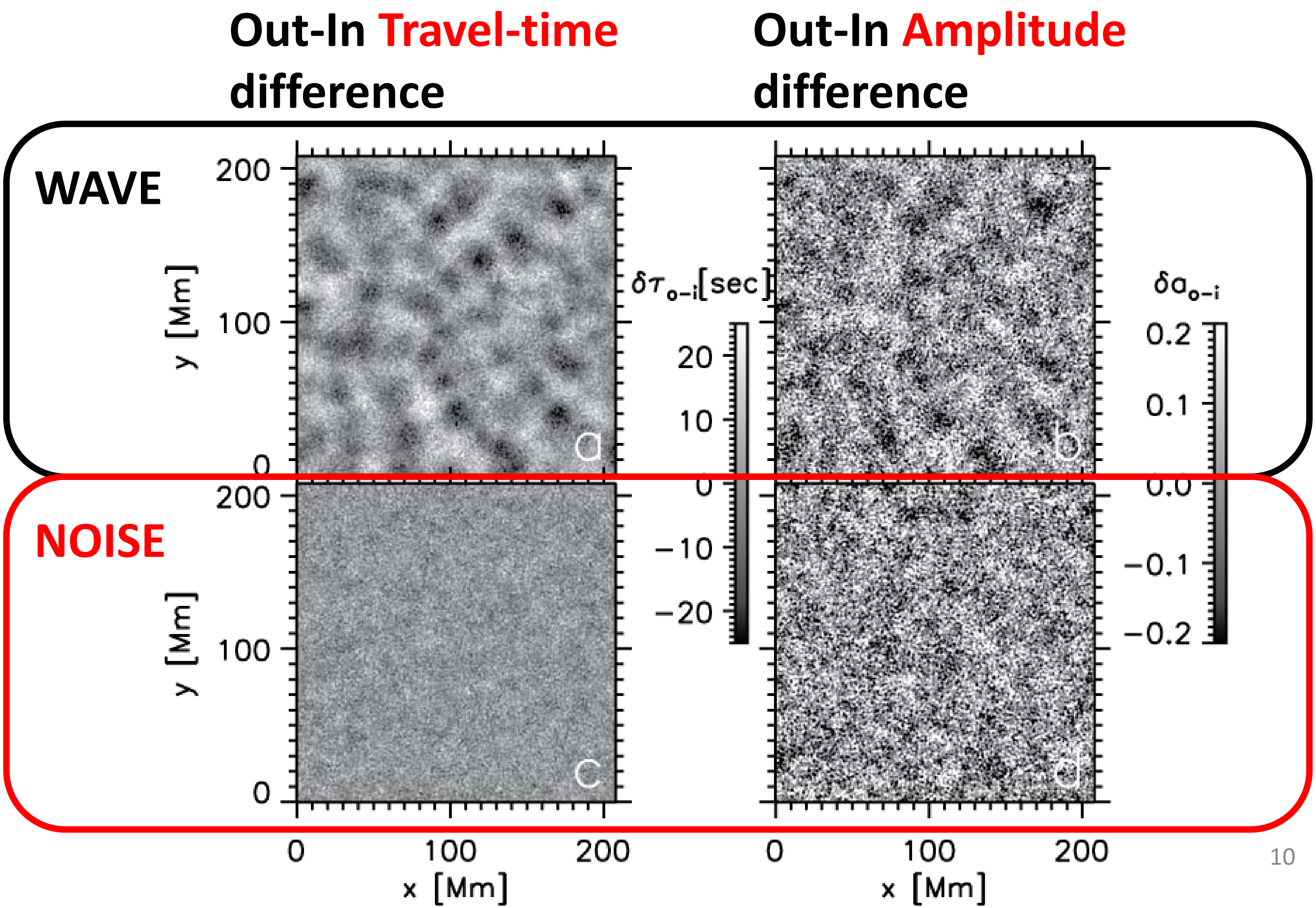


outward

inward

Estimate the S/N by creating the **noise field** by randomizing the phase of the Fourier transform of the wavefield

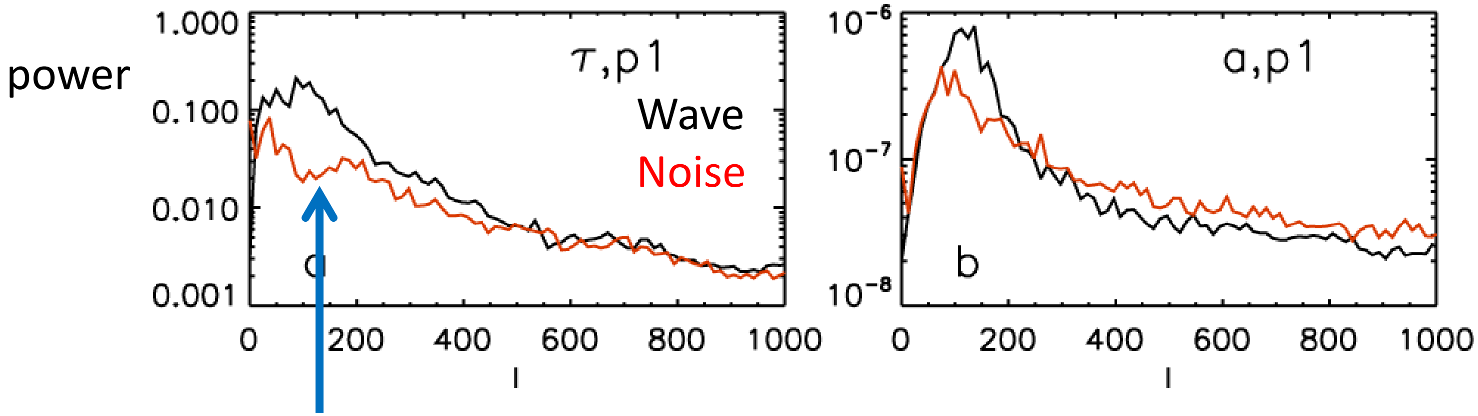
Wave & Noise


Out-In Travel-time difference
Out-In Amplitude difference
WAVE
NOISE
y [Mm]
x [Mm]
$\delta\tau_{o-i}$[sec]
δa_{o-i}
a
b
c
d
0
100
200
20
10
0
−10
−20
0.2
0.1
0.0
−0.1
−0.2

S/N ratio of the Supergranular-scale signal

$*\ell = kR_{\text{sun}}$

Out-In Travel-time difference | **Out-In Amplitude difference**



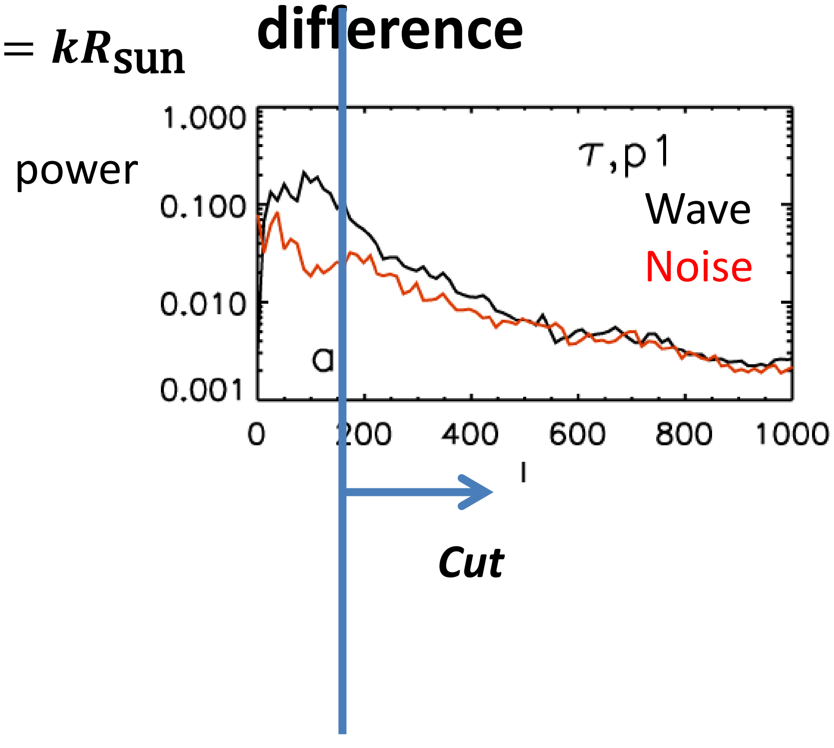
Supergranular-scale signal
($\ell = 150, \lambda = 30\text{Mm}$)
S/N ~ 30

S/N ~ a few?

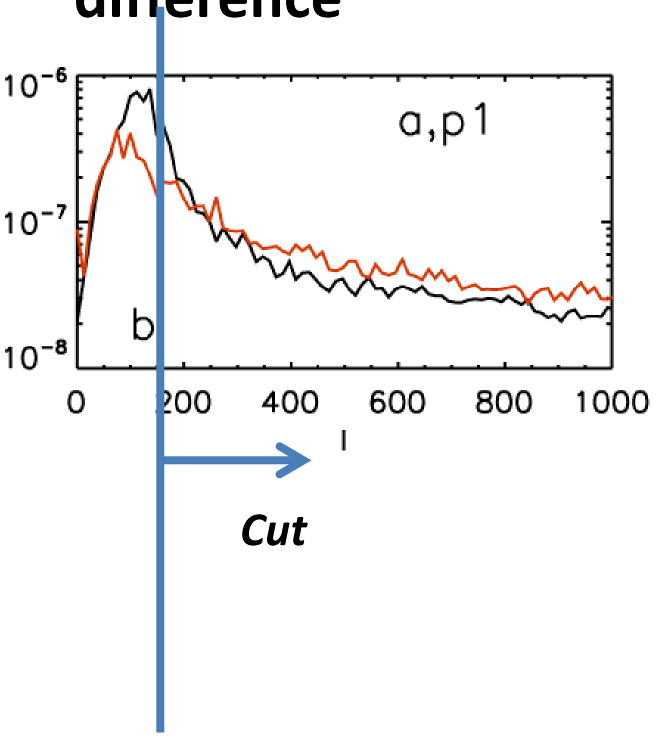
Cut the noise (high-l components) using the low-pass filter

$*\ell = kR_{\text{sun}}$

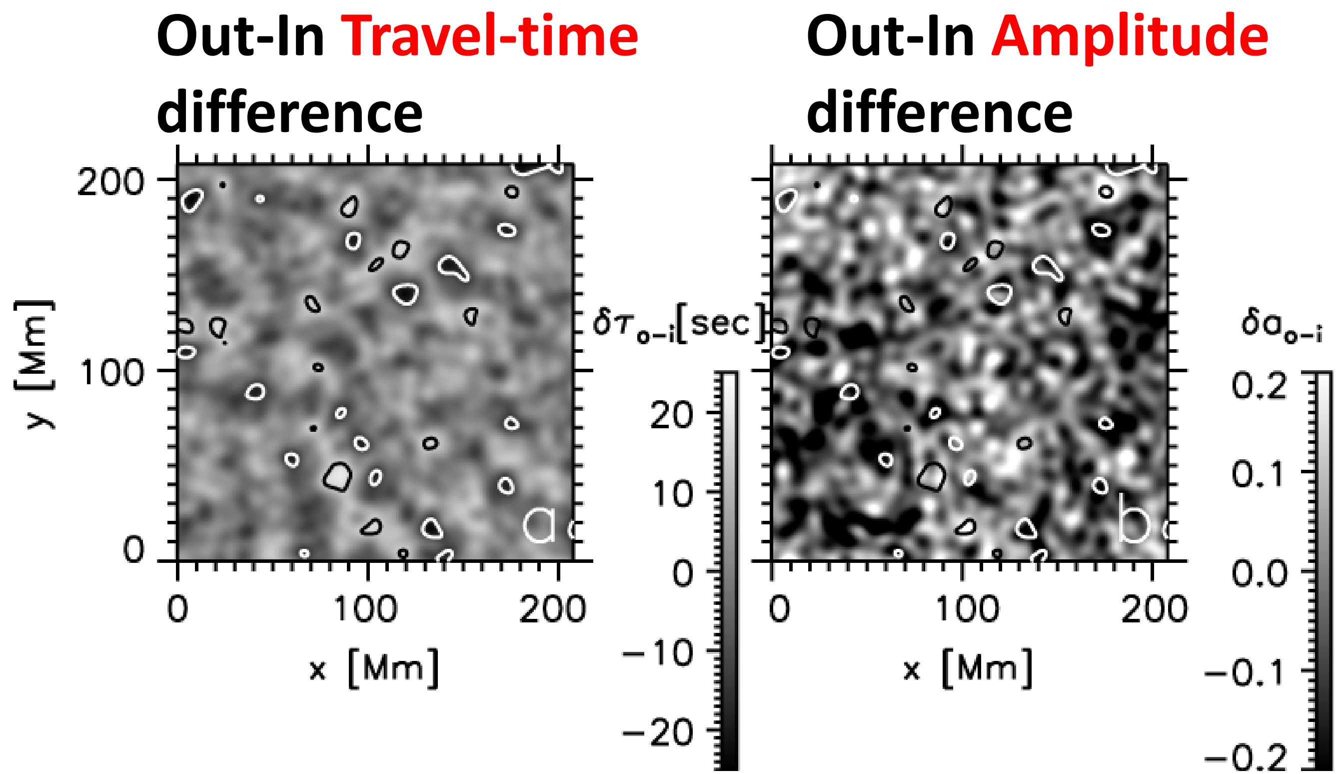
Out-In Travel-time difference



Out-In Amplitude difference



Cut the noise (high-l components) using the low-pass filter



Correlation coefficient between the travel-time and amplitude difference: +0.35

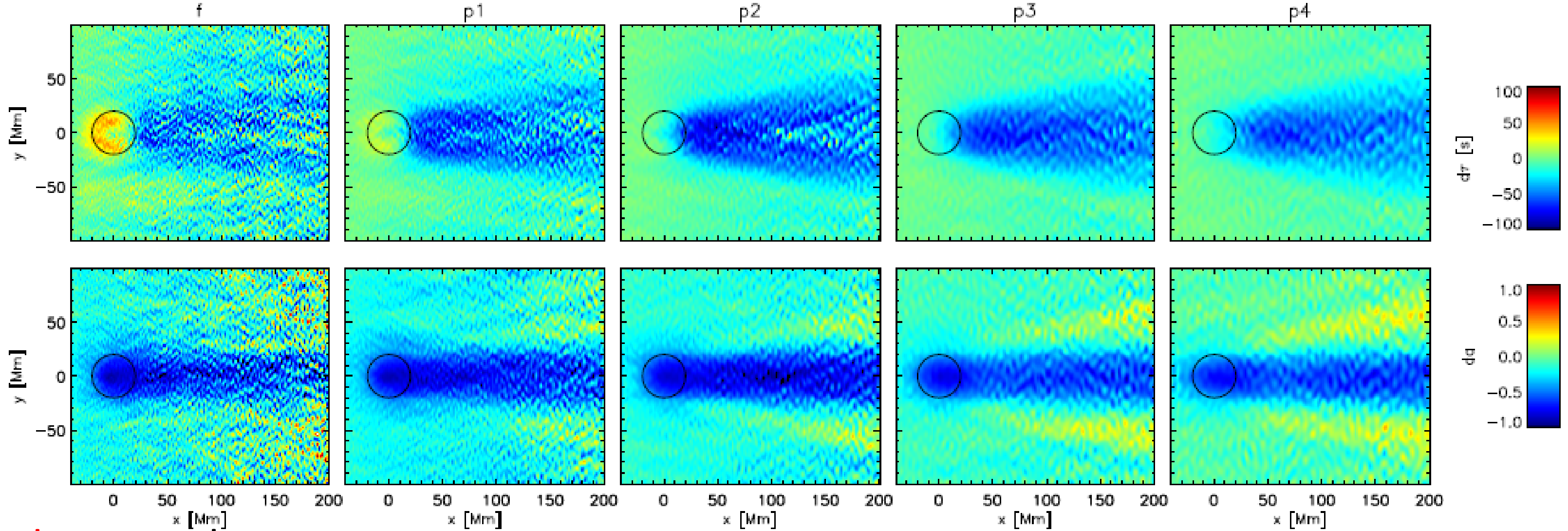
(cf., Noise data: -0.02)

Though noisy, the amplitude also shows the supergranular structure?

Example 2: around the AR

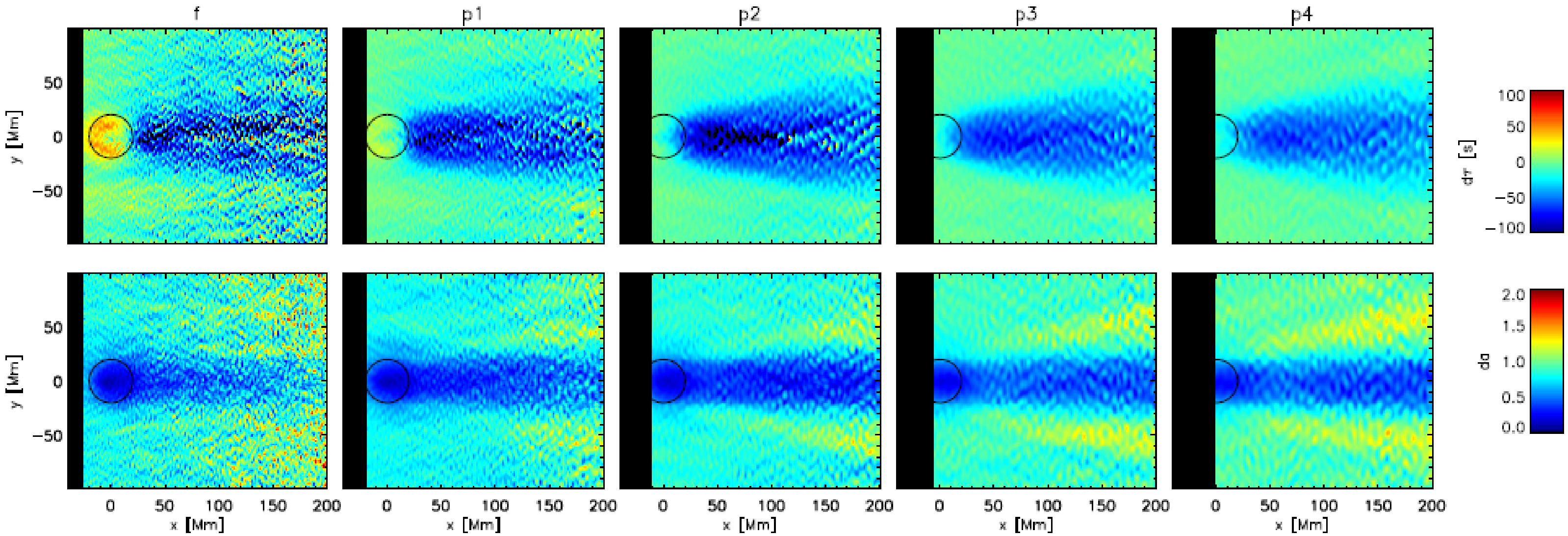
- Liang et al. (2013) reported the amplitude perturbation around a sunspot.
  - They measure “amplitude” by the Hilbert transform
- Using the same data, we measure the amplitude (and travel time) -> consistent result (more stable?)

Travel time anomaly



Amplitude anomaly

Liang et al. 2013 Fig.2 (replotted)



Summary & Perspective

- In the time-distance helioseismology analysis, so far we extract only "travel times" from the cross-covariance function of the solar seismic wave.
- Can we extract more information of the cross-covariance function?
  - Full wave form inversion might widen the local-helioseismology application
    - So far: travel time -> flow/ sound speed anomaly
    - wave excitation/attenuation? And ....?
  - Here we focus on the amplitude, and measure amplitude and travel time.
    - The out-in amplitude difference shows supergranular patterns, although noisier than the out-in travel-time difference.
    - Which parameter else is important?

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