European Solar Telescope Science Meeting, Sicily, 8:45-9:15, Tuesday, 12/06/2018

Torsional Alfvén waves in magnetic flux tubes of the solar atmosphere

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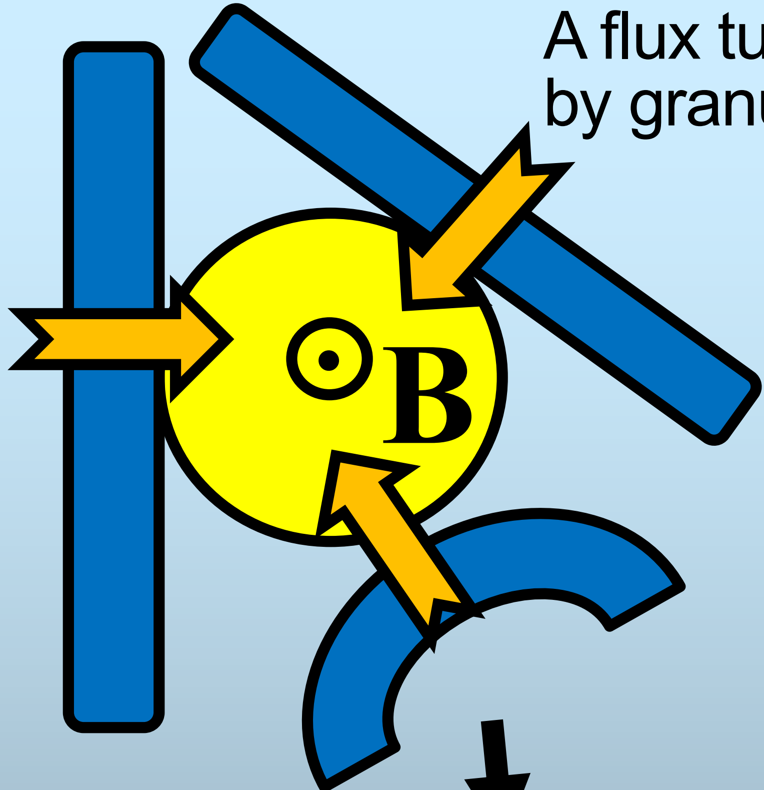
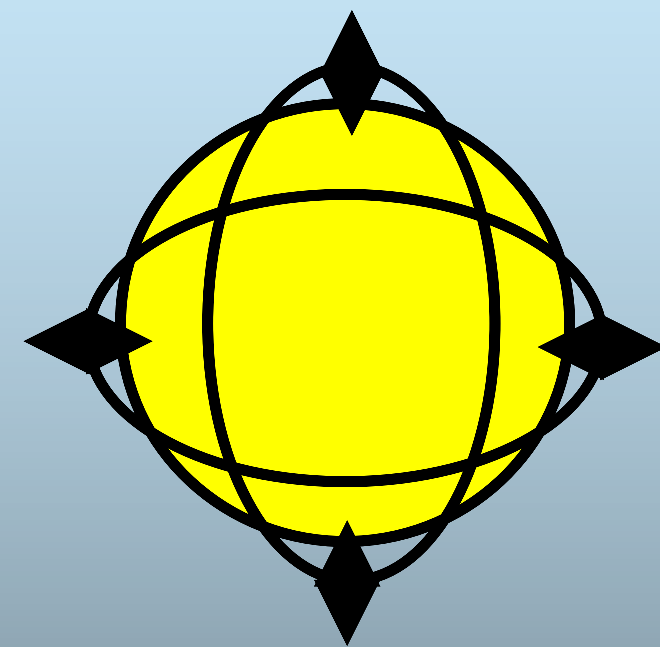
Why are we interested in Alfvén waves?

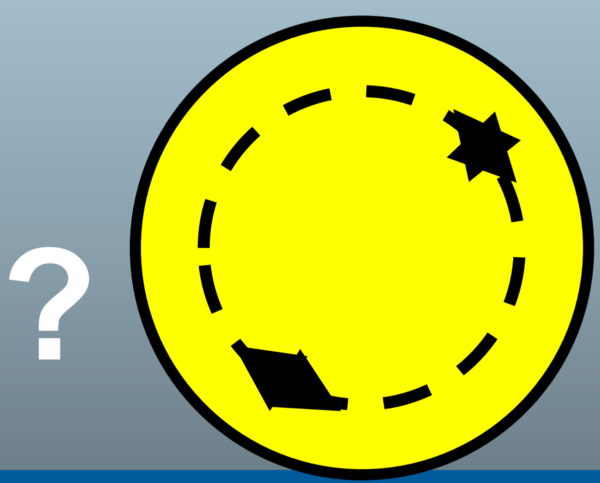
- They are dispersionless: no cutoffs, the most natural carrier of the energy from low layers to the corona.
- Non-linear effects are cubic, not quadratic – more difficult to result in shocks.
- Dissipate by shear viscosity, not volume viscosity and thermal conduction. (But, Ohmic heating – Alfvén waves are essentially an AC current; effects of partial ionisation, the readily formed steep gradients by phase mixing, …).
- Easy to excite **(?)**
- Because they are observed **(???)**

A flux tube buffeted by granulation cells



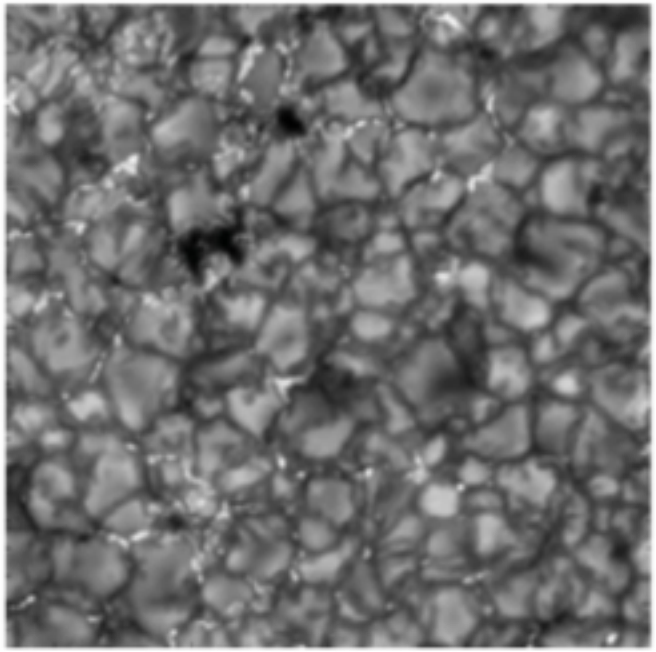
Higher-m collective (magnetoacoustic) modes seem to get excited easier:

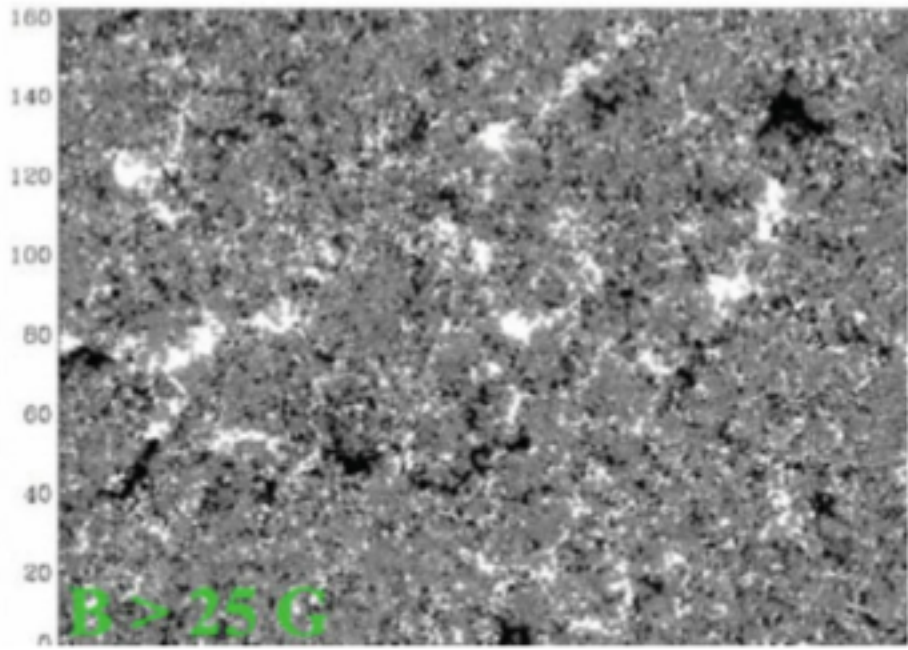




But, the above-mentioned properties of Alfvén waves are still of great interest

In the solar atmosphere (especially the corona), **Alfvén waves** with realistic periods (**> 10 s** - the period range consistent with the characteristic time of the dynamical processes, and <u>observed</u> in radio and EUV bands as magnetoacoustic waves) must be of the **torsional symmetry**.





Otherwise, if the wave front is plane, perpendicular wavelength becomes larger than the size of an AR.

Two common misconceptions:

Misconception 1. "Kink modes must are Alfvén waves too"

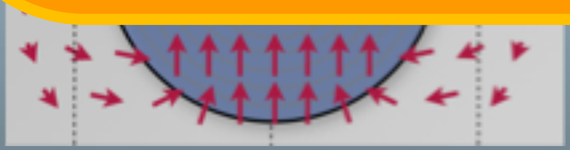
No. Kink n
waves gui
aligned pla

- Plasma
  mode

Kink waves are essentially not Alfvén, ("Alfvenic", "Alfvenish", "Alfvenicish"…) as they are:

- Compressive
- Dispersive
- Collective
- Have a cutoff in the stratified media

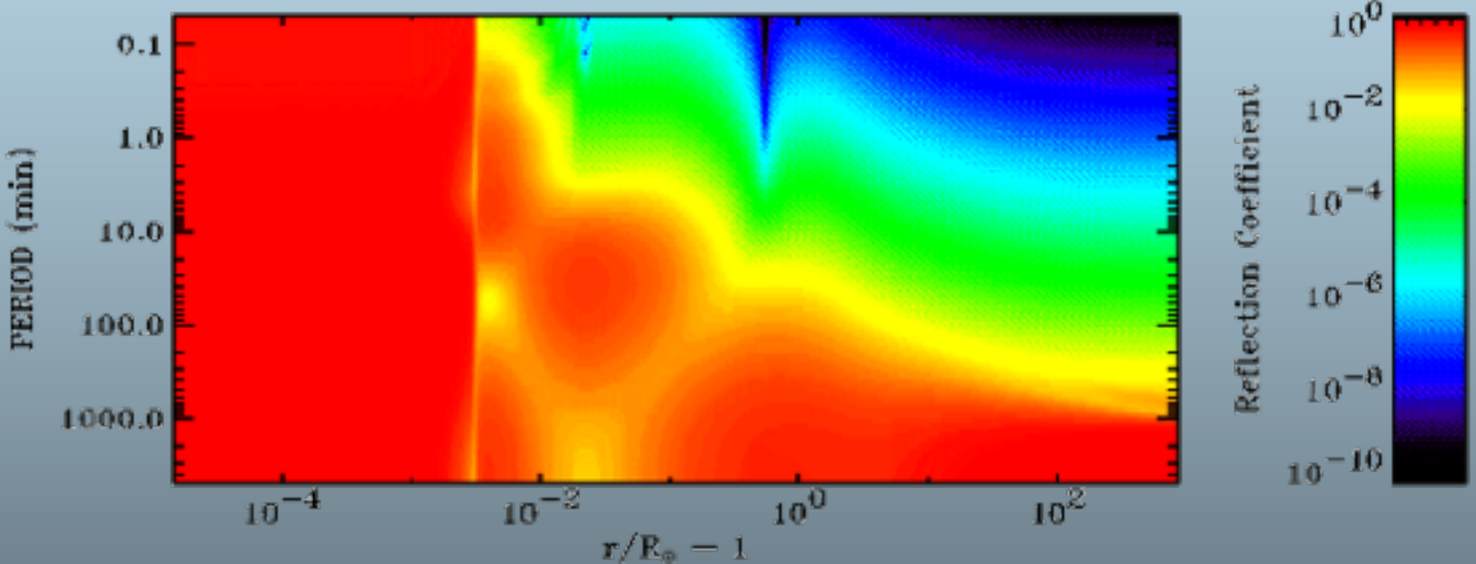
Errors in energy flux estimations > 20 times (Goossens et al. 2013)



Misconception 2. "Alfvén waves propagate through the lower solar atmospheric layers without reflection"

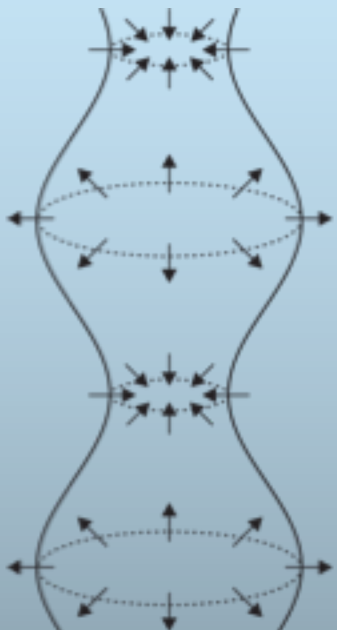
No. Even an incompressive Alfvén wave **experiences significant reflection** in the chromosphere and TR:

Reflection coefficient of Alfven waves, $(A_{up}/A_{down})^2$, vs period:

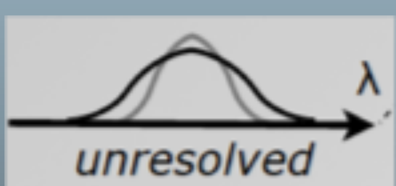


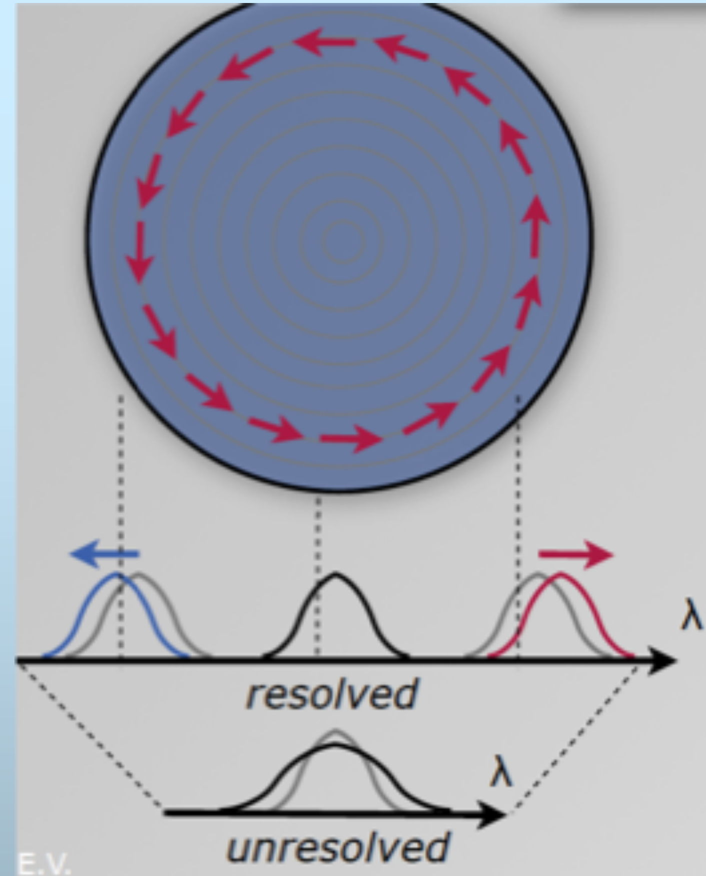
Cranmer & van Ballegooijen (2005)

Torsional waves could be detected by spectrometers:



But, if the pixel size is comparable to the tube diameter, sausage oscillations give the same spectral signature: the intensity perturbation in a sausage mode could be almost zero, as the same amount of plasma emits in all phases of the oscillation





Curtesy: E. Verwichte

The decisive evidence: coherent **spatially-resolved** anti-phase Doppler shift

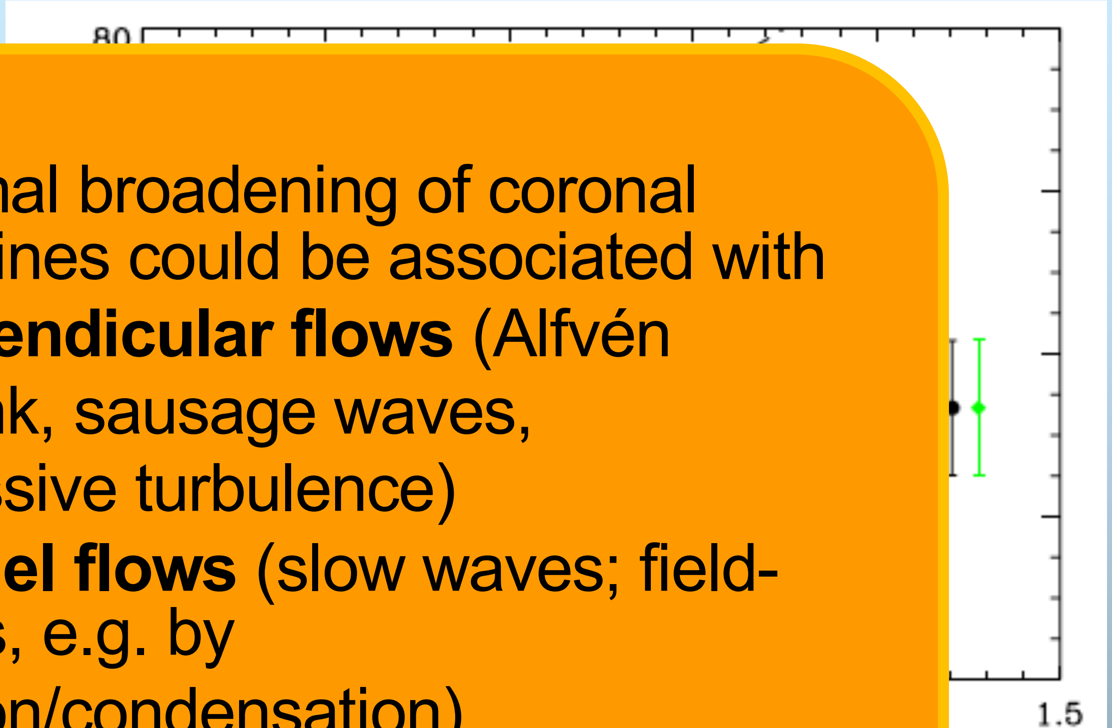
Possible observational evidence of torsional Alfvén waves in the corona: Nonthermal broadening of coronal emission lines.

$$\Delta\lambda = \frac{\lambda}{c}\left(2\right.$$

the $1/\mathrm{e}$ wi
emission li

δv^2 is the
LoS line

Non-thermal broadening of coronal emission lines could be associated with *both* **perpendicular flows** (Alfvén waves, kink, sausage waves, incompressive turbulence)
and **parallel flows** (slow waves; field-align flows, e.g. by evaporation/condensation)



om the
spectral
polar coro-
nal hole. The dashed line shows the $n_e^{-1/4}$ dependence expected for undamped WKB Alfvén waves © AAS Reproduced with permission from [*Hahn and Savin*, 2013].

An essential feature of Alfvén waves is the effect of **phase mixing**

Example: shear Alfvén waves in a non-uniform medium

Consider a 1D non-uniformity of the Alfven speed $C_A(x)$ across the magnetic field:

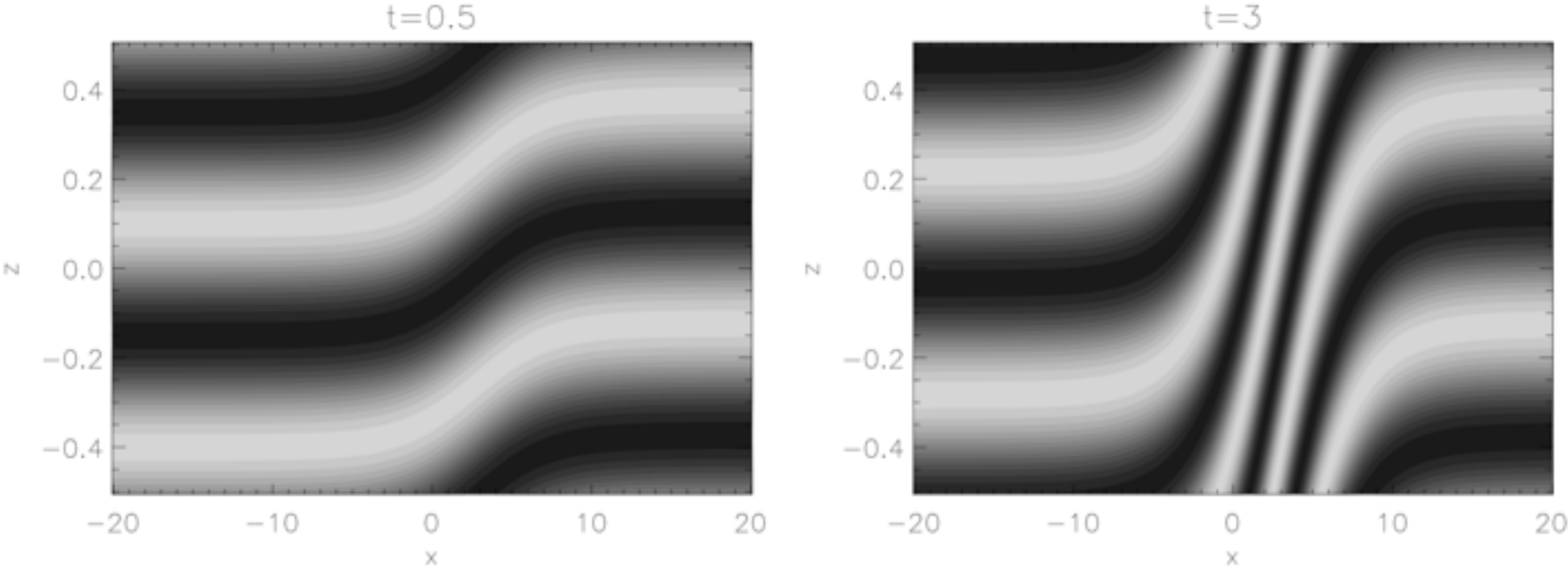
The linear Alfvén wave Eq.:

$$\left(\frac{\partial^2}{\partial t^2} - C_A^2(x)\frac{\partial^2}{\partial z^2}\right)V_y = 0.$$

$$V_y = \Psi(x) f(z \mp C_A(x)t)$$

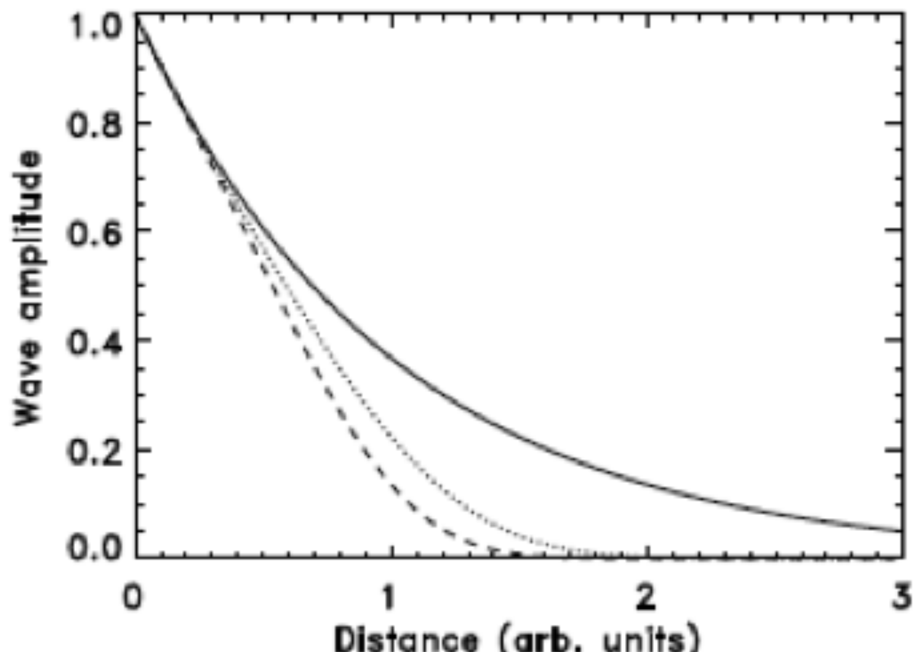
Phase mixing:

$$k_z = \text{const}, \qquad k_x \to \infty$$



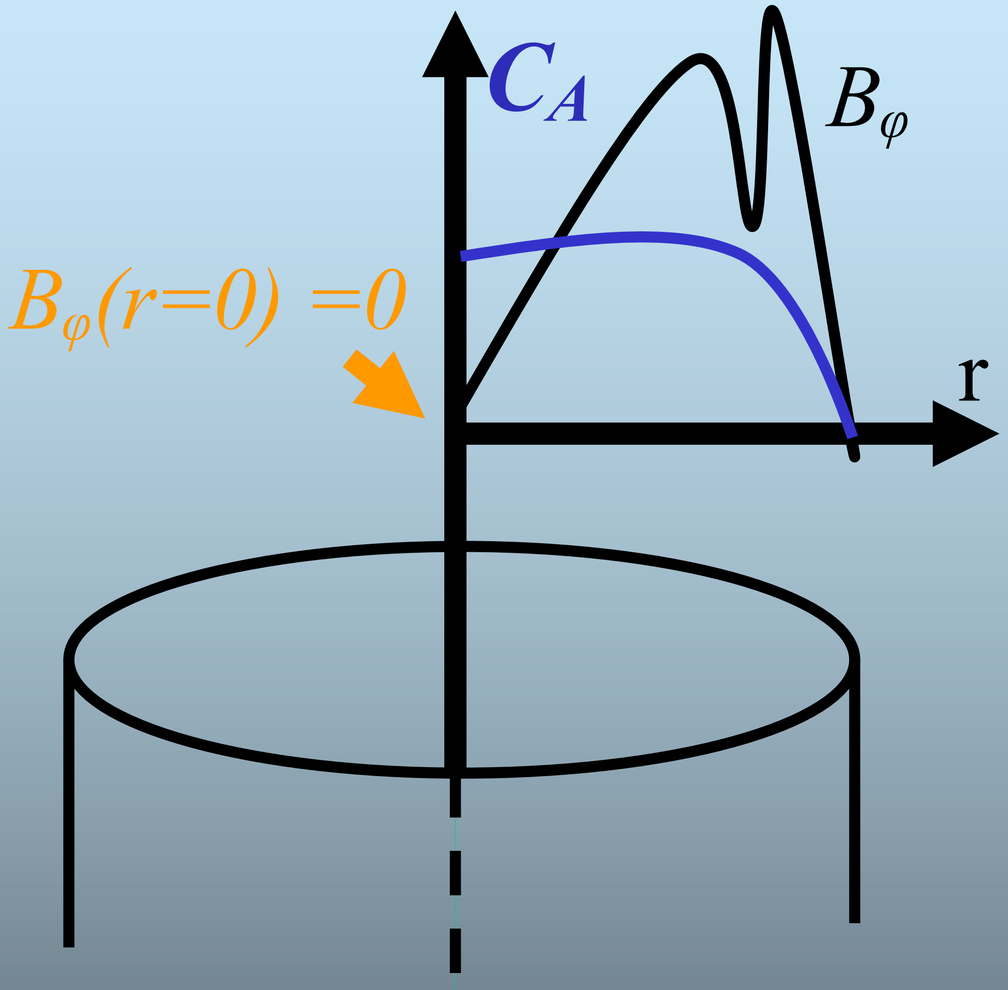
$$\text{dissipative term} = \nu \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) V_y \propto \nu (k_x^2 + k_z^2) V_y.$$

$$V_y(z) \propto V_y(0) \exp \left\{ -\frac{\nu \omega^2}{6 C_A^5(x)} \left[\frac{\mathrm{d}C_A(x)}{\mathrm{d}x} \right]^2 z^3 \right\}$$



solid curve – $dC./dx$=0. dotted – 1.. dashed – 2.

The effect of phase mixing is intrinsic for **torsional** Alfvén waves, as they are **essentially non-uniform** across the field:



Azimuthal perturbations of neighbouring magnetic surfaces are totally independent of each other.

Nonlinearly induced compressive flows:

Ponderomotive force

$$\mathbf{B}_0 \,||\, \mathbf{e}_z$$

$$\frac{\partial^2 V_z}{\partial t^2} = -\frac{1}{\rho_0}\left[\frac{\partial}{\partial t}\left(B_y \frac{\partial B_y}{\partial z}\right)\right]$$

$$\beta = 0$$

B_y – perturbation in Alfvén wave

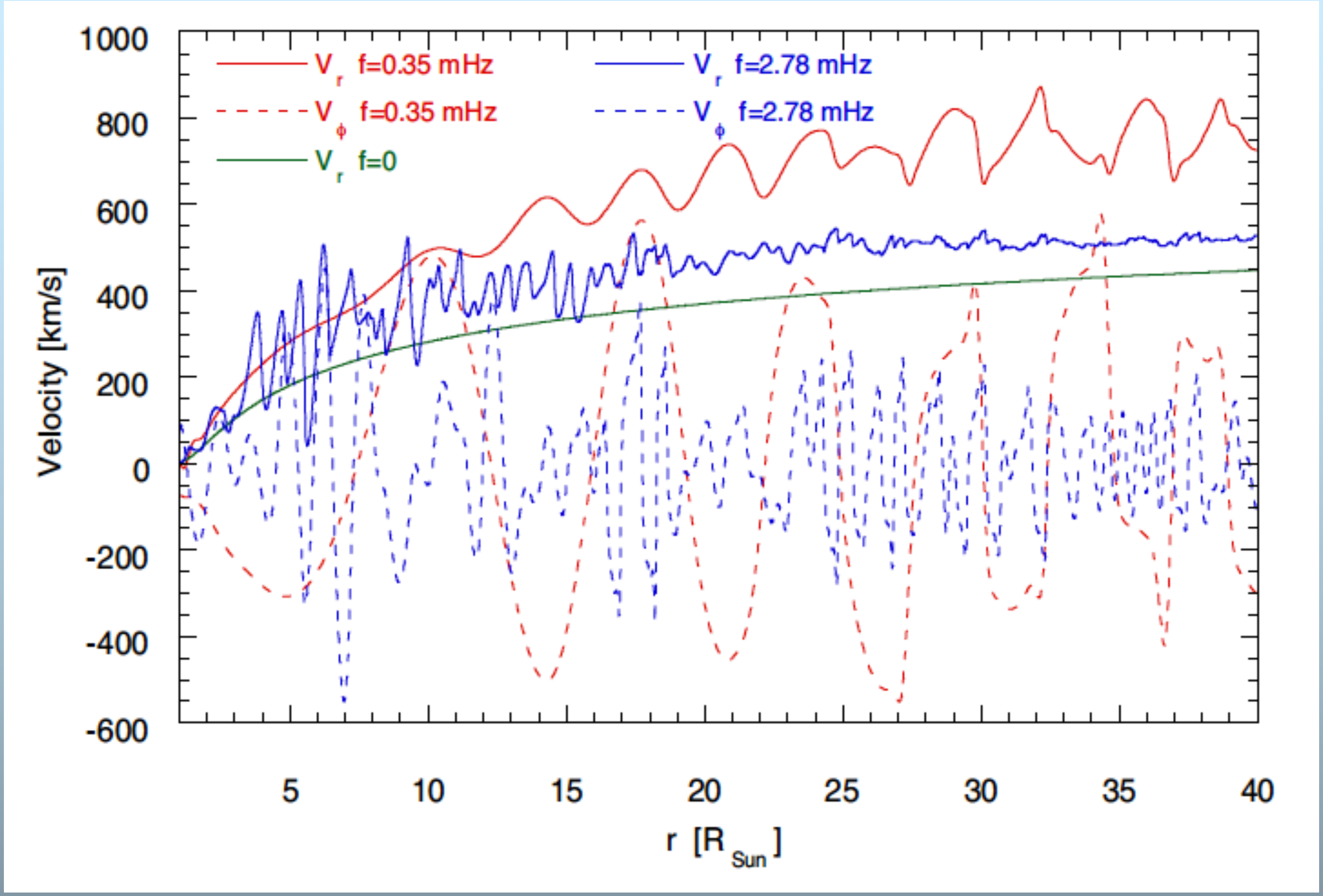
Induced parallel flows:

→ Modification of the density

→ Modification of the Alfven speed

→ Self-interaction of Alfven waves

“**Alfvénic wind**” driven by **plane** Alfvén waves



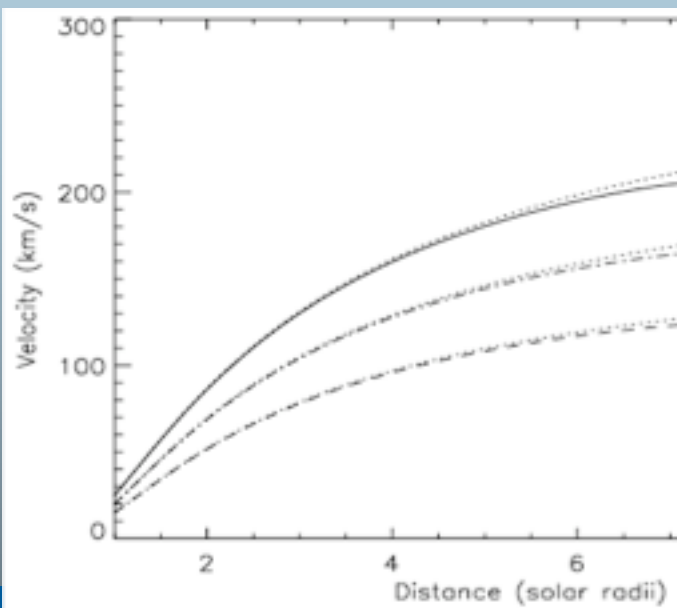
Ofman & Davila, JGR 103, 23677, 198; Torkellson & Boynton, 1998; Nakariakov et al. 2000; Suzuki 2004-2010

$$\frac{\partial V_\phi}{\partial R} - \frac{1}{4HR^2}V_\phi - \frac{1}{4C_A(C_A^2 - C_s^2)}\frac{\partial V_\phi^3}{\partial \tau}$$

$$- \frac{\bar{\nu}}{2C_A^3} \ldots$$

Spherical Cohen-Kulsrud Burgers Eq

Evolution of the Alfve



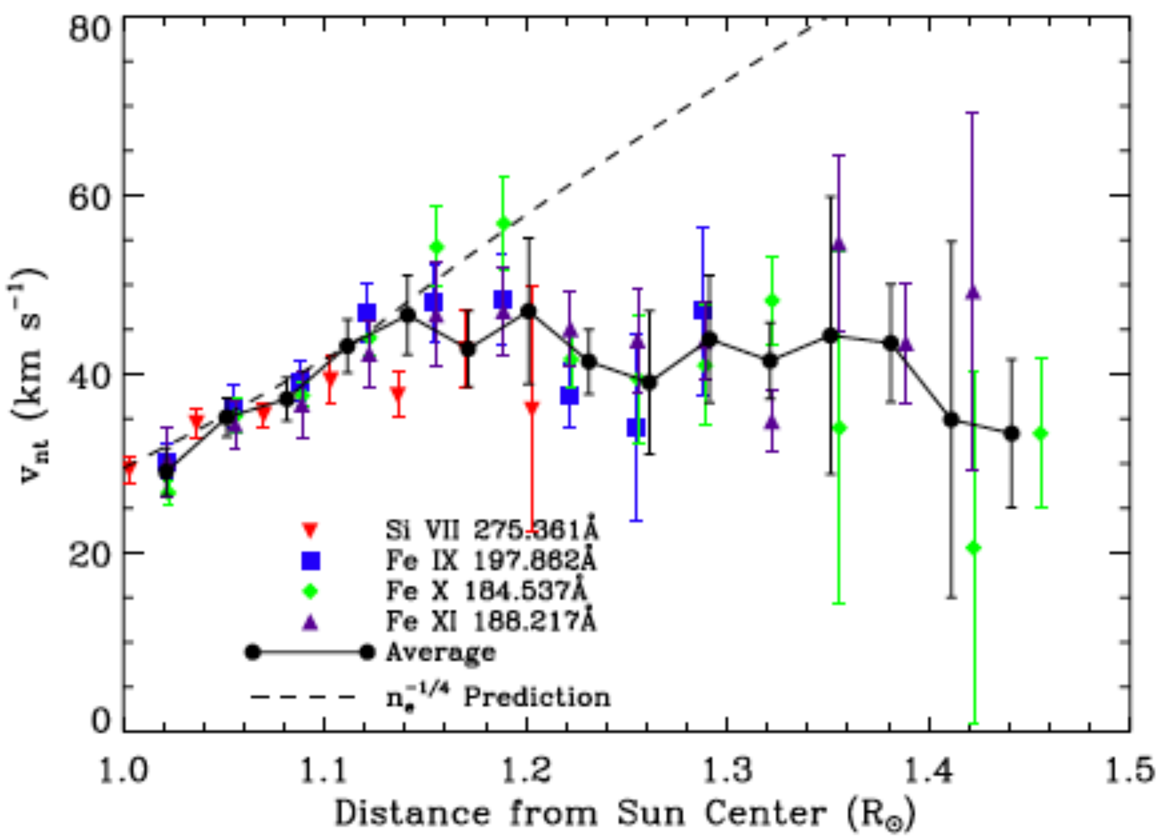


Figure 2. The averaged non-thermal velocity from the strongest observed lines with Hinode/EIS using spectral line emission data from the various ions in a polar coronal hole. The dashed line shows the $n_e^{-1/4}$ dependence expected for undamped WKB Alfvén waves © AAS Reproduced with permission from [*Hahn and Savin*, 2013].

Compressive flows induced by by the **centrifugal**, **magnetic tension** and **ponderomotive** forces:

$\Omega = V_\phi/r$
- vorticity

$J = B_\phi/r$
– twist.

$$(C_s^2 + C_A^2) D_T \rho = \frac{A_0}{2\pi}\frac{\partial^2}{\partial t^2}\left(\frac{J^2}{4\pi} - \rho_0 \Omega^2\right) + \frac{R^2 C_A^2}{4\pi}\frac{\partial}{\partial z}\left(J\frac{\partial J}{\partial z}\right),$$

In a propagating torsional wave:

$$\frac{J^2}{4\pi} - \rho_0\Omega^2 = \left(\frac{j_a^2}{4\pi} - \rho_0\frac{(-j_a)^2}{4\pi\rho_0}\right)\cos^2(\omega t - kz) = 0.$$

The effects of nonlinear magnetic twist and plasma rotation in the travelling wave **cancel** out each other, and do not add new effects.

Fundamental nonlinear effect: induced density perturbations in a **plane Alfvén wave:**

$$\rho = \frac{B_{ya}^2 k^2}{16\pi(\omega^2 - C_A^2 k^2 \beta)} \cos(2\omega t - 2kz)$$

Induced density perturbations in a **long-wavelength torsional Alfvén wave:**

$$\rho = \frac{R^2 j_a^2}{16\pi C_A^2} \cos[2(\omega t - kz)]$$

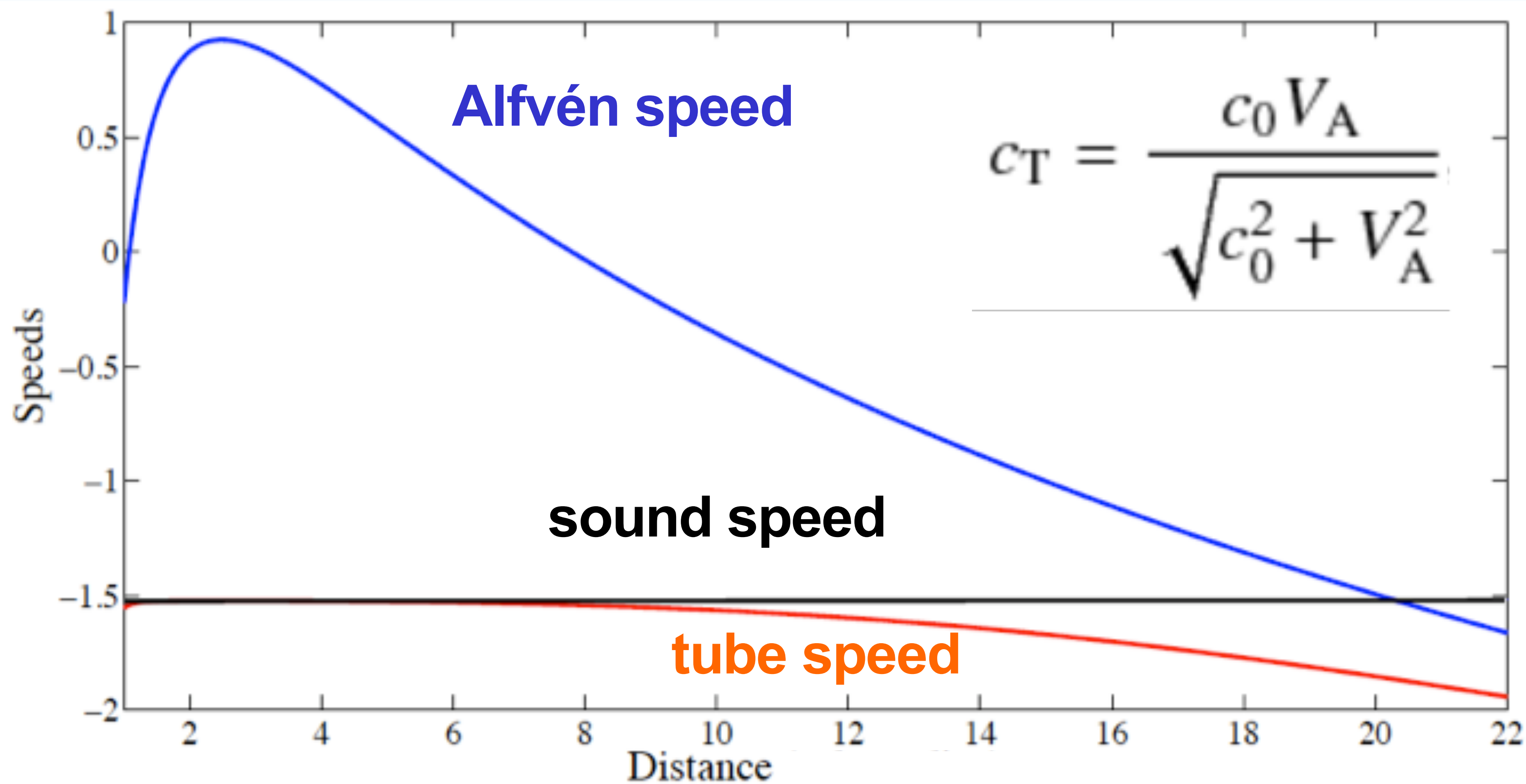
Hence, the parallel nonlinear cascades in torsional waves and in shear Alfvén waves are different!

Thin flux tube formalism:

Vasheghani Farahani et al., A&A **526**, A80, 2011

In a **plane (shear) Alfvén** wave, the induced density perturbation propagates **at the sound speed**.

In a **torsional Alfven** wave, they propagate **at the tube speed**



$$c_T = \frac{c_0 V_A}{\sqrt{c_0^2 + V_A^2}}$$

The Cohen-Kulsrud Equation for long-wavelength **torsional** Alfvén waves:

$$\frac{\partial J}{\partial \tau} + \frac{3R^2}{16\pi\rho_0 C_{\mathrm{A}}} J^2 \frac{\partial J}{\partial \xi} = 0.$$

c.f. with the Cohen-Kulsrud Equation for **plane shear** Alfvén waves:

$$\frac{\partial B_y}{\partial \tau} + \frac{3C_{\mathrm{A}}}{16\pi\rho_0 (C_{\mathrm{A}}^2 - C_s^2)} B_y^2 \frac{\partial B_y}{\partial \xi} = 0$$

By the way, both Eqs. have analytical implicit solutions:

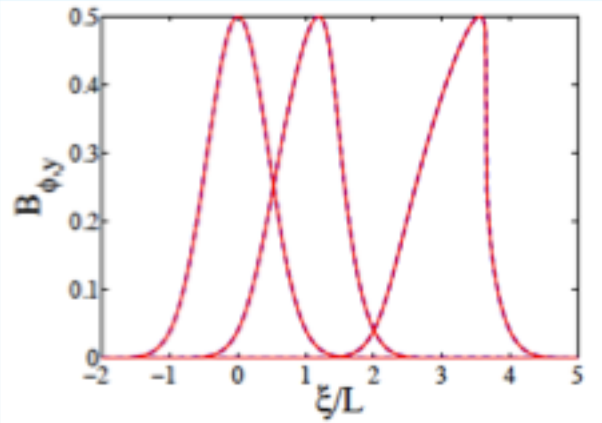
$$\frac{B_\varphi}{B_{z0}} = aRf\left(\frac{\xi}{L} - \frac{3}{4}\frac{C_\mathrm{A}}{L}\left(\frac{R^2 J}{B_{z0}}\right)^2 \tau\right)$$

$$\frac{B_y}{B_{z0}} = bf\left(\frac{\xi}{L} - \frac{3}{4}\frac{C_\mathrm{A}^3}{L(C_\mathrm{A}^2 - C_s^2)}\left(\frac{B_y}{B_{z0}}\right)^2 \tau\right)$$

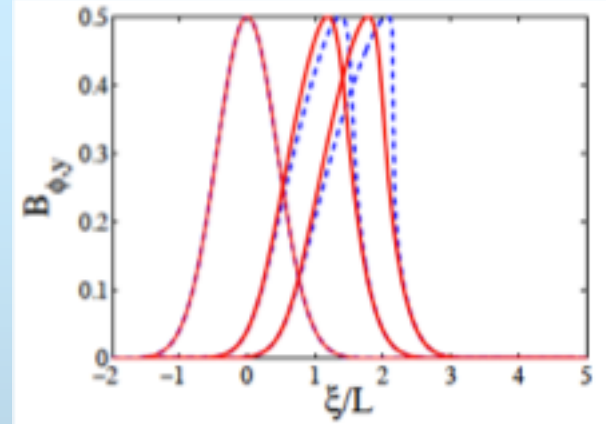
where $f(\xi)$ is the initial profile of the perturbation

Blue – **shear** wave; **Red** - **torsional** wave

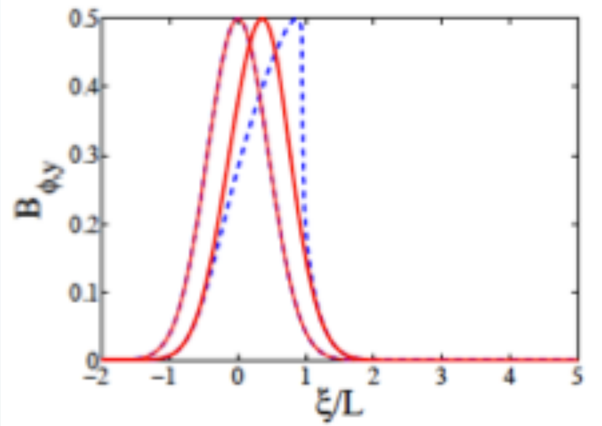
$\beta = 0$



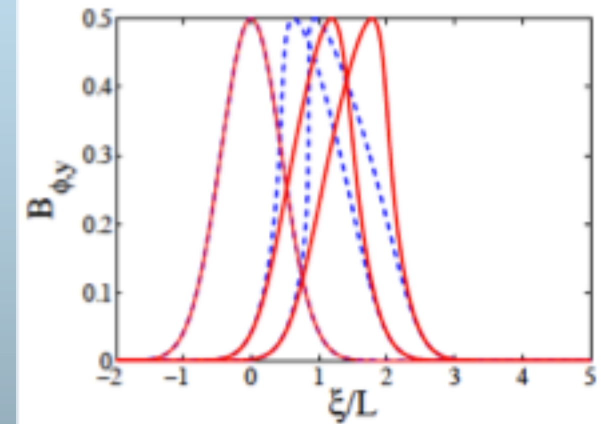
$\beta = 0.5$



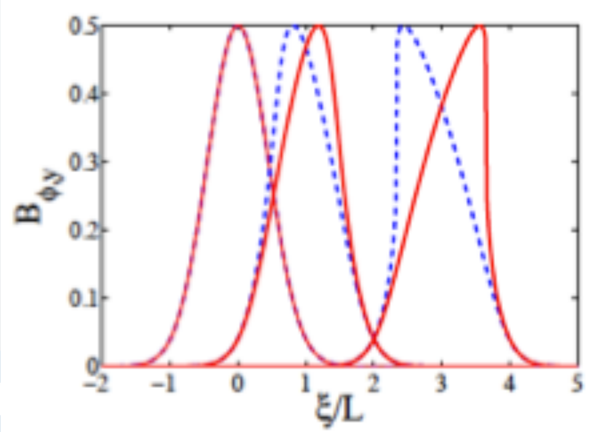
$\beta = 0.9$



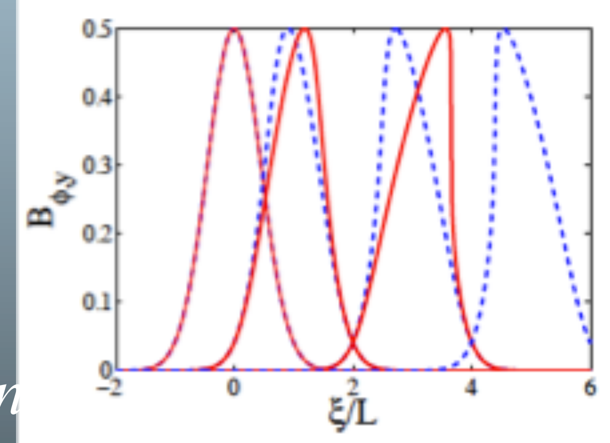
$\beta = 1.5$



$\beta = 2$

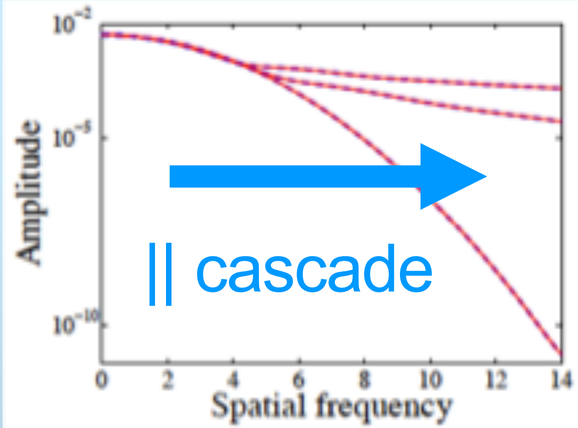


$\beta = 3$

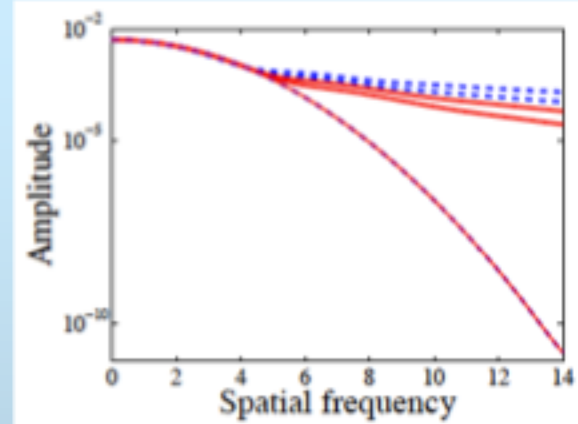


Blue – **shear** wave; **Red** – long-wavelength **torsional** wave

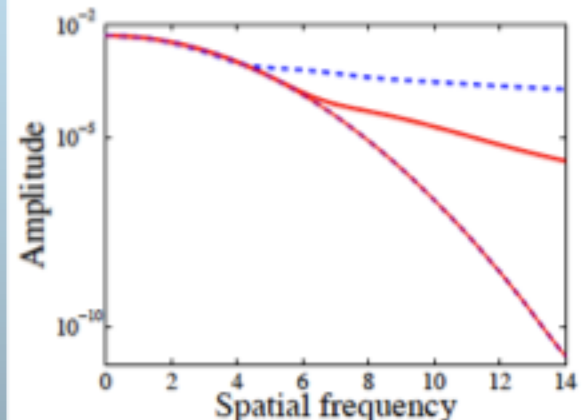
$\beta = 0$



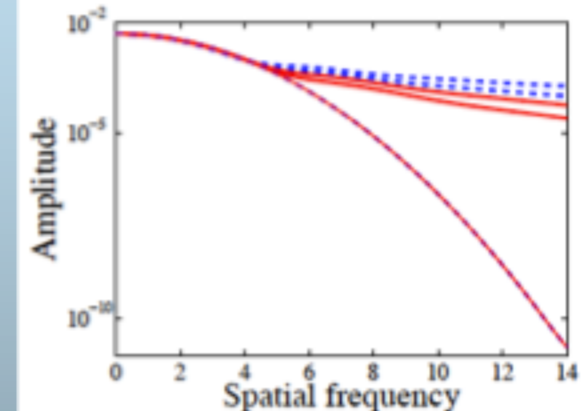
$\beta = 0.5$



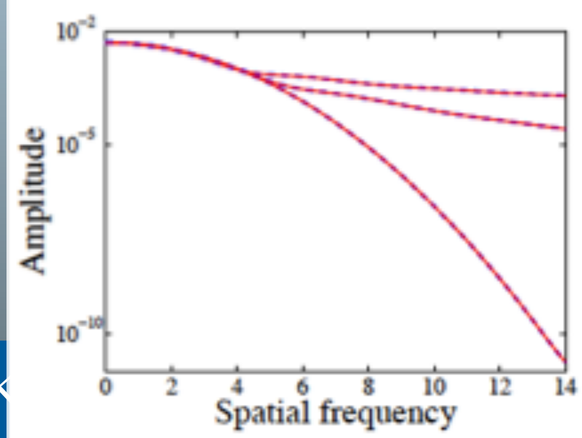
$\beta = 0.9$



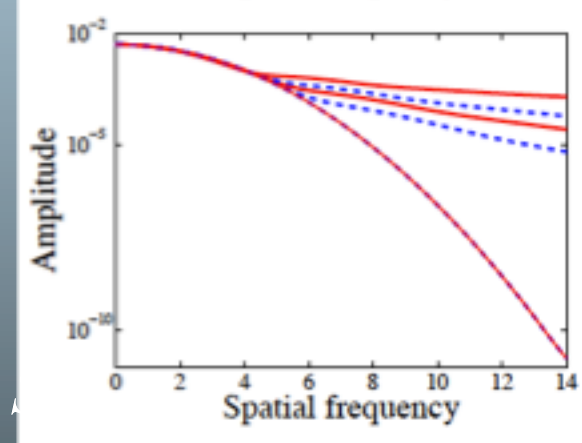
$\beta = 1.5$



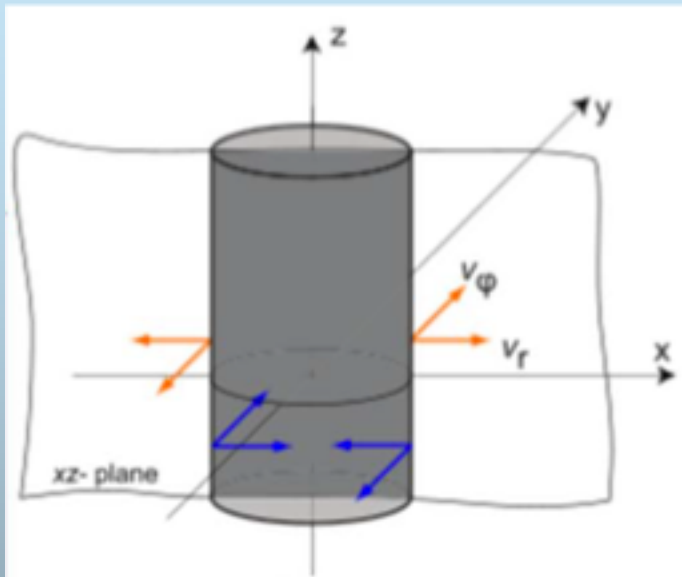
$\beta = 2$

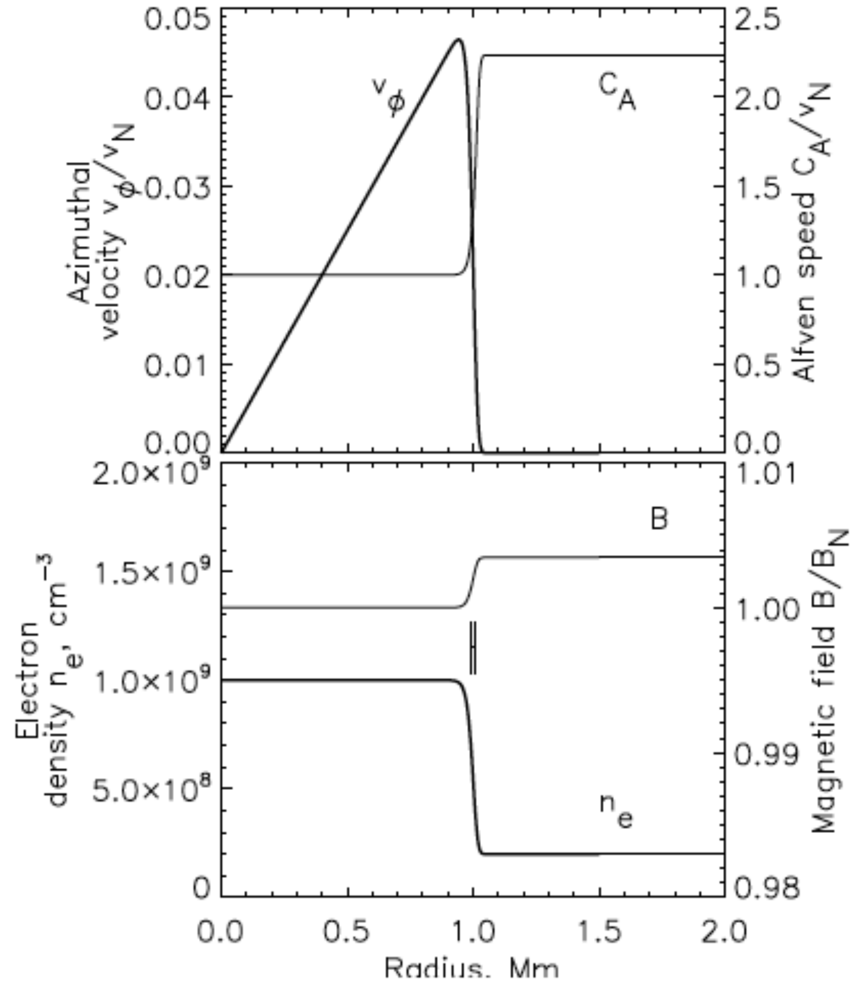


$\beta = 3$



What about the effect of finite parallel wavelength?





The Astrophysical Journal, 840:64 (11pp), 2017 May 10
https://doi.org/10.3847/1538-4357/aa6c65

Nonlinear Evolution of Short-wavelength Torsional Alfvén Waves

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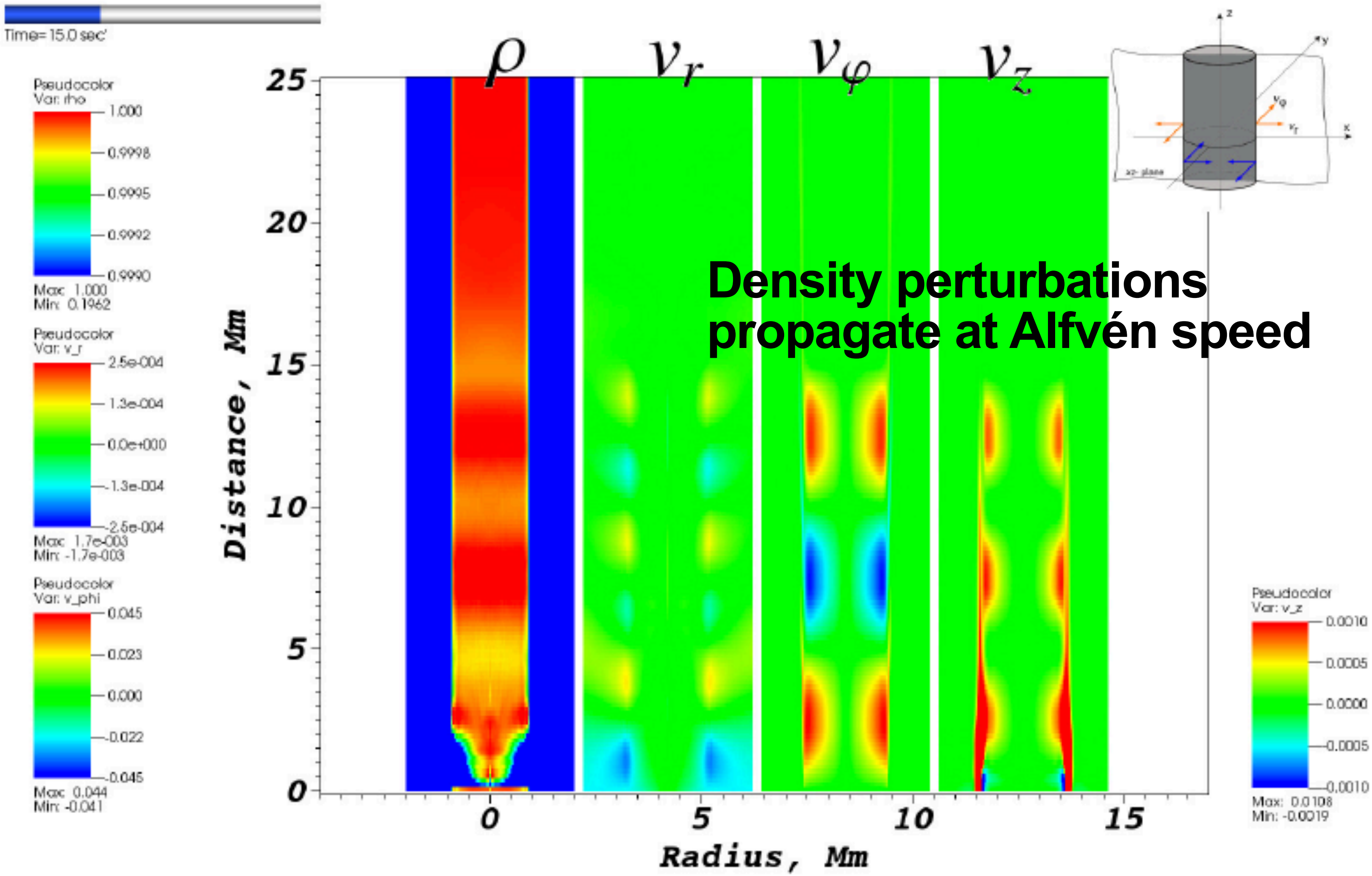
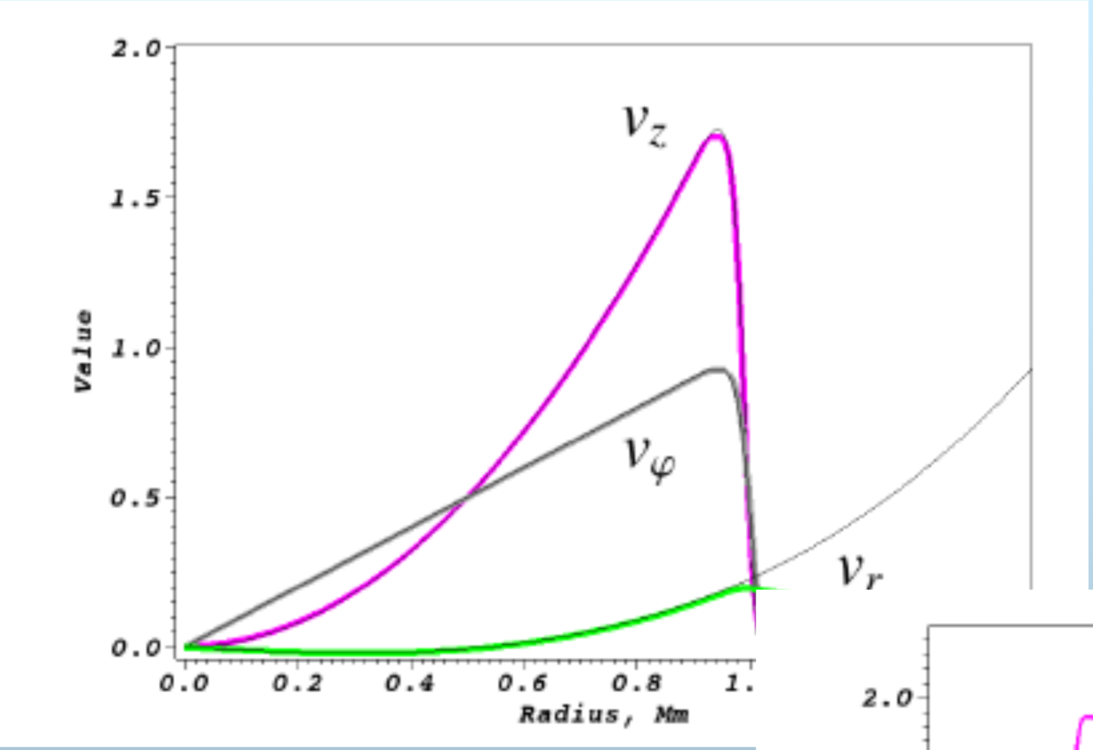
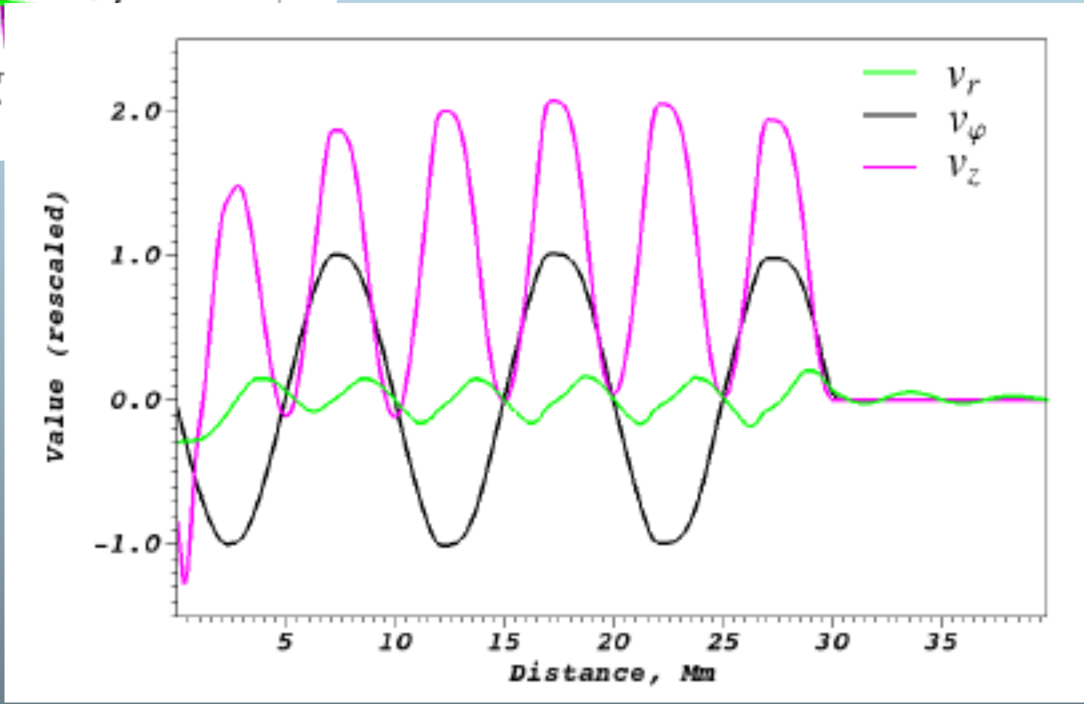
Density perturbations propagate at Alfvén speed





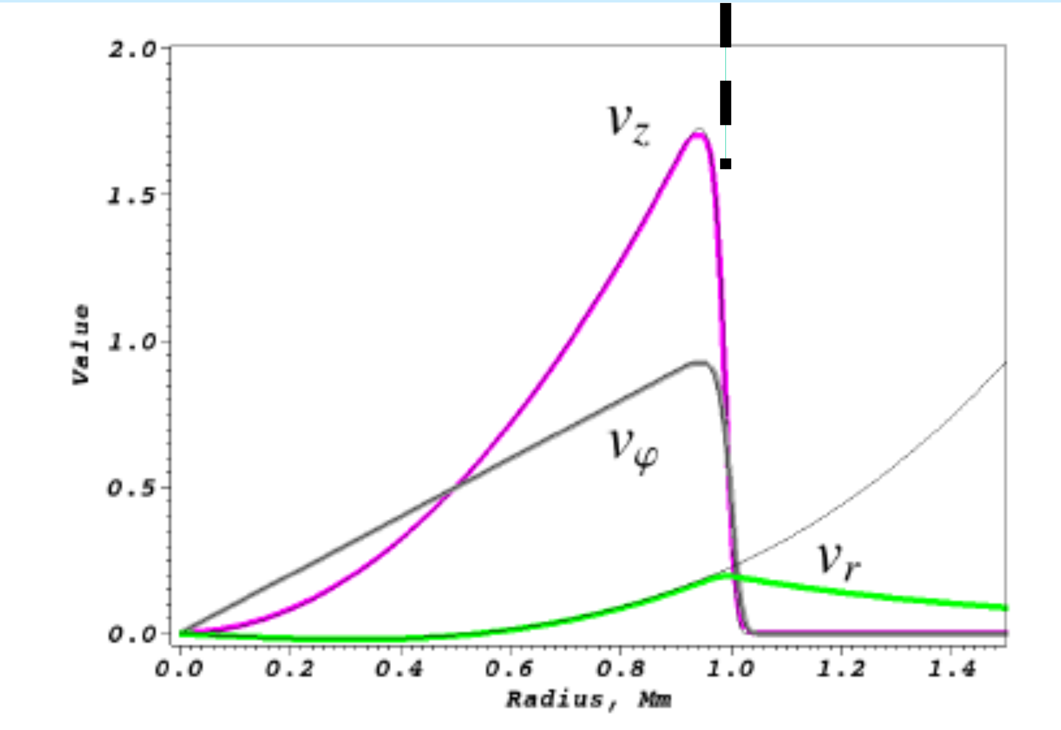
The amplitudes of different components of the velocity have different normalisations!



Role of the perpendicular nonuniformity of the wave front (**intrinsic** for torsional waves):

$\beta = 0$

$\mathbf{B}_0 \parallel \mathbf{e_z}$

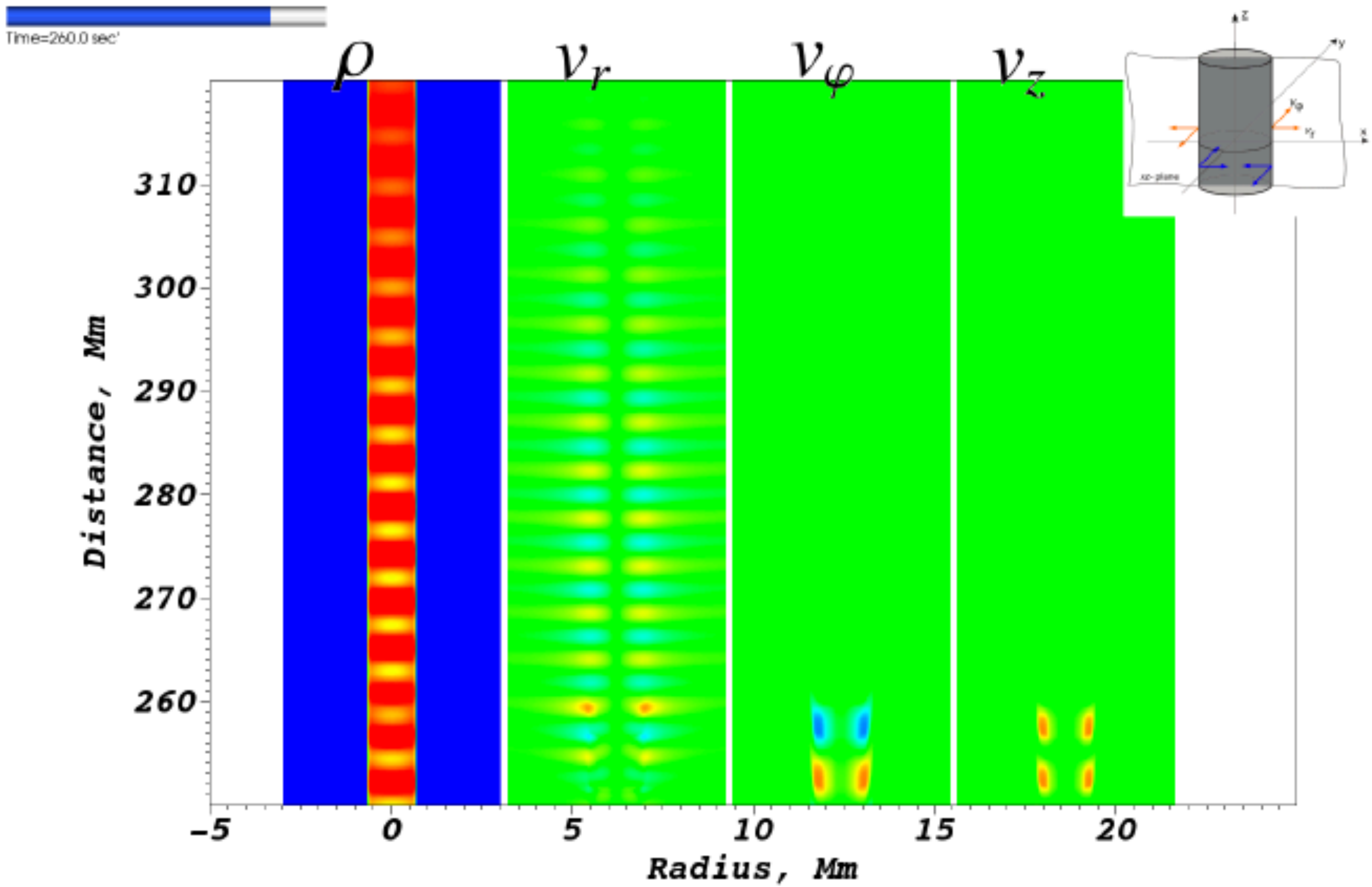


$\parallel \mathbf{B}_0$

$$\frac{\partial^2 V_z}{\partial t^2} = -\frac{1}{\rho_0}\left[\frac{\partial}{\partial t}\left(B_y \frac{\partial B_y}{\partial z}\right)\right],$$

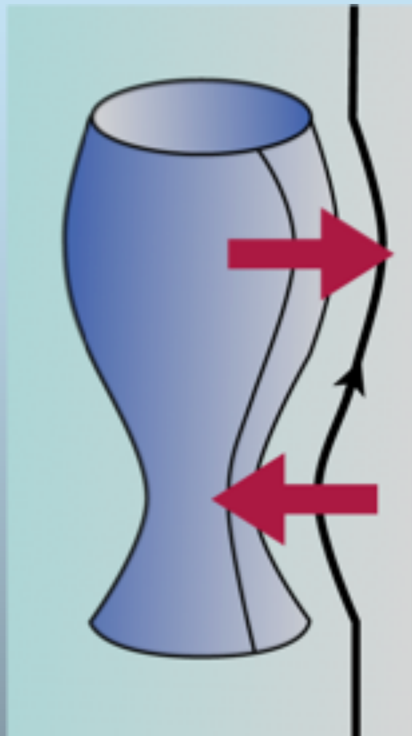
$\perp \mathbf{B}_0$

$$\frac{\partial^2 V_x}{\partial t^2} - C_A^2(x)\left(\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial z^2}\right) = -\frac{1}{\rho_0}\left[\frac{\partial}{\partial t}\left(B_y \frac{\partial B_y}{\partial x}\right)\right].$$


Time=260.0 sec'
ρ
v_r
v_φ
v_z
310
300
290
280
270
260
Distance, Mm
-5
0
5
10
15
20
Radius, Mm

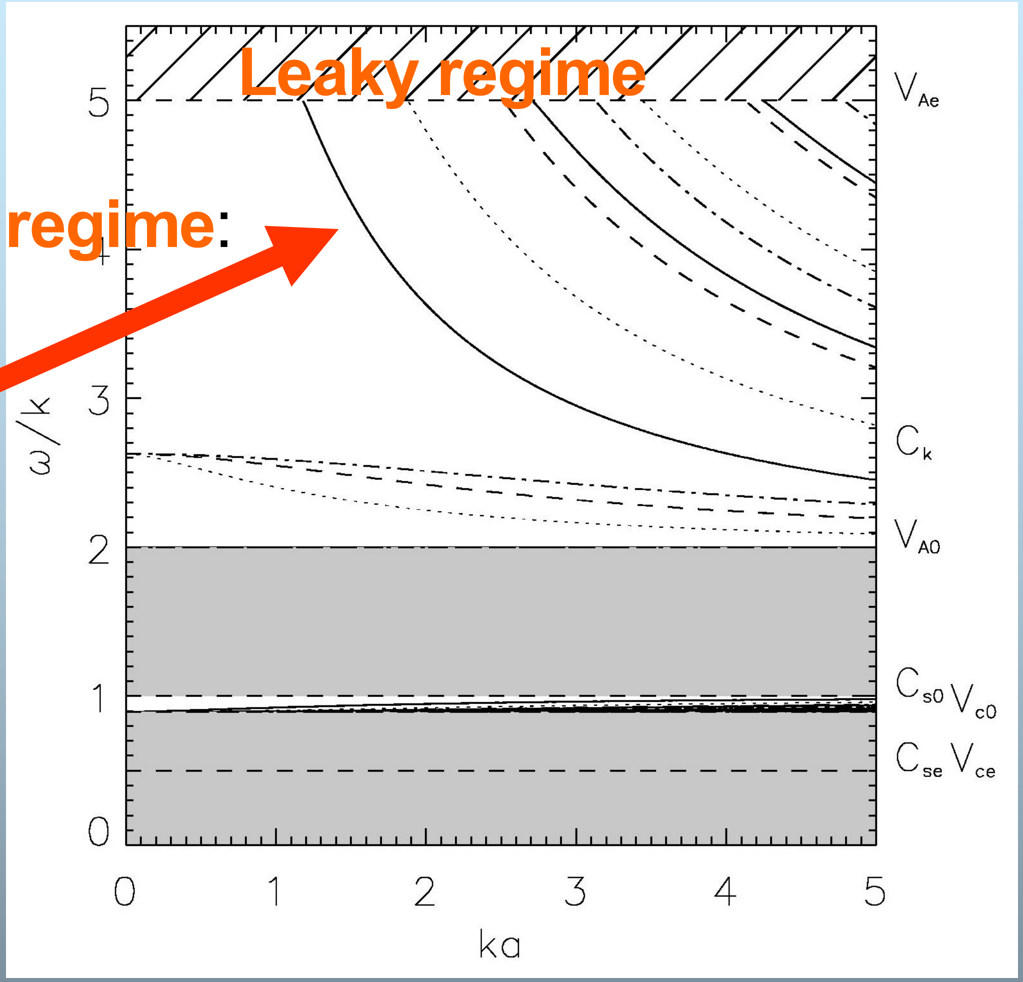
What are the perturbations of V_r?

Sausage mode:



m=0 mode

Trapped regime:



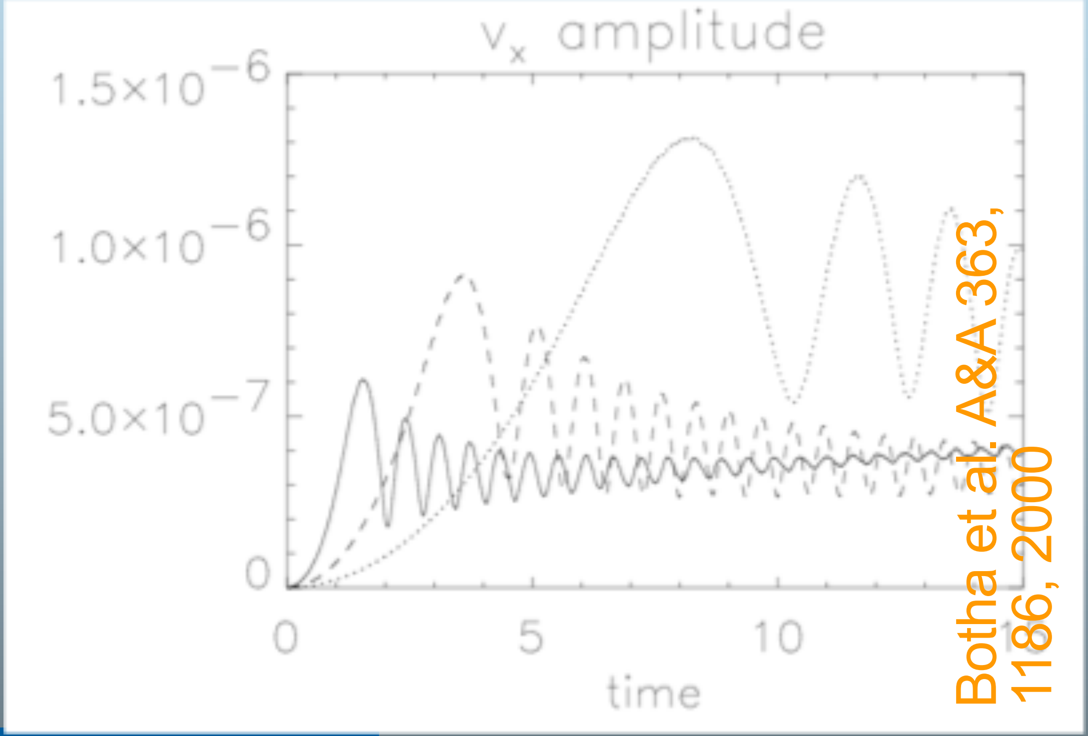
Why is the excitation of the sausage perturbations (fast magnetoacoustic modes) by Alfvén wave phase mixing much less effective than of the parallel flows (slow magnetoacoustic)?

On one hand:

$$\frac{v_x}{c_A} \sim \frac{1}{4}\frac{dc_A}{dx}\left(\frac{B_y}{B_0}\right)^2 t.$$

Nakariakov et al. Solar Phys. **175**, 93, 1997

On the other hand:



Botha et al. A&A 363, 1186, 2000

But, induced fast waves could spread the energy (heat the plasma!) across the field…

Conclusions

- Alfvén waves of realistic periods (> 1 s) in the corona **cannot be plane**, and **it is important**.
- Even in the thin fluxtube regime the **nonlinear cascades** in plane and torsional Alfvén waves **are different**: need for revision of 1D models of the solar wind acceleration.
- The intrinsic perpendicular variation of the wave amplitude is important too:
  - compressive parallel flows are induced in **annuli** where the Alfvén wave amplitude is highest (i.e. the **induced Alfvénic wind is nonuniform** in the horizontal direction – ***a macaroni flow pattern***.
  - sausage fast magnetoacoustic waves are excited.