Hilbert–Huang transform analysis of periodicities in the last two solar activity cycles

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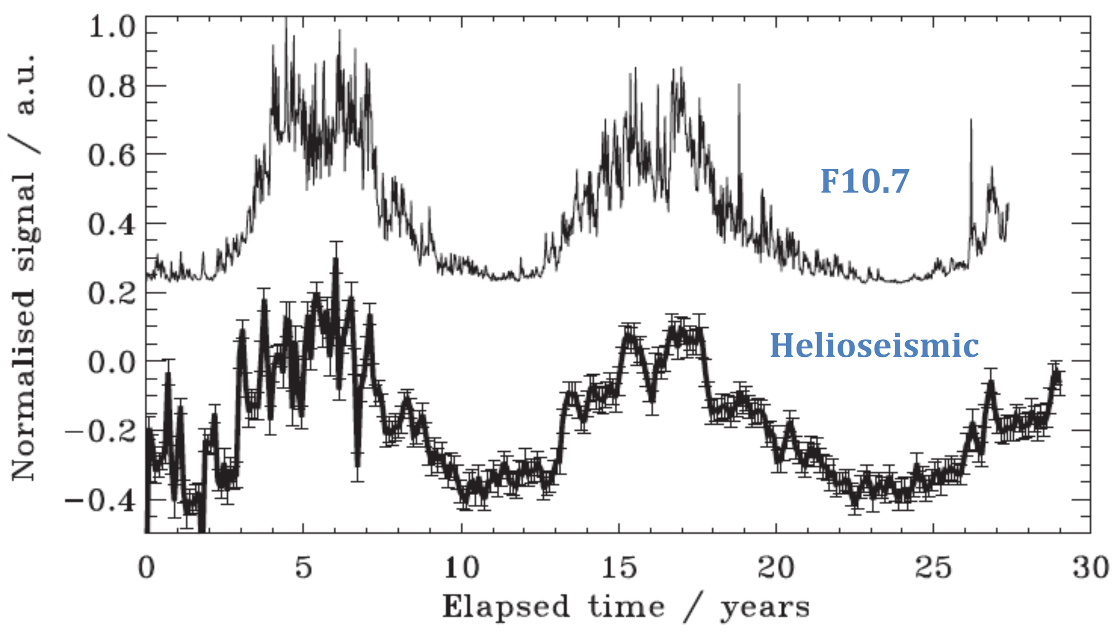
Introduction

The Sun's magnetic activity varies primarily on a time-scale of 11 yr. However, both longer and shorter term periodicities are also frequently associated with solar magnetic activity [1]. In this work, we determine activity-related periodicities that can be extracted from data recorded over the last two-and-a-half solar cycles (from 1985 to 2014). We therefore concentrate on periodicities less than (but including) 11 yr. Solar activity periodicities are hard to characterize: even the 11 yr cycle is known to vary in both the amplitude and length. Usually, for the spectral analysis of long-term solar activity, Fourier transforms and other more advanced Fourier-based techniques are used. However, all of them have disadvantages associated with the a priori prescription of the basis function, e.g. the harmonic one. Hence, such methods are highly restricted in the sense of their application to non-stationary, rather anharmonic signals. In this work, we present an analysis of periodicities observed in the last two solar activity cycles with the novel Hilbert–Huang transform (HHT) technique. The main idea of the HHT is a combination of the empirical mode decomposition (EMD) of a given signal in to a set of intrinsic mode functions (IMF), based on searching for the local time-scales naturally appearing in a signal, and the subsequent Hilbert transformation of the identified IMFs [2]. The presented work was recently published in [3].

Data



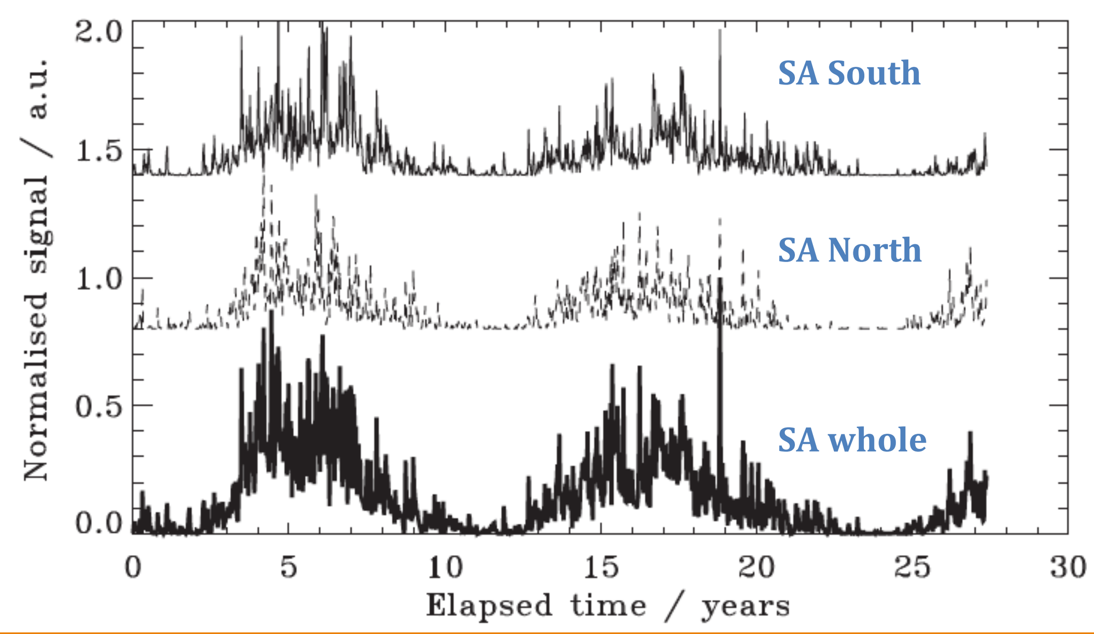
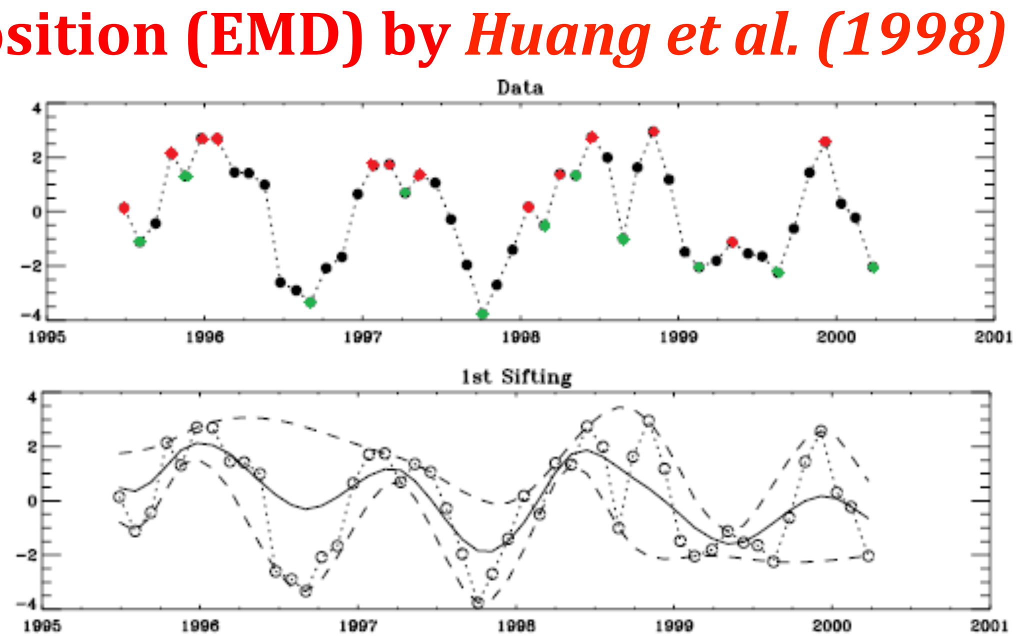


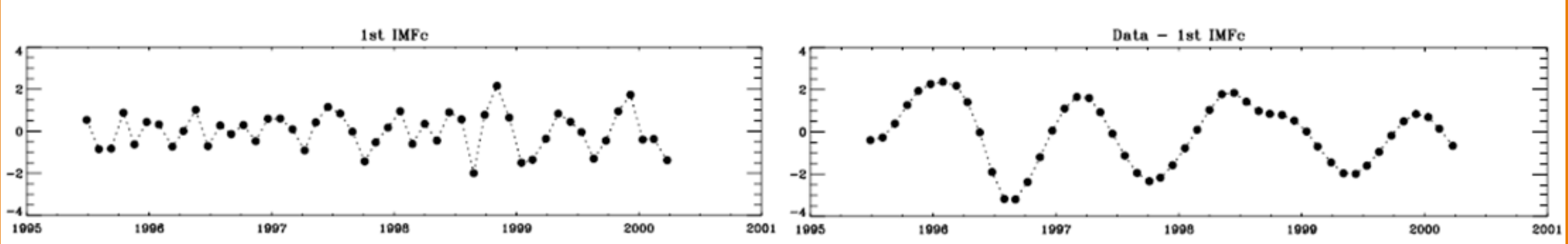
Fig. 1 *Top panel*: 10.7 cm radio flux intensity, by the NGDC; and the helioseismic frequency shift with uncertainties, by the BiSON. *Bottom panel*: sunspot area signals, recorded from the whole Sun disc, from the Northern hemisphere, and from the Southern hemisphere, by the Royal Observatory, Greenwich. The elapsed time is measured from 1985 January 1. The signals are normalised to their maximum values and shifted up and down for a better visualization. The helioseismic modes with degrees $0 \le l \le 2$ are only considered and taken by subsets of length 108 d with start times that are separated by 36 d. The 10.7 cm radio flux and sunspot area signals were averaged using a boxcar of width 10 d.

Empirical mode decomposition (EMD) by *Huang et al. (1998)*

1. Identifying of all local extrema (red points for maxima, green – for minima) of original signal *X(t)*. The illustrative example is taken from [2].
2. Define upper/lower envelopes (dashed lines) connecting all maxima/minima and their mean m_1 (solid line). Then $h_1(t)=X(t)-m_1(t)$



3. This "sifting process" must be repeated k times until the number of extrema in residual $h_{1k}(t)$ differs from the number of zero crossings by not more than 1. Finally we obtain the first intrinsic mode function (IMF):



The first IMF — Original signal after subtracting the 1st IMF

Hilbert transformation for each IMF:

$$Z(t) = X(t) + \mathrm{i}Y(t) = a(t)\mathrm{e}^{\mathrm{i}\theta(t)},$$

where $$Y(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{X(t')}{t-t'}\,\mathrm{d}t'$$

for instantaneous amplitude and frequency:

$$a(t) = [X^2(t) + Y^2(t)]^{1/2}, \omega = \frac{\mathrm{d}\theta(t)}{\mathrm{d}t}.$$

Average periodicities

The results, obtained with the HHT analysis of the raw data signals, are summarized in Table 1. Intrinsic periodicities were found simultaneously in different proxies and were separated into three distinct groups: short-term variations (with periods shorter than 0.5 yr), QBO (with typical periods from 0.5 yr up to 3.9 yr), and longer periodicities, e.g. such as the 11 yr cycle. These mean periods were estimated from the probability histograms of the corresponding Hilbert spectra (see particular example in Fig. 2). Such a histogram is calculated as the number of times each period appears in the Hilbert spectrum.

10.7 cm radio flux		Frequency	Sunspot area recorded from			Average
108 d smooth Period (d)/ σ_{peak}	10 d smooth Period (d)/ σ_{peak}	shift Period (d)/ σ_{peak}	whole Sun Period (d)/ σ_{peak}	Northern hemisphere Period (d)/ σ_{peak}	Southern hemisphere Period (d)/ σ_{peak}	period Period (d)
	$22^{+21}_{-2}/5.5$		$23^{+11}_{-2}/5.9$	$25^{+15}_{-5}/4.8$	$28^{+6}_{-7}/6.6$	25^{+7}_{-2}
	$46^{+22}_{-8}/6$		$43^{+24}_{-10}/5.6$	$40^{+19}_{-7}/6.1$	$48^{+9}_{-13}/6.6$	44^{+10}_{-5}
			$80^{+52}_{-24}/6.5$	$75^{+48}_{-24}/5.1$	$71^{+45}_{-16}/5.4$	75^{+28}_{-13}
	$96^{+18}_{-29}/5.8$	$101^{+36}_{-11}/5.5$		$98^{+106}_{-28}/5.6$		98^{+38}_{-14}
			$128^{+70}_{-28}/5.2$		$125^{+32}_{-41}/5.5$	127^{+38}_{-25}
	$140^{+120}_{-40}/5.6$				$160^{+30}_{-25}/6.9$	150^{+62}_{-24}
		$216^{+162}_{-50}/4.4$	$198^{+105}_{-31}/6.4$	$214^{+39}_{-70}/7$	$222^{+72}_{-24}/8.3$	213^{+52}_{-24}
$256^{+6}_{-37}/6.4$	$290^{+81}_{-95}/7.4$			$294^{+141}_{-47}/8.7$		292^{+81}_{-53}
$360^{+122}_{-77}/5.4$	$328^{+149}_{-15}/10$	$378^{+74}_{-35}/6$	$408^{+47}_{-148}/8.3$	$435^{+154}_{-145}/8.8$	$426^{+9}_{-98}/10.5$	395^{+46}_{-46}
$708^{+215}_{-163}/4.4$	$690^{+80}_{-200}/13$		$625^{+65}_{-240}/9.5$	$563^{+178}_{-128}/10.6$		626^{+69}_{-113}
$885^{+177}_{-200}/7.6$	$833^{+417}_{-63}/14$	$965^{+46}_{-233}/7.4$	$930^{+180}_{-190}/11.1$	$952^{+477}_{-82}/14.5$	$833^{+76}_{-33}/17.7$	903^{+133}_{-64}
$1180^{+337}_{-214}/7.1$	$1110^{+710}_{-340}/11.3$	$1250^{+520}_{-239}/6.8$	$1538^{+280}_{-538}/12.9$	$1818^{+182}_{-151}/18.4$	$1400^{+268}_{-223}/15.8$	1423^{+196}_{-146}
$4248^{+2832}_{-1213}/7$	$4000^{+2670}_{-665}/23.2$	$4250^{+2830}_{-710}/12.2$	$4500^{+2170}_{-1160}/17.3$	$4000^{+1000}_{-667}/23.9$	$4000^{+2667}_{-667}/19.1$	4150^{+1058}_{-357}

Table 1. Average intrinsic periodicities of the signals of five solar cycle proxies, estimated with HHT technique. The periods are measured in days, and their significances σ_{peak} are estimated in units of the standard deviations, calculated separately for each probability histogram of each Hilbert spectrum.

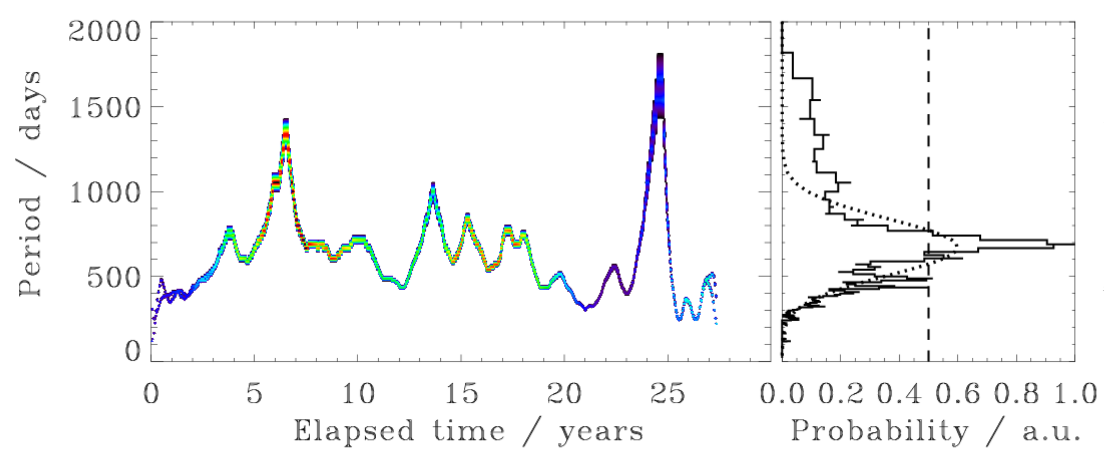


Fig. 2 *Left hand panel*: Hilbert spectrum, instant period versus time, of mode 7 of the radio flux signal with a mean period of about 690 d (≈ 2 yr), i.e. a QBO. *Right hand panel*: histogram, representing the most probable mean period of the Hilbert spectrum. Colour scheme is used for the instant spectral power.

Time variability of the empirical modes

The QBO of the sunspot area proxies with the average period of 626 d were found only in the signals from the whole Sun and from the Northern hemisphere (see Table 1), and are absent in the South. Accordingly, lines 1 and 2 in the top panel of Fig. 3 demonstrate in-phase behaviour. Correlation analysis of these lines showed that the cross-correlation coefficient varies from 0.8 to 0.9 at time lags of 0.1–0.2 yr.

The middle panel of Fig. 3 allows comparison of one of the intrinsic modes found in both the 10 d (solid) and 108 d (dot-dashed, correlated data) smoothed 10.7 cm flux data. It is clear that the agreement is predominantly very good. This indicates that the ensemble EMD technique is not adversely affected by the fact that correlated data were used.

The 11 yr cycle is clearly detected in the all proxies and is shown in the bottom panel of Fig. 3. The signatures of the so-called extended last minimum are well pronounced in these curves too. It is also clearly seen that 11 yr maxima of the sunspot area signals recorded separately from the Northern and Southern hemispheres are slightly shifted with respect to each other.

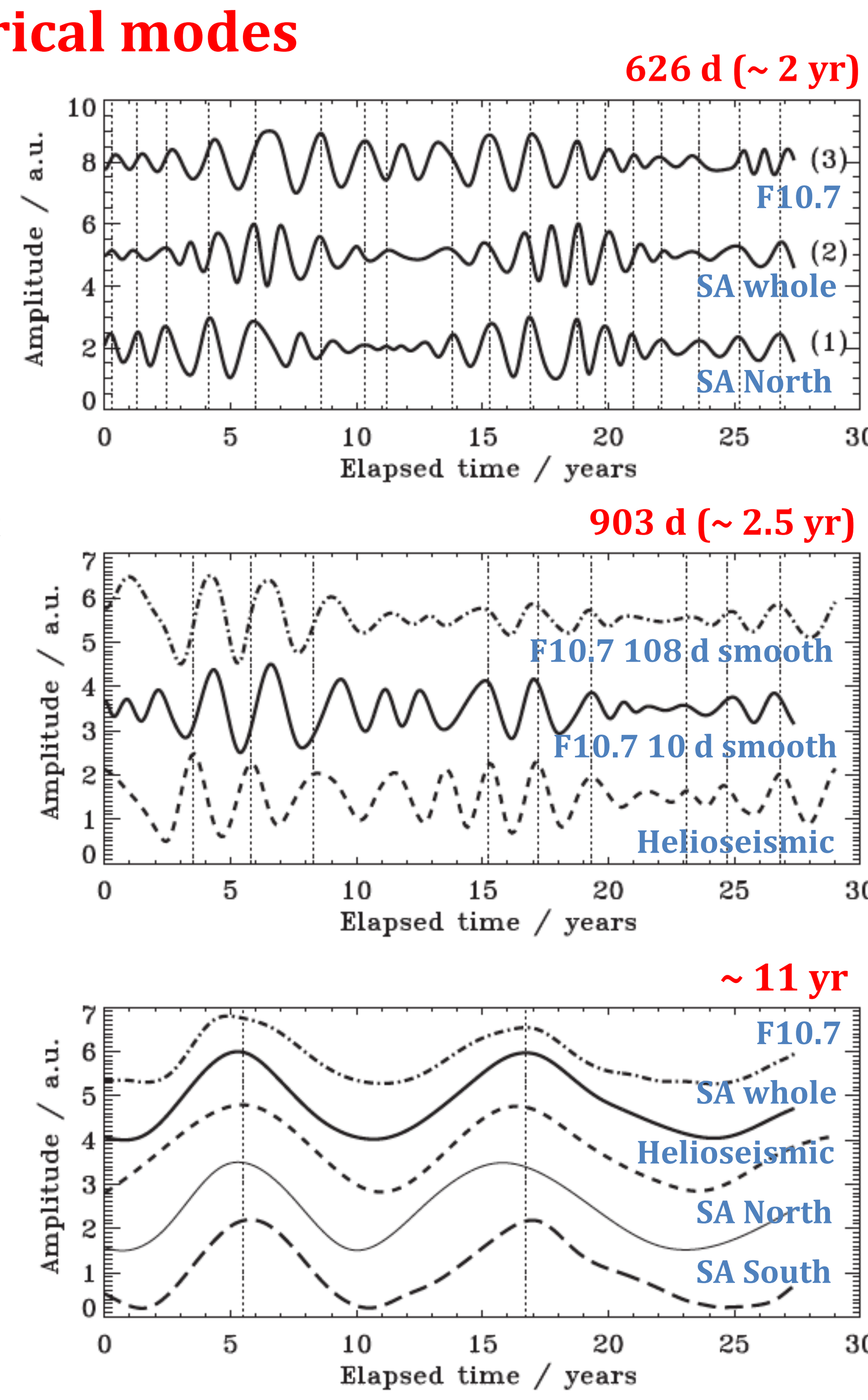


Fig. 3 Examples of empirical intrinsic modes of the raw signals shown in Fig. 1, obtained with the ensemble EMD.

References

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[2] N. E. Huang, Z. Shen, S. R. Long et al., 1998, Royal Society of London Proceedings Series A, 454, 903

[3] D. Y. Kolotkov, A.-M. Broomhall, V. M. Nakariakov, 2015, MNRAS, 451, 4, 4360